Stuart **Russell**

Peter **Norvig**

Artificial Intelligence

A Modern Approach

Third Edition

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Third Edition

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Artificial Intelligence
A Modern Approach
Third Edition

Stuart J. Russell and Peter Norvig

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Library of Congress Cataloging-in-Publication Data on File

Prentice Hall
is an imprint of

www.pearsonhighered.com

10 9 8 7 6 5 4 3 2 1
ISBN-13: 978-0-13-604259-4
ISBN-10: 0-13-604259-7

For Loy, Gordon, Lucy, George, and Isaac — S.J.R.

For Kris, Isabella, and Juliet — P.N.

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Preface

Artificial Intelligence (AI) is a big field, and this is a big book. We have tried to explore the full breadth of the field, which encompasses logic, probability, and continuous mathematics; perception, reasoning, learning, and action; and everything from microelectronic devices to robotic planetary explorers. The book is also big because we go into some depth.

The subtitle of this book is "A Modern Approach." The intended meaning of this rather empty phrase is that we have tried to synthesize what is now known into a common framework, rather than trying to explain each subfield of AI in its own historical context. We apologize to those whose subfields are, as a result, less recognizable.

New to this edition

This edition captures the changes in AI that have taken place since the last edition in 2003. There have been important applications of AI technology, such as the widespread deployment of practical speech recognition, machine translation, autonomous vehicles, and household robotics. There have been algorithmic landmarks, such as the solution of the game of checkers. And there has been a great deal of theoretical progress, particularly in areas such as probabilistic reasoning, machine learning, and computer vision. Most important from our point of view is the continued evolution in how we think about the field, and thus how we organize the book. The major changes are as follows:

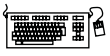
- We place more emphasis on partially observable and nondeterministic environments, especially in the nonprobabilistic settings of search and planning. The concepts of *belief state* (a set of possible worlds) and *state estimation* (maintaining the belief state) are introduced in these settings; later in the book, we add probabilities.
- In addition to discussing the types of environments and types of agents, we now cover in more depth the types of *representations* that an agent can use. We distinguish among *atomic* representations (in which each state of the world is treated as a black box), *factored* representations (in which a state is a set of attribute/value pairs), and *structured* representations (in which the world consists of objects and relations between them).
- Our coverage of planning goes into more depth on contingent planning in partially observable environments and includes a new approach to hierarchical planning.
- We have added new material on first-order probabilistic models, including *open-universe* models for cases where there is uncertainty as to what objects exist.
- We have completely rewritten the introductory machine-learning chapter, stressing a wider variety of more modern learning algorithms and placing them on a firmer theoretical footing.
- We have expanded coverage of Web search and information extraction, and of techniques for learning from very large data sets.
- 20% of the citations in this edition are to works published after 2003.
- We estimate that about 20% of the material is brand new. The remaining 80% reflects older work but has been largely rewritten to present a more unified picture of the field.

Overview of the book

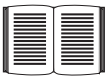
The main unifying theme is the idea of an **intelligent agent**. We define AI as the study of agents that receive percepts from the environment and perform actions. Each such agent implements a function that maps percept sequences to actions, and we cover different ways to represent these functions, such as reactive agents, real-time planners, and decision-theoretic systems. We explain the role of learning as extending the reach of the designer into unknown environments, and we show how that role constrains agent design, favoring explicit knowledge representation and reasoning. We treat robotics and vision not as independently defined problems, but as occurring in the service of achieving goals. We stress the importance of the task environment in determining the appropriate agent design.

Our primary aim is to convey the *ideas* that have emerged over the past fifty years of AI research and the past two millennia of related work. We have tried to avoid excessive formality in the presentation of these ideas while retaining precision. We have included pseudocode algorithms to make the key ideas concrete; our pseudocode is described in Appendix B.

This book is primarily intended for use in an undergraduate course or course sequence. The book has 27 chapters, each requiring about a week's worth of lectures, so working through the whole book requires a two-semester sequence. A one-semester course can use selected chapters to suit the interests of the instructor and students. The book can also be used in a graduate-level course (perhaps with the addition of some of the primary sources suggested in the bibliographical notes). Sample syllabi are available at the book's Web site, `aima.cs.berkeley.edu`. The only prerequisite is familiarity with basic concepts of computer science (algorithms, data structures, complexity) at a sophomore level. Freshman calculus and linear algebra are useful for some of the topics; the required mathematical background is supplied in Appendix A.



