

# Introduction to **Information Retrieval**

CS276: Information Retrieval and Web Search  
Christopher Manning and Pandu Nayak

Lecture 4: Index Compression

# Last lecture – index construction

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- Sort-based indexing
  - Naïve in-memory inversion
  - Blocked Sort-Based Indexing (BSBI)
    - Merge sort is effective for hard disk–based sorting (avoid seeks!)
- Single-Pass In-Memory Indexing (SPIMI)
  - No global dictionary
    - Generate separate dictionary for each block
  - Don't sort postings
    - Accumulate postings in postings lists as they occur
- Distributed indexing using MapReduce
- Dynamic indexing: Multiple indices, logarithmic merge

# Today

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BRUTUS → [1 | 2 | 4 | 11 | 31 | 45 | 173 | 174]

CAESAR → [1 | 2 | 4 | 5 | 6 | 16 | 57 | 132 | ...]

CALPURNIA → [2 | 31 | 54 | 101]

- Collection statistics in more detail (with RCV1)
  - How big will the dictionary and postings be?
- Dictionary compression
- Postings compression

# Why compression (in general)?

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- Use less disk space
  - Save a little money; give users more space
- Keep more stuff in memory
  - Increases speed
- Increase speed of data transfer from disk to memory
  - [read compressed data | decompress] is faster than [read uncompressed data]
- Premise: Decompression algorithms are fast
  - True of the decompression algorithms we use

# Why compression for inverted indexes?

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- Dictionary
  - Make it small enough to keep in main memory
  - Make it so small that you can keep some postings lists in main memory too
- Postings file(s)
  - Reduce disk space needed
  - Decrease time needed to read postings lists from disk
  - Large search engines keep a significant part of the postings in memory.
    - Compression lets you keep more in memory
- We will devise various IR-specific compression schemes

# Recall Reuters RCV1

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■ symbol	statistic	value
■ N	documents	800,000
■ L	avg. # tokens per doc	200
■ M	terms (= word types)	~400,000
■	avg. # bytes per token (incl. spaces/punct.)	6
■	avg. # bytes per token (without spaces/punct.)	4.5
■	avg. # bytes per term	7.5
■	non-positional postings	100,000,000

# Index parameters vs. what we index

(details IIR Table 5.1, p.80)

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size of	word types (terms)			non-positional postings			positional postings		
	dictionary			non-positional index			positional index		
	Size (K)	Δ%	cumul %	Size (K)	Δ %	cumul %	Size (K)	Δ %	cumul %
Unfiltered	484			109,971			197,879		
No numbers	474	-2	-2	100,680	-8	-8	179,158	-9	-9
Case folding	392	-17	-19	96,969	-3	-12	179,158	0	-9
30 stopwords	391	-0	-19	83,390	-14	-24	121,858	-31	-38
150 stopwords	391	-0	-19	67,002	-30	-39	94,517	-47	-52
stemming	322	-17	-33	63,812	-4	-42	94,517	0	-52

Exercise: give intuitions for all the '0' entries. Why do some zero entries correspond to big deltas in other columns? 7

# Lossless vs. lossy compression

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- Lossless compression: All information is preserved.
  - What we mostly do in IR.
- Lossy compression: Discard some information
- Several of the preprocessing steps can be viewed as lossy compression: case folding, stop words, stemming, number elimination.
- Chapter 7: Prune postings entries that are unlikely to turn up in the top  $k$  list for any query.
  - Almost no loss of quality in top  $k$  list.

# Vocabulary size vs. collection size

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- How big is the term vocabulary?
  - That is, how many distinct words are there?
- Can we assume an upper bound?
  - Not really: At least  $70^{20} = 10^{37}$  different words of length 20
- In practice, the vocabulary will keep growing with the collection size
  - Especially with Unicode ☺

# Vocabulary size vs. collection size

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- Heaps' law:  $M = kT^b$
- $M$  is the size of the vocabulary,  $T$  is the number of tokens in the collection
- Typical values:  $30 \leq k \leq 100$  and  $b \approx 0.5$
- In a log-log plot of vocabulary size  $M$  vs.  $T$ , Heaps' law predicts a line with slope about  $\frac{1}{2}$ 
  - It is the simplest possible (linear) relationship between the two in log-log space
    - $\log M = \log k + b \log T$
  - An empirical finding ("empirical law")

