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Black Hole Information Paradox

The black hole information paradox is a widely debated topic and has been since the time of Einstein. It appeared when physicists first tried to unify the fields of relativity and quantum mechanics. The paradox boils down to the future of information after it has gone into a black hole and whether it is conserved or destroyed as the black hole evaporates. If black holes do destroy information, then it would break something that quantum mechanics calls unitarity. Unitary states that a wave function can be created at any point in time. If the information is destroyed, then you can no longer determine the past of said information's wave function. In the field of thermodynamics this would be explained as a pure quantum state being transformed into one that is mixed after being evaporated from the black hole. In order to be in accordance with thermodynamics, as matter enters a black hole the entropy of the black hole must also increase.

Black holes emit a type of thermal emission called Hawking radiation and it is how black holes evaporate as well as carry information. The rate of Hawking radiation released by the black hole is also independent of the amount of mass that the black hole has [2]. Hawking believed that information is destroyed as a black hole reaches the size of a Planck mass, which is about 2.176434×10^{-8} kg. At this mass he believed that the black hole will not have sufficient energy to carry the information in the form of thermal radiation [1]. In the terms of thermodynamics, Hawking believed that black holes are "not governed by the usual laws of quantum mechanics: Rather, pure states evolve into mixed states" as described in [2]. This means that the remaining information stored in the black hole would be destroyed upon its evaporation.

Within thermodynamics, systems are described using microstates and macrostates. Microstates are the arrangement of the individual particles within a system and there are many variations of microstates that form a macrostate, which is the physical likeness of microstates that we perceive. All the microstates are of equal probability, so the system is constantly attempting different microstates in order to maintain an equilibrium. These microstates are used in order to determine the entropy of the system which I will explain later in this paper. All possible microstates of our system of particles within and around the black hole are located within the Hilbert space.

Don N. Page, a former student and researcher under Hawking, disagreed with Hawking's proposal to solve the black hole information paradox. Page believes that black holes emit information over their entire emission process and, at the beginning, release it at an incredibly slow rate. Its emission rate is governed by the entropy of that black hole's thermal radiation as well as the entropy of the black hole itself [1].

The second law of thermodynamics states that the entropy of the universe should always increase. Entropy itself can be defined as the extent to which each property of a particle is spread out within a system of particles. When I say properties, I mean every value that defines that particle (position, velocity, or any other degree of freedom). One way to define entropy is by using the Boltzmann equation:

$$S = k_b \log \Omega$$

Where S is the entropy of the macrostate, Ω is the number of microstates, and k_b is the Boltzmann constant. The type of entropy that concerns us within this paper is the Von Neumann Entropy which is also called the entanglement entropy which is the entropy of quantum systems.

Quantum entanglement can be described by a system of two coins, if the coins are quantumly entangled then when both coins are flipped, one will always land heads and the other tails. This can also be done with fundamental particles using the property of spin, if two electrons are entangled then one will always show up spin and the other down spin. Our pair of electrons has a wave function that governs their spin, and it is in a pure quantum state, meaning the entanglement entropy of this wave function would equal zero. However, if part of the wave function is obstructed revealing only the wave function of a single electron, then it is now in a mixed state where it could have either up or down spin (a superposition of spin). In the form of the wave function it is in a mixed state, the entanglement entropy is now greater than zero.

Within Page's paper [1], he shows that the entropy of the blackhole system is controlled by a set of density matrices. Density matrices are matrices that describe the physical state of a system and all properties about it. It is difficult for us to interact with a real-life system of many particles because of how many microstates there are, so we must use a density matrix that is described by a Hilbert space in order to learn more about the system.

Page describes in [1] that there are two density matrices that govern our black hole system, and they are ρ_h and ρ_r. $\rho_h$is the density matrix of the black hole itself and ρ_r is the density matrix of the thermal radiation that the black hole releases. When separated each of these density matrices are in a mixed state just like our pair of electrons, meaning their information is hidden from us. However together we can learn about how much information our system retains. Page relates the density matrices using the following equation:

$$\rho_r = \operatorname*{tr}_h \rho_{rh}, \quad \rho_h = \operatorname*{tr}_r \rho_{rh}$$

This equation says that the trace, or the values of the diagonal of the matrix, of the density matrix of both radiation and the black hole is equal to the density matrix that is the opposite of the one used in the trace.

