INFO8006 Introduction to Artificial Intelligence

Reasoning over time

Learning outcomes

At the end of this exercise session you should be able to:

- Define a Markov Model
- Define what is Filtering/Prediction/Smoothing/Most Likely Explanation in general and how it is done in the context of Markov Model
- Explain the simplified matrix representation of HMM (Hidden Markov Model).
- Define what is a Kalman filter and be able to use it in the context of a dynamic random process.

1 Exercise 1: Umbrella World (AIMA, Ex 15.2)

In this exercise, we examine what happens to the probabilities in the umbrella world in the limit of long time sequences.

R_{t-1}	$P(r_t \vert R_{t-1})$
T	0.7
F	0.3

R_t	$P(u_t \vert R_t)$
T	0.9
F	0.2

Table 1: Transition and Observation probability tables.

1. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases monotonically toward a fixed point. Calculate this fixed point.
2. Now consider forecasting further and further into the future, given just the first two umbrella observations. Compute the exact value of this fixed point.

Exercise 2: The coin

You are in a room containing a table, on this table are placed 3 very precious biased coins (named A, B and C). Suddenly another person enters the room and takes the coins. He throws a coin 4 times and then tells you the following information: First he tells you that he drew the first coin uniformly at random and did not throw it. Then, to select the next coin for each following throw, he either kept the same coin with a probability 2/3 or replaced it by another coin with equal probabilities. When you entered the room you inspected the coins and noticed that the coins A, B and C have a head probability of 80%, 50% and 20% respectively. The result of the throws are head, head, tail and head. If you answer right to his questions he will give you the coins. His four questions are as follows:

1. Provide a hidden Markov model that describes the sequence of throws.
2. What are the probabilities of the last coin given the sequence of evidence ?
3. What are the probabilities of the first coin chosen given the sequence evidence? And of the first coin thrown?
4. What is the most likely sequence of coins thrown?

Exercise 3: Hyperloop

This year ULiège has decided to get into the Hyperloop competition (`https://www.spacex.com/hyperloop`). Briefly, what you should do to win this competition is to build the fastest and most reliable autonomous pod. One of the most important engineering problem to build the pod is to be able to compute a robust estimation of the state of the pod (its position and speed) given many noisy sensors.

This morning you received an email asking you what would be your solution to this estimation problem. The message contains information about the sensors they plan to put in the pod. They say that they will use 3 unbiased speed sensors with a 99.7% accuracy[1] of 0.1m/s and a GPS sensor (also unbiased) which provides the pod position (in one dimension) with

[1] It means that 99.7% of the value measured will get a smaller error. e.g. (`https://math.stackexchange.com/questions/1412683/3-sigma-approximation`)

a 99.7% precision of 1 meter. After some research on the web you find out that you should use a Kalman filter to solve this task.

Define the components of your Kalman filter in the context of the state estimation of the pod. You can assume that the acceleration a is distributed normally around $\mu_a m/s^2$ with a variance equal to σ_a^2.

$\star$ Exercise 4: September 2019 (AIMA, Ex: 15.13 + 15.14)

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Answer the following questions:

1. Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Provide the conditional probability tables.
2. Then reformulate the dynamic Bayesian network as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.
3. For the evidence values e_1 = 'not red eyes, not sleeping in class', e_2 = 'red eyes, not sleeping in class' and e_3 = 'red eyes, sleeping in class' compute the following conditional probability distributions:
   (a) $P(\text{EnoughSleep}_t | e_{1:t})$ for $t = 1, 2, 3$.
   (b) $P(\text{EnoughSleep}_t | e_{1:3})$ for $t = 1, 2, 3$.

Supplementary materials

Berkeley Handout 6