# Heaps' Law

For RCV1, the dashed line

$$\log_{10} M = 0.49 \log_{10} T + 1.64$$

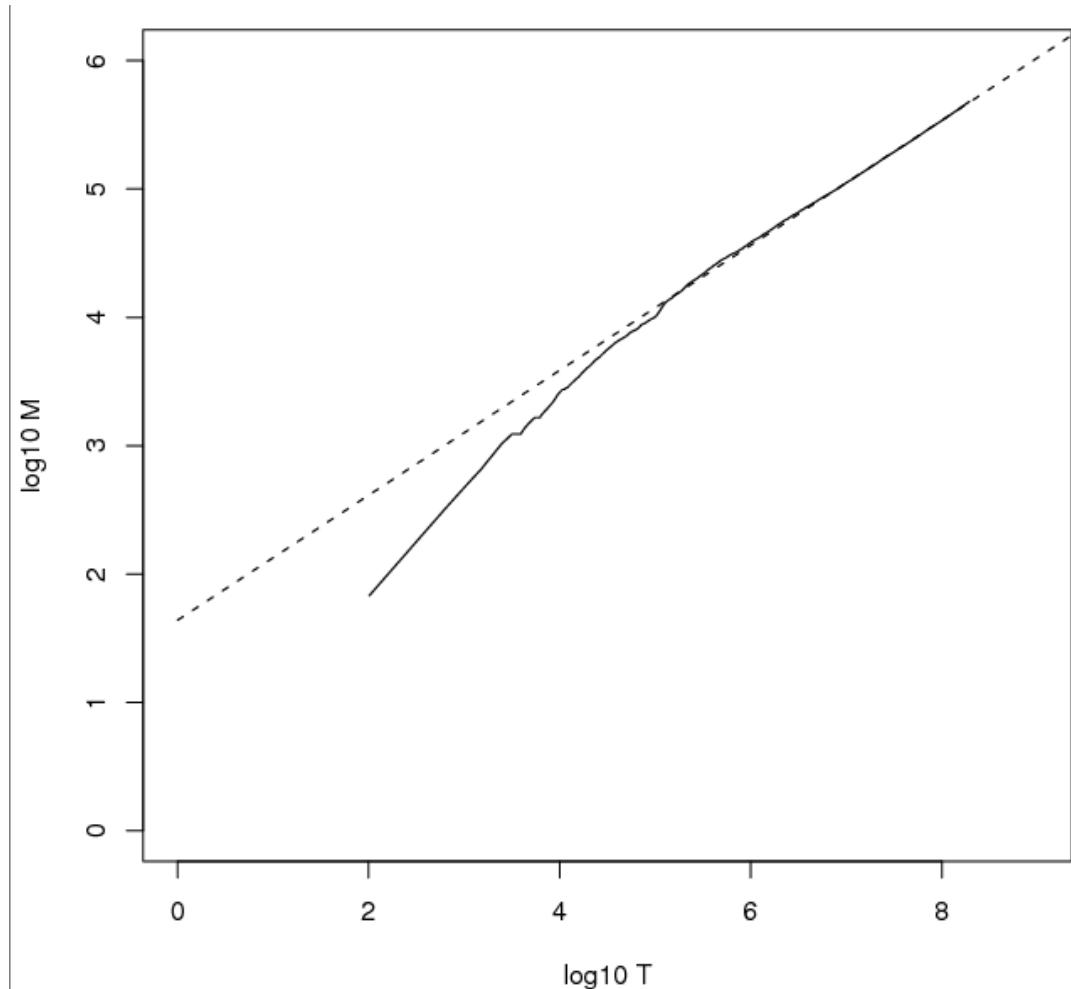
is the best least squares fit.

Thus,  $M = 10^{1.64} T^{0.49}$  so  $k = 10^{1.64} \approx 44$  and  $b = 0.49$ .

Good empirical fit for  
Reuters RCV1 !

For first 1,000,020 tokens,  
law predicts 38,323 terms;  
actually, 38,365 terms

Fig 5.1 p81



# Exercises

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- What is the effect of including spelling errors, vs. automatically correcting spelling errors on Heaps' law?
- Compute the vocabulary size  $M$  for this scenario:
  - Looking at a collection of web pages, you find that there are 3000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
  - Assume a search engine indexes a total of  $20,000,000,000$  ( $2 \times 10^{10}$ ) pages, containing 200 tokens on average
  - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

