Algorithm 2: State Evolution of DS-AMP Algorithm

Input: The noise variance σ_w^2, the sparsity level $\lambda = \frac{K_a}{K}$, the number of MAPs N_t, the frame length J, the order of QAM, the variance of the elements in the measurement matrix γ, the number of Monte Carlo simulations N_{MC}, the maximum SE iterations T_{SE}, and ε.

Output: The theoretically predicted MSE $\widehat{e}$.

$\forall m \in [N_{\text{MC}}]$: Generate N_{MC} realizations of transmit signals $\mathbf{X}^m \in \mathbb{C}^{KN_t \times J}$, based on the *a prior* distribution (9).

$\forall m, k$: Define $\mathbf{e}^1 = \mathbf{0}_{N_{\text{MC}} \times 1}$ and $\mathbf{v}^1 = \mathbf{0}_{N_{\text{MC}} \times 1}$ to record the predicted MSE and average variance of the m-th Monte Carlo realization. We initialize the iteration number $t = 1$, the predicted MSE $e^1 = 1$, the average variance $v^1 = 1$, and the activity indicators for the m-th signal realization $a_{k,m}^1 = 0.5$;

for $t = 1$ to T_{SE} **do**

  for $m = 1$ to N_{MC} **do**

    $\forall i, j, k$: $r_{l,j}^{m,t} = \left[\mathbf{x}_{k,j}^m\right]_i + \sqrt{\frac{\sigma_w^2 + \gamma K N_t e^t}{N_r \gamma}} z, \ \ \phi_{l,j}^{m,t} = \frac{\sigma_w^2 + \gamma K N_t v^t}{N_r \gamma}$;

    $\forall i, j, k$: $\left[\widehat{\mathbf{x}}_{k,j}^m\right]_i = f_m(r_{l,j}^{m,t}, \phi_{l,j}^{m,t}), \ \ \left[\widehat{\mathbf{v}}_{k,j}^m\right]_i = f_v(r_{l,j}^{m,t}, \phi_{l,j}^{m,t})$;

    $\forall k$: $a_{k,m}^{t+1} = f_a(r_{l,j}^{m,t}, \phi_{l,j}^{m,t}; a_{k,m}^t)$;

    Calculating $\left[\mathbf{e}^{t+1}\right]_m$ and $\left[\mathbf{v}^{t+1}\right]_m$ referring to (27) and (28), respectively;

  end for

  $e^{t+1} = \frac{1}{N_{\text{MC}}} \sum_{m=1}^{N_{\text{MC}}} \left[\mathbf{e}^{t+1}\right]_m, \ \ v^{t+1} = \frac{1}{N_{\text{MC}}} \sum_{m=1}^{N_{\text{MC}}} \left[\mathbf{v}^{t+1}\right]_m$;

  $\widehat{e} = e^{t+1}$;

  if $\left|e^{t+1} - e^t\right| < \varepsilon$ **then**

    break; {End the SE iterations}

  end if

end for