Study 3 Feb 21

SoS Intro

$\sum g_i^2 \geq 0$

History Class

Quintic Eqs → Galois / Abel → group theory.

↓

characterization

$x^{17} = 1$ → Gauss

Motivates the study

generic frameworks

SoS → "meta-algos"

SoS proof system

(1) is strong

(2) avoids the "curse of completeness"

to can prove lower bounds

SoS

method for solving polynomial systems

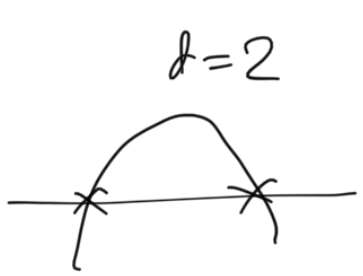
Interested:

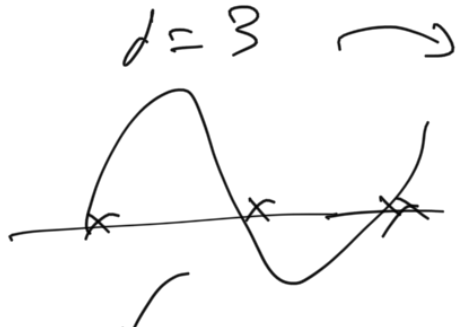
(1) Solving over $\mathbb{R}$.

(2) polynomials are low-degree.

(3) approx. sols are ok.

Need to be careful:



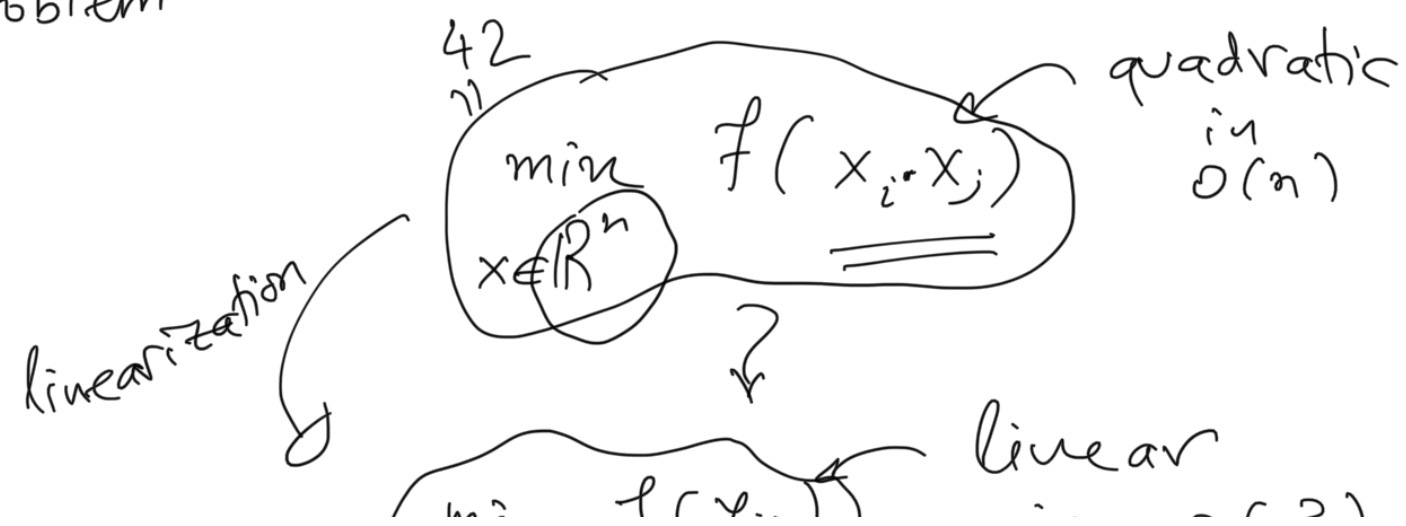


→ non-convexity

Local search → exp. many local minima of "energy function"

