

# Range-Based Set Synchronization

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## Abstract

Set reconciliation is the problem of computing the union between two sets stored on different machines. Set mirroring is the problem of updating one of these sets to be equal to the other. Optimal communication complexity requires transmitting a number of bits proportional to  $n_\Delta$ , the number of items occurring in exactly one of the two sets. While there are sophisticated set reconciliation protocols achieving this optimum, they require computation time proportional to the size of the sets and computation space proportional to  $n_\Delta$ . Furthermore, they cannot be modified to solve the set mirroring problem.

Range-based synchronization is a simpler approach based on comparing hashes of ranges over the set and recursing if the two nodes arrive at different hashes, solving both the set reconciliation and the set mirroring problem. The number of bits transmitted is worse than the optimum by a factor of the logarithm of the size of the sets, but in exchange the computation time is only proportional to  $n_\Delta$  and the computation space is constant.

## 1 Introduction

One of the problems that needs to be solved when designing a distributed system is how to efficiently synchronize data between nodes. Two nodes may each hold a particular set of data, and may then wish to exchange the ideally minimum amount of information until they both reach the same state. Typical ways in which this can happen are one node taking on the state of the other, or both nodes ending up with the union of information available between the two of them.

Distributed version control systems can be regarded as an example of the latter case: users independently create new objects which describe changes to a directory, and when connecting to each other, they both fetch all new updates from the other in order to obtain a (more) complete version history. Regarded more abstractly, the two nodes compute the union of the sets of objects they store. A version control system might attempt to leverage structured information about those objects, such as a happened-before relation, but this does not lead to good worst-case guarantees. The set reconciliation protocol we give guarantees the exchange to only take a number of rounds logarithmic in the number of objects.

A different example are peer-to-peer publish-subscribe systems such as Secure Scuttlebutt [TLMT19]. A node in the system can subscribe to any number of topics, and nodes continuously synchronize all topics in common with other nodes encountered on an overlay network. Scuttlebutt achieves efficient synchronization by enforcing a linear happened-before relation between messages published to the same topic, i.e. each message is assigned a unique sequence number that is one greater than the sequence number of the previous message. When two nodes share interest in a topic, they exchange the greatest sequence number they have for this topic, whichever node sent the greater one then knows which messages the other is missing.

The price to pay for the efficiency is that concurrent publishing of new messages to the same topic is forbidden, since it would lead to different messages with the same sequence number, breaking correctness of the synchronization procedure. An unordered pubsub mechanism based on set reconciliation would be able to support concurrent publishing, the other design aspects such as the overlay network could be left unchanged.

An example from a less decentralized setting are incremental software updates. A server might host a new version of an operating system, users running an old version want to efficiently download the changes. An almost identical problem is efficiently creating a backup of a file system to a server already holding an older backup. Both of these examples can abstractly be regarded as updating a map from file paths to file contents. Our protocol for mirroring maps could be used for determining

which files need to be updated. The protocol allows synchronizing the actual files via an arbitrary nested synchronization protocol, e.g. rsync [TM<sup>+</sup>96].

## 1.1 Efficiency Criteria

The most important criteria for evaluating synchronization protocols:

- number of round-trips
- total bandwidth usage
- computation time complexity per round-trip
- space complexity per round-trip
- space complexity for auxiliary data structures
- time complexity for keeping auxiliary data structure up to date as the main data structure is being modified

## 1.2 Recursively Comparing Fingerprints

We conclude the introduction with a brief sketch of a set reconciliation protocol (i.e. a protocol for computing the union of two sets on different machines) that exemplifies the core ideas. The protocol leverages the fact that sets can be partitioned into a number of smaller subsets. The protocol assumes that the sets contain elements from universe on which there is a total order based on which ranges can be defined, and that nodes can compute fingerprints for any subset of the universe.

Suppose for example two nodes A, B each hold a set of natural numbers. They can reconcile all numbers within a range as follows: A computes a fingerprint over all the numbers it holds within the range and then sends this fingerprint to B, together with the range boundaries. B then computes the fingerprint of all numbers it holds within that same range. There are three possible cases:

- B computed the same fingerprint it received, then the range has been fully reconciled and the protocol terminates.
- B has no numbers within the range, B then notifies A, A then transmits all its numbers from the range, and the range has been fully reconciled.
- Otherwise, B splits the range into two sub-ranges, such that B has a roughly equal number of numbers within each range. B then initiates reconciliation for both of these ranges, the roles A and B reverse.

Crucially, in the last case, the two recursive protocol invocations can be performed in parallel. The number of parallel sessions increases exponentially, so the original range is being reconciled in a number of rounds logarithmic in the greater number of items held by any node within that range.

