

MUSA 174: INTRODUCTION TO CATEGORY THEORY
University of California, Berkeley
Spring 2023

Instructor: Bryce Goldman
Tuesday, Thursday 12:30 PM - 2:00 PM, Evans 732

Facilitators:

Bryce Goldman (alkizar@berkeley.edu)
Rhea Kommerell (rkommerell@berkeley.edu)
Matthew Lideros (mattlideros@berkeley.edu)
Kishan Jani (kishanjani@berkeley.edu)

Faculty Sponsor:

Martin Olsson (martinolsson@berkeley.edu)

Course Email: musa174decal@gmail.com

Class Discord: <https://discord.gg/3WkD6U7Fd6>

Office Hours: Office hours will be held after class and by appointment. Also, each instructor has MUSA Office Hours in Evans 938, which can be found here: <https://musa.berkeley.edu/office.html>.

Online Availability: In addition to in-person office hours, we will be available to answer questions on the class Discord server, and encourage you to post questions there to be answered by the instructors or your classmates. The server will be moderated by the instructors, and any administrative questions or concerns should be directed towards them.

Course Material: We will primarily be following Emily Riehl's *Category Theory in Context*. A copy of the most recent version of the book can be accessed for free on her website: <https://math.jhu.edu/~eriehl/>.

Optional Texts:

While *Category Theory in Context* is the only text that will be required, some may find the following useful as supplementary material:

- *Basic Category Theory* by Tom Leinster is a slightly more beginner-friendly introduction to category theory. In particular, it assumes fewer prerequisites than Riehl's book, so it may be ideal for those with less topological/algebraic experience.
- *Categories for the Working Mathematician* by Saunders MacLane, for a more thorough treatment of category theory.
- *Topology: A Categorical Approach* by Tai-Danae Bradley, Tyler Bryson, and John Terilla, as a more gentle introduction to categories which is motivated by topology (also might be useful for those looking to learn the basics of point-set topology from a categorical perspective).
- *Algebra: Chapter 0* by Paolo Aluffi, for a categorical introduction to abstract algebra.
- *Algebra* by Dummit and Foote, as a reference for abstract algebra.

Course Overview:

Category theory is an emerging field of math which lies at the intersection of algebra, topology, and logic, and provides a common framework which unifies seemingly disparate concepts. Understanding category theory is becoming increasingly necessary for studying math at the graduate level and beyond, but it is rare that the subject is formally taught. Students are typically expected to internalize many of the ideas of category theory merely through osmosis. We believe that this is a disservice to the subject, and seek to restore balance to the universe by providing an environment for advanced undergrads to learn about category theory more formally, in preparation for pursuing a career in mathematical research.

Category theory is a language that is used frequently in graduate classes, particularly ones with an algebraic flavor. While this course covers more category theory than may be necessary for most graduate classes, the language it provides will help you see the formal connections between constructions and ideas that would otherwise seem only vaguely similar. Having an understanding of these broad structural connections allows you to then focus on the details of whatever concept you are learning without losing sight of the bigger picture. As such this course can be seen roughly as an introduction to graduate level mathematics, though that will not be its focus.

We want to encourage a welcoming and inclusive learning environment. Questions, curiosity, and collaboration are all highly encouraged, and dismissive attitudes are strongly discouraged. Math is a difficult subject, and confusion is not a sign of weakness. If students would like help outside of class, they are highly encouraged to ask the course facilitators to meet one-on-one. The course facilitators also hold office hours that can be found here, <https://musa.berkeley.edu/office.html>.

Student Learning Outcomes:

1. Students will be able to read and write category-theoretic proofs.
2. Students will be comfortable with categorical language and concepts, such as functors, natural transformations, and limits.
3. Students will be able to interpret commutative diagrams, and produce proofs by “diagram chasing.”
4. Students will learn to identify applications of category theory in other disciplines of math.

MUSA 174 Prerequisites:

Students are expected to be very comfortable reading and writing proofs at an upper-division level. They are also assumed to be familiar with the content of Math 110 and Math 113: in particular they should feel confident working with groups, rings, fields, and vector spaces. It may also be helpful, although is not strictly necessary, if students have encountered basic (algebraic) topology.

Grading Policy:

Homework (60%), Attendance (30%), Discussion Participation (10%)

Grade: Passed	70% or above
Grade: Not Passed	Below 70%

A grade of incomplete will only be given when the student is unable to complete the required work due to exceptional circumstances (illness, accident, death in the family, etc.), and their work up until that point has been satisfactory (passing).

Lecture Information and Attendance Policy:

Lectures will be held on Tuesdays and alternating with discussion section on Thursdays, and will be centered around introducing new material. Lecture Attendance is worth 30% of your grade! If you don't attend the lecture section or you are more than 10 minutes late (10 minutes after "Berkeley time") then you will be marked as absent. You will be allowed two unexcused lecture absences for the Spring term that won't be factored into your overall Attendance Grade. In the case of emergency, an absence will only be excused if you provide documentation.