# Zipf's law

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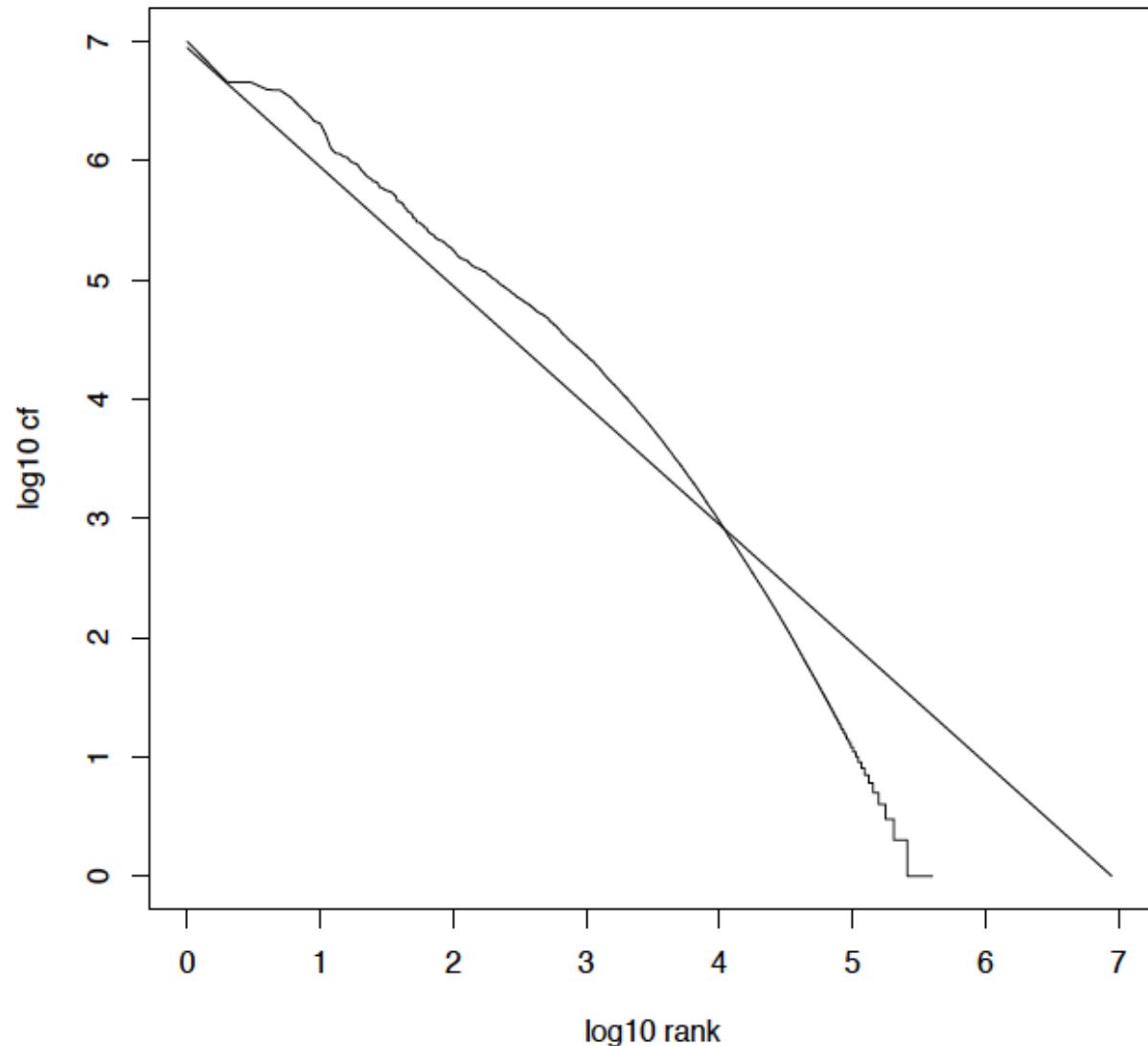
- Heaps' law gives the vocabulary size in collections.
- We also study the relative frequencies of terms.
- In natural language, there are a few very frequent terms and very many very rare terms.
- Zipf's law: The  $i^{\text{th}}$  most frequent term has frequency proportional to  $1/i$ .
- $cf_i \propto 1/i = K/i$  where  $K$  is a normalizing constant
- $cf_i$  is collection frequency: the number of occurrences of the term  $t_i$  in the collection.

# Zipf consequences

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- If the most frequent term (*the*) occurs  $cf_1$  times
  - then the second most frequent term (*of*) occurs  $cf_1/2$  times
  - the third most frequent term (*and*) occurs  $cf_1/3$  times ...
- Equivalent:  $cf_i = K/i$  where  $K$  is a normalizing factor, so
  - $\log cf_i = \log K - \log i$
  - Linear relationship between  $\log cf_i$  and  $\log i$
- Another power law relationship

# Zipf's law for Reuters RCV1



# Compression

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- Now, we will consider compressing the space for the dictionary and postings. We'll do:
  - Basic Boolean index only
  - No study of positional indexes, etc.
- But these ideas can be extended
- We will consider compression schemes

# DICTIONARY COMPRESSION

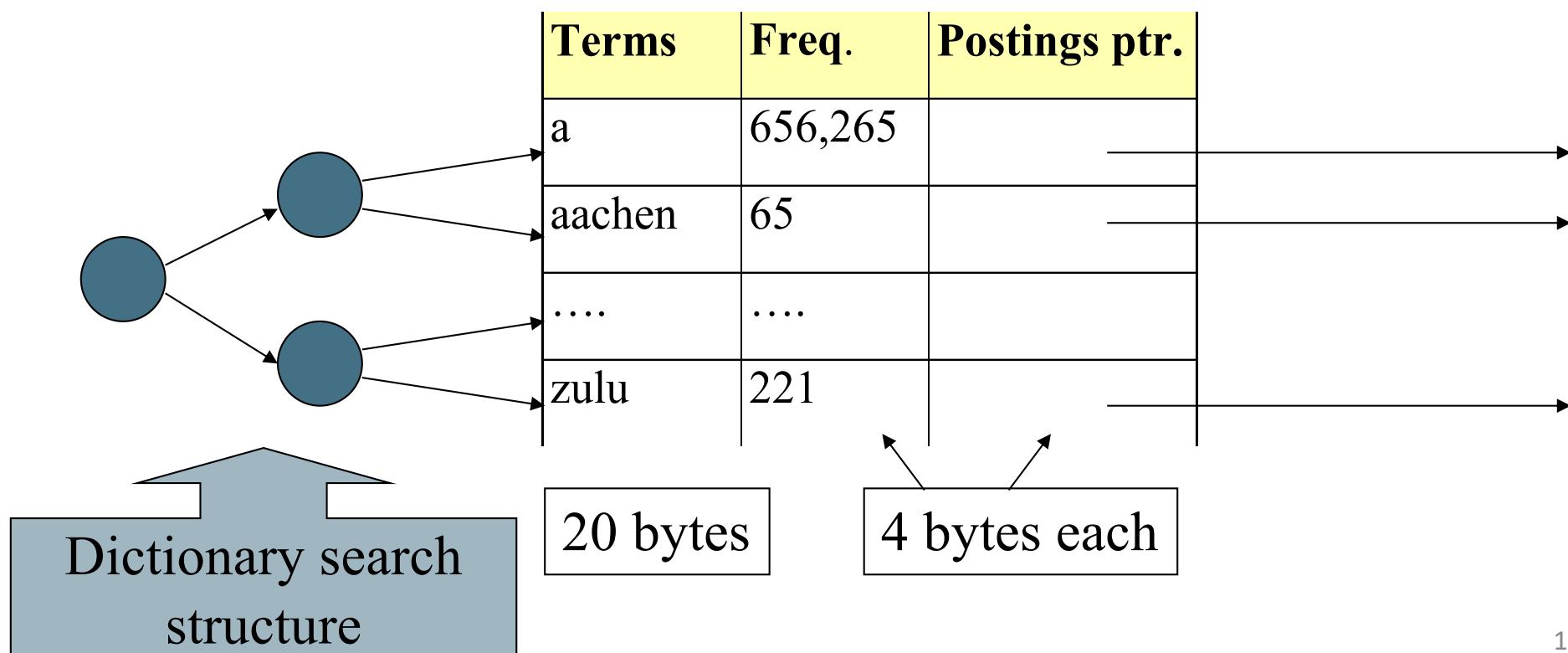
# Why compress the dictionary?