Exercises are given at the end of each chapter. Exercises requiring significant programming are marked with a **keyboard** icon. These exercises can best be solved by taking advantage of the code repository at `aima.cs.berkeley.edu`. Some of them are large enough to be considered term projects. A number of exercises require some investigation of the literature; these are marked with a **book** icon.



Throughout the book, important points are marked with a *pointing* icon. We have included an extensive index of around 6,000 items to make it easy to find things in the book. Wherever a **new term** is first defined, it is also marked in the margin.

NEW TERM

About the Web site

`aima.cs.berkeley.edu`, the Web site for the book, contains

- implementations of the algorithms in the book in several programming languages,
- a list of over 1000 schools that have used the book, many with links to online course materials and syllabi,
- an annotated list of over 800 links to sites around the Web with useful AI content,
- a chapter-by-chapter list of supplementary material and links,
- instructions on how to join a discussion group for the book,

- instructions on how to contact the authors with questions or comments,
- instructions on how to report errors in the book, in the likely event that some exist, and
- slides and other materials for instructors.

About the cover

The cover depicts the final position from the decisive game 6 of the 1997 match between chess champion Garry Kasparov and program DEEP BLUE. Kasparov, playing Black, was forced to resign, making this the first time a computer had beaten a world champion in a chess match. Kasparov is shown at the top. To his left is the Asimo humanoid robot and to his right is Thomas Bayes (1702–1761), whose ideas about probability as a measure of belief underlie much of modern AI technology. Below that we see a Mars Exploration Rover, a robot that landed on Mars in 2004 and has been exploring the planet ever since. To the right is Alan Turing (1912–1954), whose fundamental work defined the fields of computer science in general and artificial intelligence in particular. At the bottom is Shakey (1966–1972), the first robot to combine perception, world-modeling, planning, and learning. With Shakey is project leader Charles Rosen (1917–2002). At the bottom right is Aristotle (384 B.C.–322 B.C.), who pioneered the study of logic; his work was state of the art until the 19th century (copy of a bust by Lysippos). At the bottom left, lightly screened behind the authors' names, is a planning algorithm by Aristotle from *De Motu Animalium* in the original Greek. Behind the title is a portion of the CPSC Bayesian network for medical diagnosis (Pradhan *et al.*, 1994). Behind the chess board is part of a Bayesian logic model for detecting nuclear explosions from seismic signals.

Credits: Stan Honda/Getty (Kasparaov), Library of Congress (Bayes), NASA (Mars rover), National Museum of Rome (Aristotle), Peter Norvig (book), Ian Parker (Berkeley skyline), Shutterstock (Asimo, Chess pieces), Time Life/Getty (Shakey, Turing).

Acknowledgments

This book would not have been possible without the many contributors whose names did not make it to the cover. Jitendra Malik and David Forsyth wrote Chapter 24 (computer vision) and Sebastian Thrun wrote Chapter 25 (robotics). Vibhu Mittal wrote part of Chapter 22 (natural language). Nick Hay, Mehran Sahami, and Ernest Davis wrote some of the exercises. Zoran Duric (George Mason), Thomas C. Henderson (Utah), Leon Reznik (RIT), Michael Gourley (Central Oklahoma) and Ernest Davis (NYU) reviewed the manuscript and made helpful suggestions. We thank Ernie Davis in particular for his tireless ability to read multiple drafts and help improve the book. Nick Hay whipped the bibliography into shape and on deadline stayed up to 5:30 AM writing code to make the book better. Jon Barron formatted and improved the diagrams in this edition, while Tim Huang, Mark Paskin, and Cynthia Bruyns helped with diagrams and algorithms in previous editions. Ravi Mohan and Ciaran O'Reilly wrote and maintain the Java code examples on the Web site. John Canny wrote the robotics chapter for the first edition and Douglas Edwards researched the historical notes. Tracy Dunkelberger, Allison Michael, Scott Disanno, and Jane Bonnell at Pearson tried their best to keep us on schedule and made many helpful suggestions. Most helpful of all has

been Julie Sussman, P.P.A., who read every chapter and provided extensive improvements. In previous editions we had proofreaders who would tell us when we left out a comma and said *which* when we meant *that*; Julie told us when we left out a minus sign and said x_i when we meant x_j. For every typo or confusing explanation that remains in the book, rest assured that Julie has fixed at least five. She persevered even when a power failure forced her to work by lantern light rather than LCD glow.

