

Problem Set 3 – Integrate-and-fire neuron model

Computational Neuroscience Summer Program

June, 2011

In this problem set you will be building a simple integrate-and-fire neuron. You should assume a specific membrane capacitance of $c_m = 10 \text{ nF/mm}^2$, a specific membrane resistance of $r_m = 1 \text{ M}\Omega\cdot\text{mm}^2$, a resting membrane potential of $E = -70 \text{ mV}$, a reset potential of $V_{\text{reset}} = -80 \text{ mV}$, an action potential threshold of $V_{\text{threshold}} = -55 \text{ mV}$, and a cell surface area of $A = 0.025 \text{ mm}^2$. Write up your results in a text editor of your choosing. Include any relevant figures, your Matlab code, and any other calculations related to the problem set. You may work individually or in groups, but each student should hand in their own report.

Equations

$$\begin{aligned}C_m &= A \cdot c_m & R_m &= \frac{r_m}{A} \\ \tau_m &= c_m \cdot r_m & V_\infty &= E + R_m I_{\text{ext}} \\ V(t) &= V_\infty + (V(0) - V_\infty)e^{\frac{-t}{\tau_m}} & r_{\text{isi}} &= (\tau_m \ln(\frac{R_m I_{\text{ext}} + E - V_{\text{reset}}}{R_m I_{\text{ext}} + E - V_{\text{threshold}}}))^{-1} \\ \tau_m \frac{dV}{dt} &= E - V(t-1) + R_m I_{\text{ext}}\end{aligned}$$

Problems

1. Model an integrate-and-fire neuron using the equations above and the following rule: when the neuron's membrane voltage exceeds $V_{\text{threshold}}$, set the voltage in that timestep to $V_{\text{peak}} = 40 \text{ mV}$, and in the next timestep set the voltage to V_{reset} . Set $dt = 0.1 \text{ ms}$. Apply a square pulse of 0.5 nA from $t = 250 \text{ ms}$ until $t = 750 \text{ ms}$ in your simulation. Use Matlab's subplot command to plot the membrane voltage over time in the top panel and I_{ext} in the bottom panel (use the same time scale for the horizontal axis of both plots). No text is required for this question; just include a plot.
2. Compute the average firing rate (spikes per second) of the integrate-and-fire neuron for the pulse interval you used in question 1 (500 ms). Now plot simulated firing rate vs r_{isi} for several values of I_{ext} (use I_{ext} between 0 and 1 nA). How does the firing rate of the modeled neuron compare to the estimated firing rate given by r_{isi} ? Note: the equation for r_{isi} only holds if $V_\infty > V_{\text{threshold}}$; otherwise, $r_{\text{isi}} = 0$.
3. Starting from 0 nA, gradually increase the amount of external current injected into the integrate-and-fire neuron in steps of 0.01 nA. Keep the pulse duration constant at 500 ms. What is the smallest amount of current you can inject which will still result in an action potential? Is there a maximum firing rate this neuron can achieve? Why or why not?
4. **Challenge problem.** Compute firing rate as a function of pulse duration, I_{duration} , using 20 durations between 10 and 500 ms. Repeat this for several different values of I_{ext} (try using 10 log-spaced values between 0.1 and 5 nA). Explain what you see. In particular, are the firing rate curves smooth or jagged? Why?
5. Vary the resting potential, specific capacitance, specific resistance, and surface area variables. How do increases or decreases in these values affect firing rate of the integrate-and-fire neuron? Explain (try to stay at or under 1-2 sentences per variable). Include plots for each of these variables.