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- Search begins with the dictionary
- We want to keep it in memory
- Memory footprint competition with other applications
- Embedded/mobile devices may have very little memory
- Even if the dictionary isn't in memory, we want it to be small for a fast search startup time
- So, compressing the dictionary is important

# Dictionary storage – naïve version

- Array of fixed-width entries
  - ~400,000 terms; 28 bytes/term = 11.2 MB.



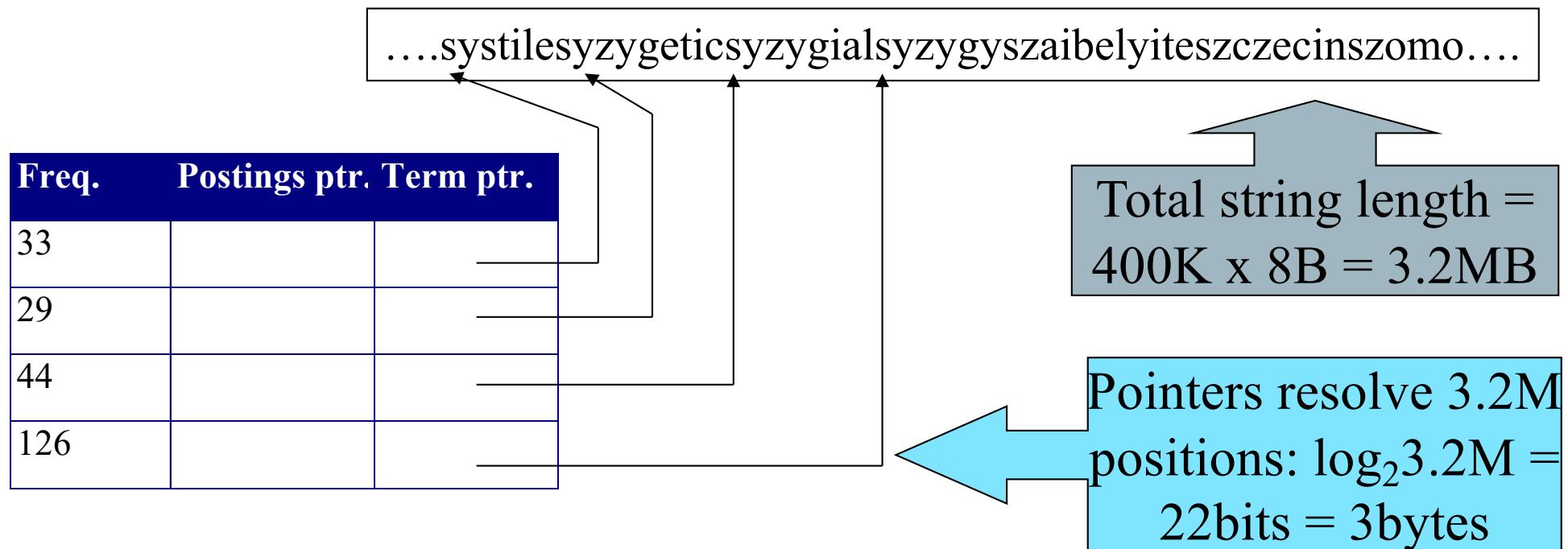
# Fixed-width terms are wasteful

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- Most of the bytes in the **Term** column are wasted – we allot 20 bytes for 1 letter terms.
  - And we still can't handle *supercalifragilisticexpialidocious* or *hydrochlorofluorocarbons*.
- Written English averages ~4.5 characters/word.
  - Exercise: Why is/isn't this the number to use for estimating the dictionary size?
- Ave. dictionary word in English: ~8 characters
  - How do we use ~8 characters per dictionary term?
- Short words dominate token counts but not type average.

