

## Classification exercise

Construct a neural network for solving the Exclusive-OR problem, showing the values of all the weights and biases of the network. You may assume that a threshold function is used for all the neurons.

The truth table of ( $x_1$  XOR  $x_2$ ):

$x_1$	$x_2$	$x_1$ XOR $x_2$
0	0	-1
0	1	1
1	0	1
1	1	-1

Hint: The solution of the XOR problem requires two hidden neurons.

$h_1$ :  $\text{sign}(x_1 + x_2 - 0.5)$

$h_2$ :  $\text{sign}(x_1 + x_2 - 1.5)$

Answer:

The equations of the two separation boundaries are given by:

$$x_1 + x_2 - 0.5 = 0$$

$$x_1 + x_2 - 1.5 = 0$$

As a result, the weights of the first hidden neuron are  $w_{11} = 1$ ,  $w_{21} = 1$ ,  $w_{01} = -0.5$ , and the weights of the second neuron are  $w_{12} = 1$ ,  $w_{22} = 1$ ,  $w_{02} = -1.5$ .

Using these two neurons, the inputs are mapped to the hidden output space as follows:

$x_1$	$x_2$	$h_1$	$h_2$
0	0	-1	-1
0	1	1	-1
1	0	1	-1
1	1	1	1

The set of transformed inputs can be separated by the boundary (1)  $h_1 - h_2 - 1 = 0$  or (2)  $-h_1 + h_2 + 1 = 0$ . Since the expression  $h_1 - h_2 - 1$  results in the correct set of outputs, the weights of the output neuron are  $u_{11} = 1$ ,  $u_{21} = -1$ ,  $u_{01} = -1$ .