Using this equation for the density matrices, we can describe the quantum entanglement entropy of the system by this equation:

$$S_r = -\operatorname*{tr}_r(\rho_r \ln \rho_r) = S_h = -\operatorname*{tr}_h(\rho_h \ln \rho_h)$$

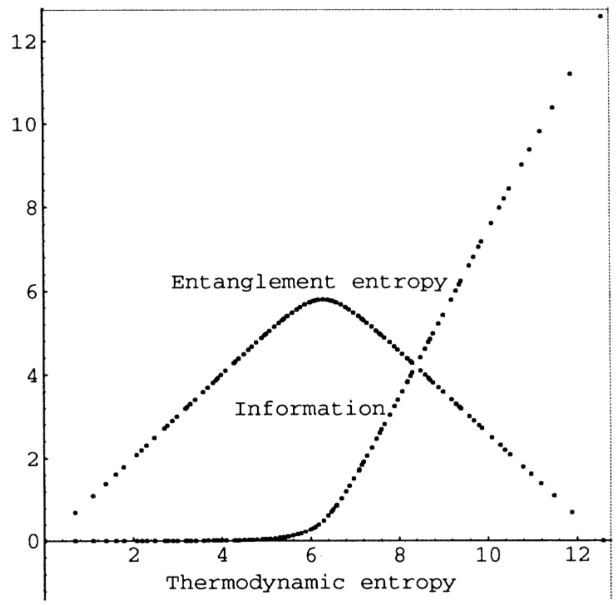
S_r being the Von Neumann entropy of the thermal radiation and S_h being the Von Neumann entropy of the black hole.

Page then defines the information within our system as the "deviation of the entanglement entropy from maximum" [1], yielding the following equation for the information of the radiation as well as the black hole:

$$I_r = \ln m - S_r \simeq s_r - S_r, \quad I_h = \ln n - S_h \simeq s_h - S_h$$

$\ln m - S_r$ and $\ln n - S_h$ are each equal to the amount of information lost in the radiation and black hole subsystems respectively. m and n are the dimensions of the subsystems and roughly correlate to the size of the Hilbert space for the radiation and the black hole, consequently setting the number of microstates of each of the subsystems.

Within [1] Page gives us a graph of the entanglement entropy ($S_{mn} = \ln m - I_{m,n}$) as well as the information (I_{mn}) in relation to the thermodynamic entropy which in this case increases as time increases, leading this figure to be an accurate representation of the change in these two quantities over the lifetime of a black hole.



The function of the entanglement entropy in this figure is called the "Page curve". As you can see from the figure, the entanglement entropy only starts to decrease as the amount of information released starts to increase.

During the period when the black hole hasn't emitted most of its energy, the number of microstates of the radiation is much smaller than the number of microstates of the black hole ($\Gamma_r \ll \Gamma_h$). This means that the Hilbert space of radiation must also be much smaller than the Hilbert space of the black hole at this time, and the size of the Hilbert space generally relates to the amount of information that the system holds [1]. The rate that information is emitted is governed by the differential equation:

$$\frac{dI}{dt} \sim e^{-4\pi/y^2}$$

Where $y = M_{plank}/M$, M_{plank} being the Planck mass, which is a constant, and M which is the mass of the black hole. As the dimension n of the black hole, and the size of the Hilbert state, starts to near m then the black hole starts to evaporate incredibly quickly and disappears.

Giddings and Nelson in [3] use a perturbative analysis to argue that when $s_r < s_h$, at the end of a black hole's life, the information released does not "come out at any finite order of the perturbation" [1], meaning that the information within that subsystem would be in a mixed state and undecipherable. However, Page disagrees with this statement and instead says that the information is encoded within the correlations between the two subsystems. This means that when the correlation of the smaller subsystem is small then the perturbation analysis is not sufficient to conclude whether the information still exists or not.

In conclusion, black holes store information rather than destroy it upon evaporation, and information is released throughout a black hole's life starting at a very slow rate. That information is encoded within the thermal radiation that it releases and is of a non-mixed form, giving the ability to find the past and future of its wavefunction, while staying consistent with unitarity.

[1] D. N. Page, "Information in black hole radiation", Phys. Rev. Lett. 71, 3743 (1993).

[2] C. G. Callan Jr., S. B. Giddings, J. A. Harvey, and A. Strominger, "Evanescent black holes," Phys. Rev. D, 45, R1005 (1992).

[3] S. B. Giddings and W. M. Nelson, "Quantum emission from two-dimensional black holes," Phys. Rev. D, 46, 2486 (1992).