1 Representing uncertain knowledge (08/11/2018)

1.1 Objectives

At the end of this repetition you should be able to:

- Apply Bayes rules, independence and marginalisation appropriately to compute probabilities
- Define, construct a Bayesian network
- Compute probabilities in the context of a simple Bayesian network.

1.2 Exercises

a $\approx$ 10 **min**

Let two events A, B of the probability space Ω; Is it possible to get $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \vee B) = 0.5$? If so, what range of probabilities would be possible for $A \wedge B$?

b $\approx$ 15 **min**

Given the probability table of Figure 1 compute the following probabilities:

1. P(toothache)
2. P(cavity)
3. P(toothache | cavity)
4. P(cavity | toothache $\vee$ catch)

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 1: Probablity Table of Toothache and Cavity

c $\approx$ 10 **min**

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

d $\approx$ 20 **min**

We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal probability of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X1, X2, and X3.

1. Draw the Bayesian network corresponding to this setup and define the necessary CPTs.
2. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

e ≈ 30 **min**

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

1. Which of the three networks in Figure 2 claim that $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?
2. Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?
3. Which of the three networks is the best description of the hypothesis?
4. Write down the CPT for the Gchild node in network (a), in terms of s and m.
5. Suppose that $P(G_{father} = l) = P(G_{mother} = l) = q$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of m and q only, by conditioning on its parent nodes.
6. Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong. hypothesis about the inheritance of handedness?

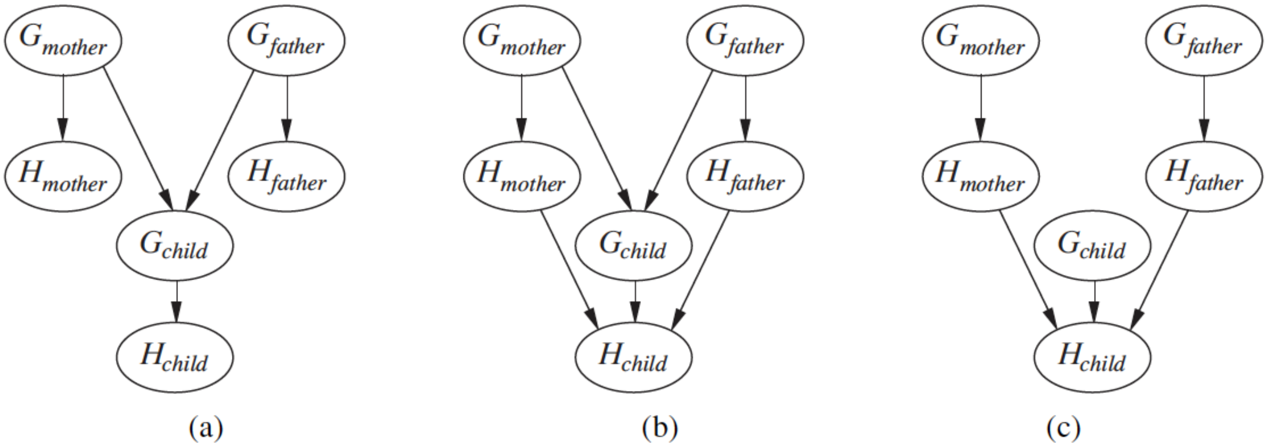

Figure 2: Possible Bayesian Networks of handedness inheritance

f ≈ 5 **min**

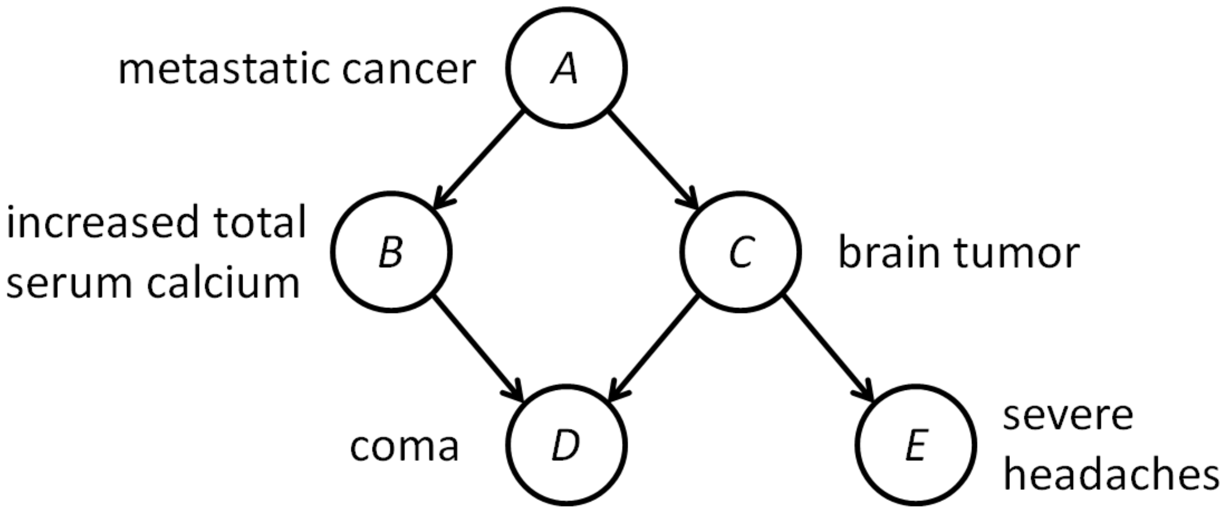

Figure 3: Bayesian Network of metastatic cancer

Consider the Bayesian network of Figure 3, which, if any, of the following are asserted by the network structure?

- $P(b, c) = P(b)P(c)$
- $P(b, c|a) = P(b|a)P(c|a)$
- $P(b, c|a, d) = P(b|a, d)P(c|a, d)$
- $P(c|a, d, e) = P(c|a, b, d, e)$
- $P(b, e|a) = P(b|a)P(e|a)$
- $P(b, e) = \sum_{a\in A, c\in C, d\in D} P(a)P(b|a)P(c|a)P(e|c)P(d|b, c)$

1.3 Supplementary material

https://www.youtube.com/watch?v=x-2uVNze56s