# Compressing the term list: Dictionary-as-a-String

- Store dictionary as a (long) string of characters:
  - Pointer to next word shows end of current word
  - Hope to save up to 60% of dictionary space

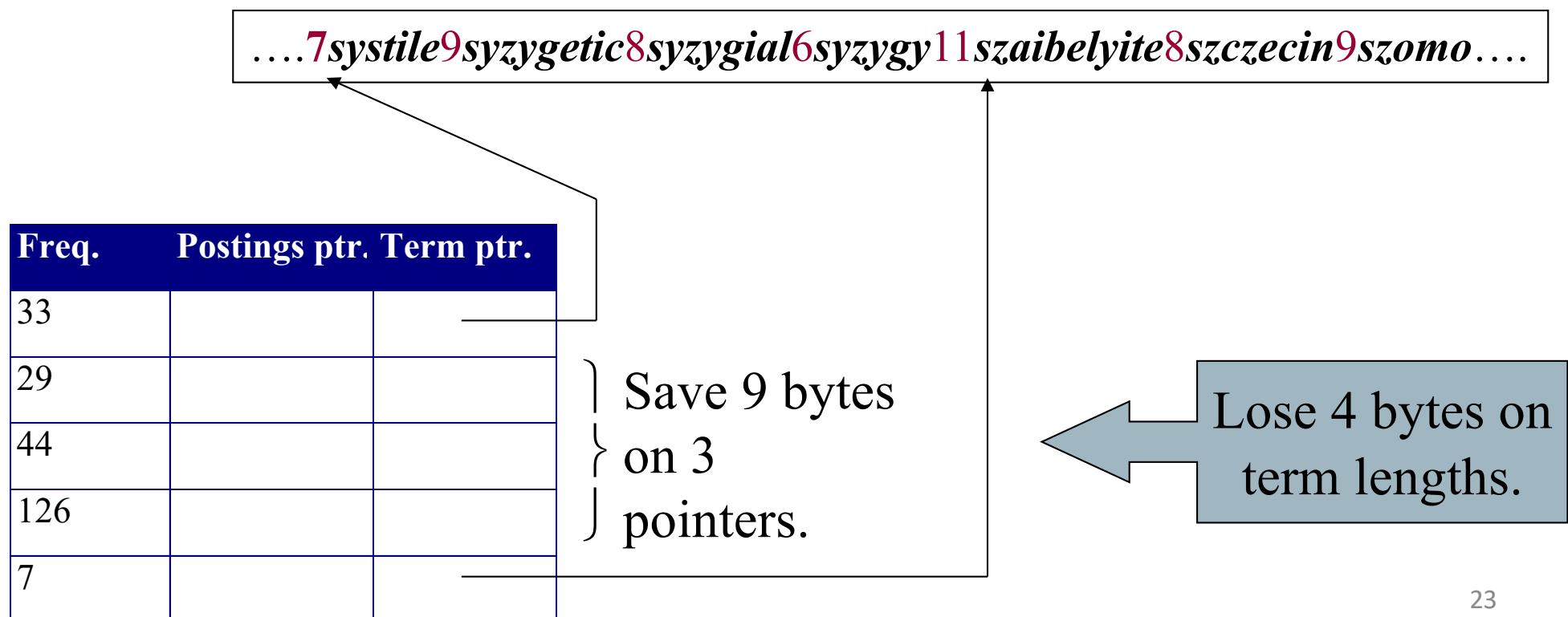


# Space for dictionary as a string

- 4 bytes per term for Freq.
  - 4 bytes per term for pointer to Postings.
  - 3 bytes per term pointer
  - Avg. 8 bytes per term in term string
  - 400K terms x 19  $\Rightarrow$  7.6 MB (against 11.2MB for fixed width)
- } Now avg. 11 bytes/term, not 20.

# Blocking

- Store pointers to every  $k$ th term string.
  - Example below:  $k=4$ .
- Need to store term lengths (1 extra byte)



# Blocking Net Gains

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- Example for block size  $k = 4$
- Where we used 3 bytes/pointer without blocking
  - $3 \times 4 = 12$  bytes,

now we use  $3 + 4 = 7$  bytes.

Shaved another ~0.5MB. This reduces the size of the dictionary from 7.6 MB to 7.1 MB.

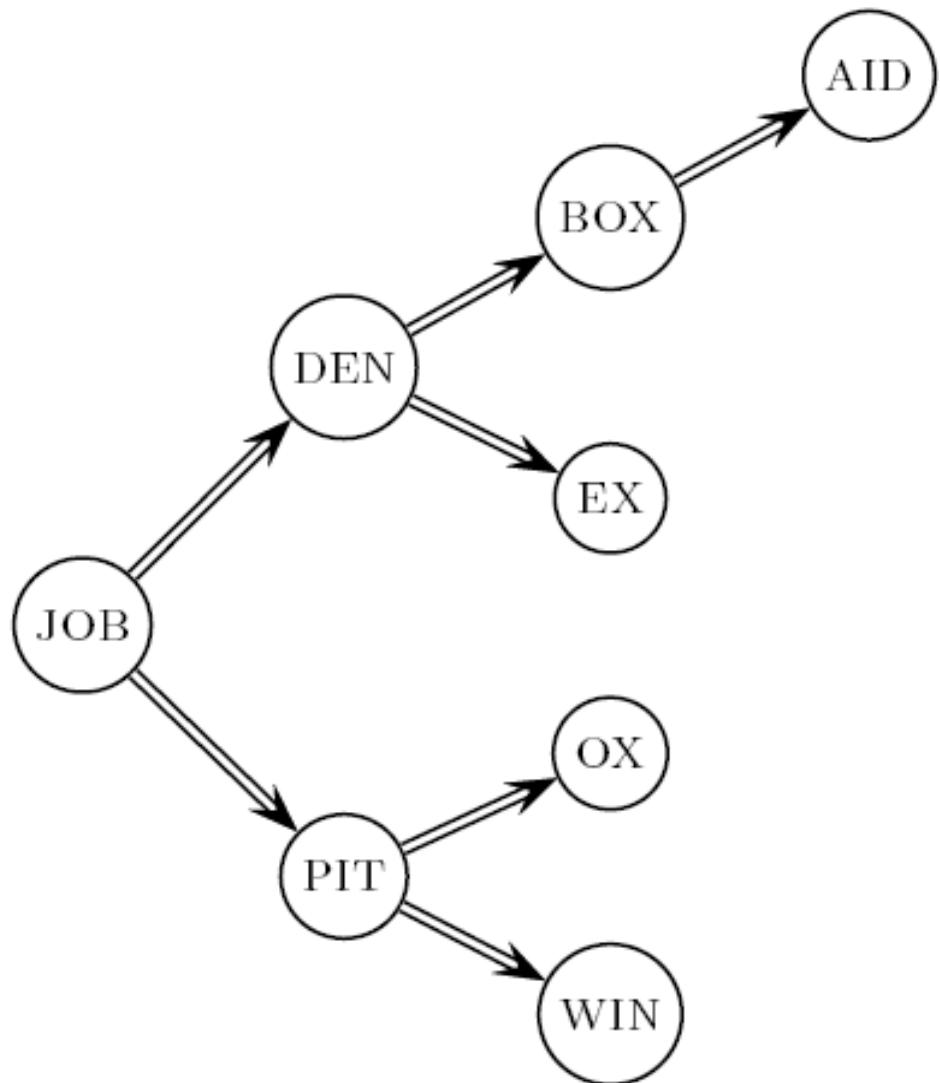
We can save more with larger  $k$ .