Tool

Non convex problem → enlarge search space

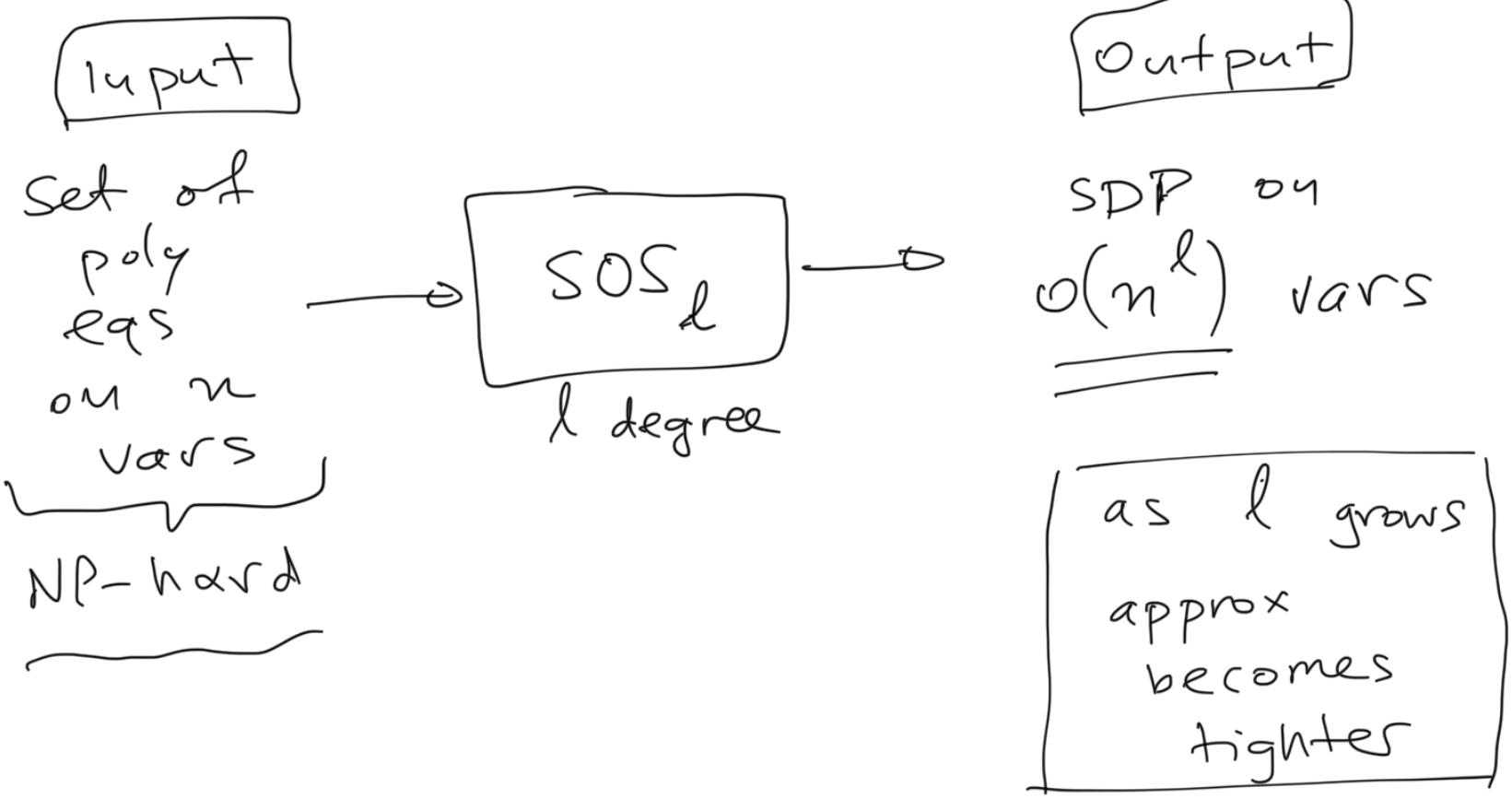


$y \in \mathbb{R}^{n^2}$ =

≥ 0

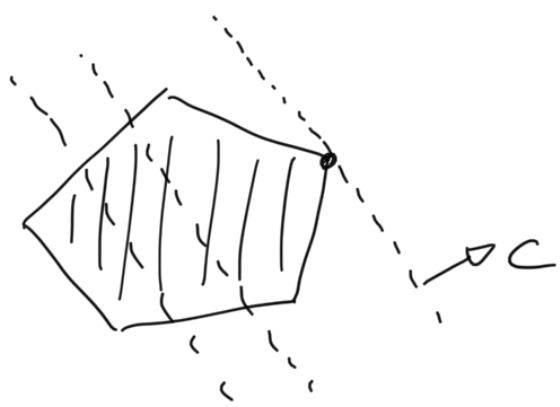
in $O(n^2)$

SoS algo
is a systematic way
of enlarging space



what is SDP?

LP



$\min \langle c, x \rangle$
s.t. $Ax \leq b$

SDP
add a constraint $M(x) \succeq \mathbb{0}$. (I)

Spectahedron = Polyhedron ∩ (⪰)

$$\min \sum c_{ij} \langle x_i, x_j \rangle$$

$$x_1, \dots, x_n \in \mathbb{R}^n$$

$$\text{s.t.} \sum A_{ijk} \langle x_i, x_j \rangle \leq b_k \quad \forall k$$

M is PSD $\iff$ $m_{ij} = \langle x_i, x_j \rangle$ $\forall i, j$

Roots of SoS

1900: Minkowski, Hilbert → 17th problem. (Remedy me)

$f \geq 0$ is it SoS? ???

1960: Motzkin

is it SoS of rational fcns

$$1 + x^4 y^2 + y^4 x^2 - 3x^2 y^2 \geq 0$$

AM-GM

$$\frac{\alpha + b + \gamma}{3} \geq \sqrt[3]{\alpha \beta \gamma}$$

$\alpha = 1$

$b = x^4 y^2$

$\gamma = y^4 x^2$

1927: Artin : Yes

1970 : Krivine :

Any unsat system of poly eqs can be certified via SoS proof.

(i.e., $\sum_{i=1}^{r} p_i^2 = -1$)

1990–2000:

Grigoriev, Parillo, Lasserre

↓

$O(n^l)$

Why Learning ?

$x_1, \ldots, x_n \sim P(\theta)$

Identifiability

SoS : Turn identifiability proof into algorithm

Problem : $f : \{0,1\}^n \to \mathbb{R}$

decide $f \geq 0$

or $\exists\, x \in \{0,1\}^n : f(x) < 0.$ ←

① Is this easy?

maxcut

$$f_G(x) = \sum_{i \sim j} (x_i - x_j)^2 = x^T L_G x$$

$$f = c - f_G$$

SoS with Problem:

- either "gives a proof" that $f(x) \geq 0$ $\forall x$ & gives $g_1, \ldots$

- or "an object that pretends to be a pt s.t. $f(x) < 0$."

s.t. $f = \sum g_r^2$

Next: SoS certificate deg $-d$