Stuart would like to thank his parents for their support and encouragement and his wife, Loy Sheflott, for her endless patience and boundless wisdom. He hopes that Gordon, Lucy, George, and Isaac will soon be reading this book after they have forgiven him for working so long on it. RUGS (Russell's Unusual Group of Students) have been unusually helpful, as always.

Peter would like to thank his parents (Torsten and Gerda) for getting him started, and his wife (Kris), children (Bella and Juliet), colleagues, and friends for encouraging and tolerating him through the long hours of writing and longer hours of rewriting.

We both thank the librarians at Berkeley, Stanford, and NASA and the developers of CiteSeer, Wikipedia, and Google, who have revolutionized the way we do research. We can't acknowledge all the people who have used the book and made suggestions, but we would like to note the especially helpful comments of Gagan Aggarwal, Eyal Amir, Ion Androutsopoulos, Krzysztof Apt, Warren Haley Armstrong, Ellery Aziel, Jeff Van Baalen, Darius Bacon, Brian Baker, Shumeet Baluja, Don Barker, Tony Barrett, James Newton Bass, Don Beal, Howard Beck, Wolfgang Bibel, John Binder, Larry Bookman, David R. Boxall, Ronen Brafman, John Bresina, Gerhard Brewka, Selmer Bringsjord, Carla Brodley, Chris Brown, Emma Brunskill, Wilhelm Burger, Lauren Burka, Carlos Bustamante, Joao Cachopo, Murray Campbell, Norman Carver, Emmanuel Castro, Anil Chakravarthy, Dan Chisarick, Berthe Choueiry, Roberto Cipolla, David Cohen, James Coleman, Julie Ann Comparini, Corinna Cortes, Gary Cottrell, Ernest Davis, Tom Dean, Rina Dechter, Tom Dietterich, Peter Drake, Chuck Dyer, Doug Edwards, Robert Egginton, Asma'a El-Budrawy, Barbara Engelhardt, Kutluhan Erol, Oren Etzioni, Hana Filip, Douglas Fisher, Jeffrey Forbes, Ken Ford, Eric Fosler-Lussier, John Fosler, Jeremy Frank, Alex Franz, Bob Futrelle, Marek Galecki, Stefan Gerberding, Stuart Gill, Sabine Glesner, Seth Golub, Gosta Grahne, Russ Greiner, Eric Grimson, Barbara Grosz, Larry Hall, Steve Hanks, Othar Hansson, Ernst Heinz, Jim Hendler, Christoph Herrmann, Paul Hilfinger, Robert Holte, Vasant Honavar, Tim Huang, Seth Hutchinson, Joost Jacob, Mark Jelasity, Magnus Johansson, Istvan Jonyer, Dan Jurafsky, Leslie Kaelbling, Keiji Kanazawa, Surekha Kasibhatla, Simon Kasif, Henry Kautz, Gernot Kerschbaumer, Max Khesin, Richard Kirby, Dan Klein, Kevin Knight, Roland Koenig, Sven Koenig, Daphne Koller, Rich Korf, Benjamin Kuipers, James Kurien, John Lafferty, John Laird, Gus Larsson, John Lazzaro, Jon LeBlanc, Jason Leatherman, Frank Lee, Jon Lehto, Edward Lim, Phil Long, Pierre Louveaux, Don Loveland, Sridhar Mahadevan, Tony Mancill, Jim Martin, Andy Mayer, John McCarthy, David McGrane, Jay Mendelsohn, Risto Miikkulanien, Brian Milch, Steve Minton, Vibhu Mittal, Mehryar Mohri, Leora Morgenstern, Stephen Muggleton, Kevin Murphy, Ron Musick, Sung Myaeng, Eric Nadeau, Lee Naish, Pandu Nayak, Bernhard Nebel, Stuart Nelson, XuanLong Nguyen, Nils Nilsson, Illah Nourbakhsh, Ali Nouri, Arthur Nunes-Harwitt, Steve Omohundro, David Page, David Palmer, David Parkes, Ron Parr, Mark