Discussion Information and Attendance Policy:

Thursday discussion sections will give you the opportunity to apply the concepts you've learned in a group setting. Each group will be assigned a different problem on a worksheet and then the group will present their solution to the class after a dedicated amount of time for solving it. Discussion and feedback is encouraged during presentation for solutions, as there are many ways to prove most statements. Your group's problem set will be graded on both completion and accuracy. If you don't attend the discussion section or you are more than 10 minutes late (10 minutes after "Berkeley time") then you will receive a zero on the group assignment.

Homework Policy:

Problem sets will be released no later than Sunday at 11:59 PM and will be due the following Sunday at 11:59 PM on Bcourses. Each assignment should be uploaded as a PDF file or PNG/JPEG image (we strongly encourage students to type their homework using LaTeX, although this is not required as long as all handwritten work is legible and formatted clearly). In addition to a handful of required problems each week (which will be graded on completion and accuracy), there will sometimes be one or two optional problems. These will not be graded, and are purely for the benefit of those with time to work on them. While we will not be providing comments on these problems when grading, students are welcome to bring their solutions to office hours for feedback. If you believe that there was a grading error on one of the assignments please let a facilitator know as soon as possible, as we will not accept regrade requests after two weeks of an assignment's due date.

Important Dates:

First Day of Class	January 17, 2022
Last Day to Add/Drop	February 3, 2022
Last Day of Class	May 5, 2022

DSP Accommodations:

If you need any type of accommodations throughout the semester, please contact one of the instructors as soon as possible and be sure to provide a copy of your DSP Accommodation Letter.

Academic Honesty:

The Mathematics Department, and in particular, the instructors in this course, expect that students in mathematics courses will not engage in cheating or plagiarism. The following has been adapted from the Math Department web page to suit our course.

What does cheating mean?

Broadly speaking, cheating means violating the policies of a course or of the university in order to gain an unfair advantage over fellow students. A particular kind of cheating is plagiarism, which means taking credit for someone else's work. Cheating and plagiarism hurts your fellow students in the short term, they hurt the cheater in the long term, and they will not be tolerated. Instructors can easily spot when problem sets look unusually similar, or have similar (wrong or correct) answers, calculations, ideas, or thought structure. If you write the correct answer to a computational problem without any justification or with a bogus justification leading to that answer, this raises strong suspicions that you cheated, on top of not receiving any credit anyways due to the lack of correct justification. We encourage MUSA 174 students to collaborate on

problem sets and seek out additional help from tutors, online resources, and other texts. If you use other resources or you collaborate with another student to complete your problem set, please indicate so on your problem set. We encourage you to work with one another and seek out additional resources; however, we won't tolerate plagiarism.

What to do in a case of cheating?

If you suspect that other students are cheating, you should immediately inform one of your instructors. Students may be cheating in ways that the instructors have never even heard of before(unlikely, but possible). Even if you don't mention any names, the sooner you inform the instructor what is going on, the sooner they can take measures to put a stop to it. You can further report any cheating at: <http://sa.berkeley.edu/conduct/reporting/academic>.

Resolution to cheating

If you are suspected of cheating, the instructors may pursue a variety of actions depending on the particular nature of the incident. If you accept responsibility for academic misconduct, the matter can often be resolved between you and the instructors with possible academic sanctions ranging from losing points on a problem set to failing the class. The instructor may also send a report to the Mathematics Department and/or Center for Student Conduct. It is not necessary for the instructor to determine whether the student(s) has a passing knowledge of the relevant factual material. It is understood that any student who knowingly aids in cheating is as guilty as the cheating student.

Course Outline

Exercises in parentheses are optional.

Week	Topics Covered	Reading	Homework
1	Basic definitions, first examples	Ch1.1	1.1.i, 1.1.ii, 1.1.iii
2	Duality	Ch1.2	1.2.ii, 1.2.iv, 1.2.vi (1.2.iii)
3	Functors and functoriality	Ch1.3	1.3.i, 1.3.ii, 1.3.viii (1.3.vi)
4	Functors and natural transformations	Ch1.4	1.4.i, 1.4.iv (1.4.v)
5	Equivalences of categories	Ch1.5	1.5.i, 1.5.iii, 1.5.iv (1.5.ii)
6	Diagram chasing	Ch1.6	1.6.i, 1.6.ii, 1.6.iii (1.6.v, 1.6.vi)
7	Representable functors	Ch2.1	2.1.ii, 2.1.iii
8	The Yoneda lemma	Ch2.2	2.2.i, 2.2.iv
9	Universal properties and the category of elements	Ch2.3, Ch2.4	2.4.ii, 2.4.viii (2.4.x)
10	Limits and cones	Ch3.1	3.1.i, 3.1.iv, 3.1.vi, 3.1.vii (3.1.iii)
11	Limits in Set, representability	Ch3.2-Ch3.4	3.2.ii, 3.2.vi, 3.3.ii
12	Complete categories, functoriality	Ch3.5, Ch3.6	3.5.i, 3.5.v
13	Adjoint functors	Ch4.1	4.1.i, 4.1.iii (4.1.iv)
14	Units and counits	Ch4.2	4.2.i, 4.2.v
15	The calculus of adjunctions, relations with limits	Ch4.4, Ch4.5	4.4.i, 4.5.i (4.5.ii)