Question: Why not go with larger  $k$ ?

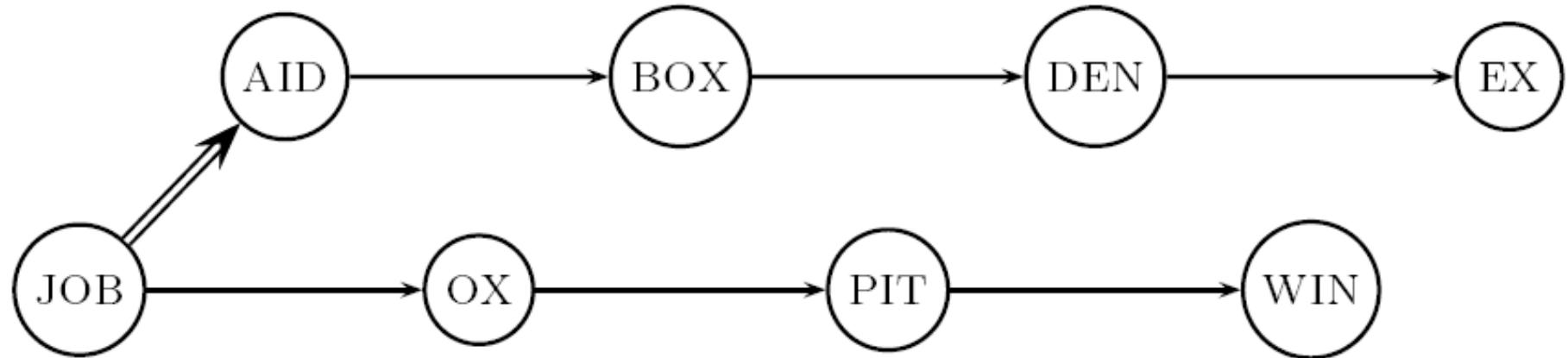
# Dictionary search without blocking

- Assuming each dictionary term equally likely in query (not really so in practice!), average number of comparisons  
$$= (1+2\cdot2+4\cdot3+4)/8 \sim 2.6$$

Exercise: what if the frequencies of query terms were non-uniform but known, how would you structure the dictionary search tree?



# Dictionary search with blocking



- Binary search down to 4-term block;
  - Then linear search through terms in block.
- Blocks of 4 (binary tree), avg. =  
$$(1+2\cdot2+2\cdot3+2\cdot4+5)/8 = 3 \text{ compares}$$

# Exercises

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- Estimate the space usage (and savings compared to 7.6 MB) with blocking, for block sizes of  $k = 4, 8$  and  $16$ .
- Estimate the impact on search performance (and slowdown compared to  $k=1$ ) with blocking, for block sizes of  $k = 4, 8$  and  $16$ .

# Front coding

- Front-coding:

- Sorted words commonly have long common prefix – store differences only
- (for last  $k-1$  in a block of  $k$ )

**8automata8automate9automatic10automation**

→ **8automat\*** *a1* ◇ *e2* ◇ *ic3* ◇ *ion*

Encodes prefix ***automat***

Extra length  
beyond ***automat***.

Begins to resemble general string compression. 28

# RCV1 dictionary compression summary

Technique	Size in MB
Fixed width	11.2
Dictionary-as-String with pointers to every term	7.6
+ blocking, $k = 4$	7.1
+ blocking + front coding	5.9

# POSTINGS COMPRESSION

# Postings compression

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- The postings file is much larger than the dictionary, factor of at least 10, often over 100 times larger
- Key desideratum: store each posting compactly.
- A posting for our purposes is a docID.
- For Reuters (800,000 documents), we would use 32 bits per docID when using 4-byte integers.
- Alternatively, we can use  $\log_2 800,000 \approx 20$  bits per docID.
- Our goal: use far fewer than 20 bits per docID.