The remainder of this thesis fleshes out details and applies the same idea to some other data structures. The viability of this approach hinges on the efficient computation of fingerprints, which is discussed and solved in chapter 2. We then give a thorough definition of the set conciliation protocol in chapter 3, and prove its correctness and its complexity guarantees. Chapter 4 gives a more concrete protocol that allows nodes to enforce limits on the amount of computational resources they spend, at the cost of increasing the number of roundtrips if these resource limits are reached. Chapter 5 shows how to apply the same basic ideas to k-d-trees, maps, tries and radix trees, and briefly discusses why it does not make sense to apply it to arrays. Chapter 6 gives an overview of related work and justifies the chosen approach. We conclude in chapter 7.

## 2 Computing Fingerprints

Let  $U$  be the set of items that can be placed inside a data structure. Let  $\leq$  be a total order on  $U$ . For  $x, y \in U$ ,  $x \leq y$  and  $A \subseteq U$ , we write  $[x, y)_A$  to denote the set  $\{a \in A \mid x \leq a < y\}$ .

Let  $h$  be a hash function from  $U$  to a smaller set of hash digests  $H$ , e.g. the set of  $k$ -bit integers for some natural number  $k$ . Let  $\oplus : H \times H \rightarrow H$  be a function such that  $(H, \oplus, \emptyset)$  form a group with neutral element  $\emptyset \in H$ , e.g. addition modulo  $k$  with neutral element 0.

We write  $f(X)$  for the fingerprint of a set of items  $X$ , which is the same as the fingerprint for the ordered list obtained by sorting  $X$  according to  $\leq$ , denoted as  $f(x_0, x_1, \dots)$  and defined as the sum over all  $h(x_i)$ :

$$\begin{aligned} f() &= \emptyset \\ f(x_0, x_1, \dots) &= h(x_0) \oplus f(x_1, \dots) \end{aligned}$$

Observe that  $f([x, y)_A) = f([min(A), y)_A) \oplus f([min(A), x)_A)$ . For efficient computation of fingerprints for arbitrary ranges it thus suffices to be able to efficiently compute the sum of hashes over arbitrary prefixes of  $A$  sorted according to  $\leq$ .

To that end, store  $A$  in a balanced search tree that holds the sum over the hashes of all descendents in every internal vertex, and the elements of  $A$  in the leaves (this can be maintained as a self-balancing tree).  $f([min(A), x)_A)$  can then be computed in  $O(\log(n))$  time by traversing from the root to  $x$  and summing over the hashes stored in the left children of all vertices encountered in the traversal.

## 3 Related Work

Most of the literature on set reconciliation is prioritizing minimizing roundtrips, at the cost of high computation times. [MTZ03] gives theoretical limits and a very clever protocol approaching them, but which requires  $O(n^3)$  computation time per round-trip. The authors acknowledge the practical infeasibility and offer [MT02], which is also the only paper that cares whether auxiliary data structures can be efficiently synchronized with the set to reconcile as it changes. They use a simpler approach highly related to error-correcting codes: Both peers send a digest, if the digests are similar enough, the union can be computed from holding both digests and one of the sets. If they are not similar enough, the set has been too large, so they recursively reconcile partitions of the set. Unfortunately, their auxiliary data structure is not self-balancing, so their complexity guarantees degrade as the set to reconcile is being modified.

More recent work such as [EGUV11] or [OAL<sup>+</sup>19] focuses on invertible bloom filters, and fully embraces taking  $O(n)$  computation time per synchronization session. The probabilistic guarantees also involve enough math with actual numbers to require some healthy suspicion.

All of the previous work assumes that the items to be synchronized all have equal and relatively small size, which our approach does not require. None of the literature approaches utilize any structure of the data, whereas we can easily reconcile certain subsets (e.g. all data from within a timeframe if the total order being used sorts by timestamp).

I am not aware of any literature at all that acknowledges the fact that the participating peers only have finite memory available, particularly a server which synchronizes with many peers concurrently has to enforce low memory consumption per session. Any implementation of a protocol not acknowledging this will simply crash at some point.

Filesystem synchronization literature usually focuses on variants of rsync to optimize the single-file case, efficiently determining which files need updating is rarely discussed. Practical implementations usually sort all filenames, concatenate them, and then run their particular rsync the variant on that string. Using map synchronization should be more efficient.

The basic idea of adding up hashes in a tree is not particularly original, e.g. the CCNx 0.8 Sync protocol [SYW<sup>+</sup>17] does the same to solve a specific goal in a specific context. This thesis highlights the concept as a standalone algorithmic solution to a general problem, examines the concept more closely (discussing choice of the group operation, security issues, etc.), adapts it to deal with bounded memory, and applies it to more data structures than just sets.

## Work Plan

- by 05.05: basic set reconciliation chapter
- by 26.05: fingerprint chapter
- by 16.06: bounded-memory set reconciliation chapter
- by 07.07: other data structures chapter
- by 28.07: conclusion, coherence, polishing
- 15.08: self-inflicted soft deadline, unless adding more content

Possibly a chapter discussing more specifics that would occur when using set reconciliation as the core of an unordered p2p pubsub mechanism.

## References

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