# Postings: two conflicting forces

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- A term like ***arachnocentric*** occurs in maybe one doc out of a million – we would like to store this posting using  $\log_2 1M \approx 20$  bits.
- A term like ***the*** occurs in virtually every doc, so 20 bits/posting  $\approx 2\text{MB}$  is too expensive.
  - Prefer 0/1 bitmap vector in this case ( $\approx 100\text{K}$ )

# Gap encoding of postings file entries

- We store the list of docs containing a term in increasing order of docID.
  - ***computer***: 33,47,154,159,202 ...
- Consequence: it suffices to store *gaps*.
  - 33,14,107,5,43 ...
- Hope: most gaps can be encoded/stored with far fewer than 20 bits.
  - Especially for common words

# Three postings entries

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	encoding	postings list						
THE	docIDs	...	283042		283043	283044	283045	...
	gaps			1		1	1	...
COMPUTER	docIDs	...	283047		283154	283159	283202	...
	gaps			107		5	43	...
ARACHNOCENTRIC	docIDs	252000	500100					
	gaps	252000	248100					

# Variable length encoding

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- Aim:
  - For *arachnocentric*, we will use  $\sim 20$  bits/gap entry.
  - For *the*, we will use  $\sim 1$  bit/gap entry.
- If the average gap for a term is  $G$ , we want to use  $\sim \log_2 G$  bits/gap entry.
- Key challenge: encode every integer (gap) with about as few bits as needed for that integer.
- This requires a *variable length encoding*
- Variable length codes achieve this by using short codes for small numbers

# Unary code

- Represent  $n$  as  $n$  1s with a final 0.
  - Unary code for 3 is 1110.
  - Unary code for 40 is

- Unary code for 80 is:

- This doesn't look promising, but....
    - Optimal if  $P(n) = 2^{-n}$
    - We can use it as part of our solution

# Gamma codes

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- We can compress better with bit-level codes
  - The Gamma code is the best known of these.
- Represent a gap  $G$  as a pair *length* and *offset*
- *offset* is  $G$  in binary, with the leading bit cut off
  - For example  $13 \rightarrow 1101 \rightarrow 101$
- *length* is the length of offset
  - For  $13$  (offset  $101$ ), this is  $3$ .
- We encode *length* with *unary code*:  $1110$ .
- Gamma code of  $13$  is the concatenation of *length* and *offset*:  $1110101$

# Gamma code examples

number	length	offset	$\gamma$ -code
0			none
1		0	0
2		10	0
3		10	1
4	110	00	110,00
9	1110	001	1110,001
13	1110	101	1110,101
24	11110	1000	11110,1000
511	11111110	11111111	11111110,11111111
1025	111111110	0000000001	1111111110,0000000001

# Reminder: bitwise operations

- For compression, you need to use bitwise operators

The screenshot shows a course website for "Computer Organization & Systems" in "Week 2". The main title is "Computer Organization & Systems". Below it, under "Week 2", is a section for "Lecture 3 (Mon 4/8): Bits and Bitwise Operators". The description states: "We'll dive further into bits and bytes, and how to manipulate them using bitwise operators." To the right, there are links for "Lecture 3 Slides" (B&O Ch 2.1) and "Assignments" with "In: assign0" and "Out: assign1".

- Python (and most everything else):
  - & bitwise and; | bitwise or; ^ bitwise xor; ~ ones complement
  - << left shift bits, >> right shift; LACKS >>> zero fill right shift
  - Recipes:
    - Extract 7 bits: `a & 0x7f00 >> 8`; if take high-order bit add: `& 0x7f`
    - Combine 3 5-bit numbers: `a | (b << 5) | (c << 10)`
    - Lookup tables rather than decoding can be faster, yet still small

# Gamma code properties

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- $G$  is encoded using  $2 \lfloor \log G \rfloor + 1$  bits
  - Length of offset is  $\lfloor \log G \rfloor$  bits
  - Length of length is  $\lfloor \log G \rfloor + 1$  bits
- All gamma codes have an odd number of bits
- Almost within a factor of 2 of best possible,  $\log_2 G$
  
- Gamma code is uniquely prefix-decodable, like VB
- Gamma code can be used for any distribution
  - Optimal for  $P(n) \approx 1/(2n^2)$
- Gamma code is parameter-free

# Gamma seldom used in practice

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- Machines have word boundaries – 8, 16, 32, 64 bits
  - Operations that cross word boundaries are slower
- Compressing and manipulating at the granularity of bits can be too slow
- All modern practice is to use byte or word aligned codes
  - Variable byte encoding is a faster, conceptually simpler compression scheme, with decent compression

# Variable Byte (VB) codes

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- For a gap value  $G$ , we want to use close to the fewest bytes needed to hold  $\log_2 G$  bits
- Begin with one byte to store  $G$  and dedicate 1 bit in it to be a continuation bit  $c$
- If  $G \leq 127$ , binary-encode it in the 7 available bits and set  $c = 1$
- Else encode  $G$ 's lower-order 7 bits and then use additional bytes to encode the higher order bits using the same algorithm
- At the end set the continuation bit of the last byte to 1 ( $c = 1$ ) – and for the other bytes  $c = 0$ .

# Example

docIDs	824	829	215406
gaps		5	214577
VB code	00000110 10111000	10000101	00001101 00001100 10110001

Postings stored as the byte concatenation

000001101011100010000101000011010000110010110001

Key property: VB-encoded postings are uniquely prefix-decodable.

For a small gap (5), VB uses a whole byte.

# RCV1 compression

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, k = 4	7.1
with blocking & front coding	5.9
collection (text, xml markup etc)	3,600.0
collection (text)	960.0
Term-doc incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, $\gamma$ -encoded	101.0

# Other variable unit codes

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- Variable byte codes are used by many real systems
  - Good low-tech blend of variable-length coding and sensitivity to computer memory alignment matches
- Byte alignment wastes space if you have many small gaps – as gap encoding often makes
- More modern work mainly uses the ideas:
  - Be word aligned (32 or 64 bits; even faster)
  - Encode several gaps at the same time
  - Often assume a maximum gap size, perhaps with an escape

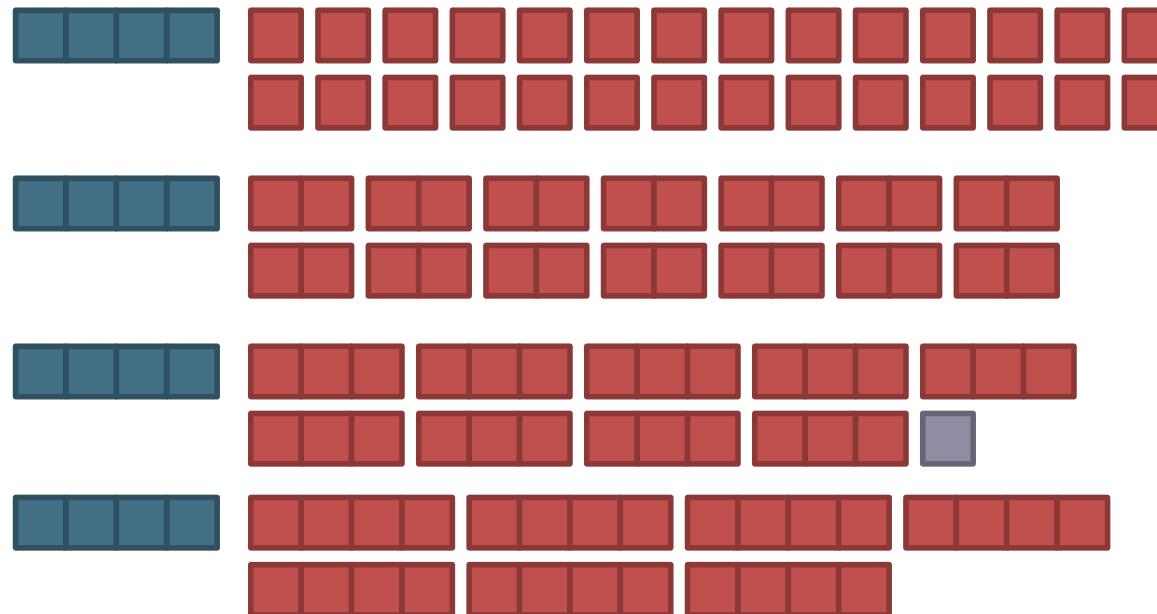
# Group Variable Integer code

- Used by Google around turn of millennium....
  - Jeff Dean, keynote at WSDM 2009 and presentations at CS276
- Encodes 4 integers in blocks of size 5–17 bytes
- First byte: four 2-bit binary length fields
- $L_1 | L_2 | L_3 | L_4$ ,  $L_j \in \{1, 2, 3, 4\}$
- Then,  $L_1 + L_2 + L_3 + L_4$  bytes (between 4–16) hold 4 numbers
  - Each number can use 8/16/24/32 bits. Max gap length ~4 billion
- It was suggested that this was about twice as fast as VB encoding
  - Decoding gaps is much simpler – no bit masking
  - First byte can be decoded with lookup table or switch

# Simple-9 [Anh & Moffat, 2004]

A word-aligned, multiple number encoding scheme

How can we store several numbers in 32 bits with a format selector?



# Simple9 Encoding Scheme [Anh & Moffat, 2004]

- Encoding block: 4 bytes (32 bits)
- Most significant nibble (4 bits) describe the layout of the 28 other bits as follows:
  - 0: a single 28-bit number
  - 1: two 14-bit numbers
  - 2: three 9-bit numbers (and one spare bit)
  - 3: four 7-bit numbers
  - 4: five 5-bit numbers (and three spare bits)
  - 5: seven 4-bit numbers
  - 6: nine 3-bit numbers (and one spare bit)
  - 7: fourteen two-bit numbers
  - 8: twenty-eight one-bit numbers
- Simple16 is a variant with 5 additional (uneven) configurations
- Efficiently decoded with hand-coded decoder, using bit masks
- Extended Simple Family – idea applies to 64-bit words, etc.

Layout (4 bits)	n numbers of b bits each
	$n * b \leq 28$

# Index compression summary

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- We can now create an index for highly efficient Boolean retrieval that is very space efficient
  - Only 4% of the total size of the collection
  - Only 10-15% of the total size of the text in the collection
- 
- We've ignored positional information
  - Hence, space savings are less for indexes used in practice
    - But techniques substantially the same

# Resources for today's lecture

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- *IIR* 5
- *MG* 3.3, 3.4.
- F. Scholer, H.E. Williams and J. Zobel. 2002.  
Compression of Inverted Indexes For Fast Query Evaluation. *Proc. ACM-SIGIR 2002*.
  - Variable byte codes
- V. N. Anh and A. Moffat. 2005. Inverted Index Compression Using Word-Aligned Binary Codes. *Information Retrieval* 8: 151–166.
  - Word aligned codes