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About the Authors

Stuart Russell was born in 1962 in Portsmouth, England. He received his B.A. with first-class honours in physics from Oxford University in 1982, and his Ph.D. in computer science from Stanford in 1986. He then joined the faculty of the University of California at Berkeley, where he is a professor of computer science, director of the Center for Intelligent Systems, and holder of the Smith–Zadeh Chair in Engineering. In 1990, he received the Presidential Young Investigator Award of the National Science Foundation, and in 1995 he was cowinner of the Computers and Thought Award. He was a 1996 Miller Professor of the University of California and was appointed to a Chancellor's Professorship in 2000. In 1998, he gave the Forsythe Memorial Lectures at Stanford University. He is a Fellow and former Executive Council member of the American Association for Artificial Intelligence. He has published over 100 papers on a wide range of topics in artificial intelligence. His other books include *The Use of Knowledge in Analogy and Induction* and (with Eric Wefald) *Do the Right Thing: Studies in Limited Rationality.*

Peter Norvig is currently Director of Research at Google, Inc., and was the director responsible for the core Web search algorithms from 2002 to 2005. He is a Fellow of the American Association for Artificial Intelligence and the Association for Computing Machinery. Previously, he was head of the Computational Sciences Division at NASA Ames Research Center, where he oversaw NASA's research and development in artificial intelligence and robotics, and chief scientist at Junglee, where he helped develop one of the first Internet information extraction services. He received a B.S. in applied mathematics from Brown University and a Ph.D. in computer science from the University of California at Berkeley. He received the Distinguished Alumni and Engineering Innovation awards from Berkeley and the Exceptional Achievement Medal from NASA. He has been a professor at the University of Southern California and a research faculty member at Berkeley. His other books are *Paradigms of AI Programming: Case Studies in Common Lisp* and *Verbmobil: A Translation System for Face-to-Face Dialog* and *Intelligent Help Systems for UNIX*.

Contents

order even with an inadmissible heuristic. The idea of keeping track of the best alternative path appeared earlier in Bratko's (1986) elegant Prolog implementation of A* and in the DTA* algorithm (Russell and Wefald, 1991). The latter work also discusses metalevel state spaces and metalevel learning.

The MA* algorithm appeared in Chakrabarti *et al.* (1989). SMA*, or Simplified MA*, emerged from an attempt to implement MA* as a comparison algorithm for IE (Russell, 1992). Kaindl and Khorsand (1994) have applied SMA* to produce a bidirectional search algorithm that is substantially faster than previous algorithms. Korf and Zhang (2000) describe a divide-and-conquer approach, and Zhou and Hansen (2002) introduce memory-bounded A* graph search and a strategy for switching to breadth-first search to increase memory-efficiency (Zhou and Hansen, 2006). Korf (1995) surveys memory-bounded search techniques.

The idea that admissible heuristics can be derived by problem relaxation appears in the seminal paper by Held and Karp (1970), who used the minimum-spanning-tree heuristic to solve the TSP. (See Exercise 3.30.)

The automation of the relaxation process was implemented successfully by Prieditis (1993), building on earlier work with Mostow (Mostow and Prieditis, 1989). Holte and Hernadvolgyi (2001) describe more recent steps towards automating the process. The use of pattern databases to derive admissible heuristics is due to Gasser (1995) and Culberson and Schaeffer (1996, 1998); disjoint pattern databases are described by Korf and Felner (2002); a similar method using symbolic patterns is due to Edelkamp (2009). Felner *et al.* (2007) show how to compress pattern databases to save space. The probabilistic interpretation of heuristics was investigated in depth by Pearl (1984) and Hansson and Mayer (1989).

By far the most comprehensive source on heuristics and heuristic search algorithms is Pearl's (1984) *Heuristics* text. This book provides especially good coverage of the wide variety of offshoots and variations of A*, including rigorous proofs of their formal properties. Kanal and Kumar (1988) present an anthology of important articles on heuristic search, and Rayward-Smith *et al.* (1996) cover approaches from Operations Research. Papers about new search algorithms—which, remarkably, continue to be discovered—appear in journals such as *Artificial Intelligence* and *Journal of the ACM*.

PARALLEL SEARCH

The topic of **parallel search** algorithms was not covered in the chapter, partly because it requires a lengthy discussion of parallel computer architectures. Parallel search became a popular topic in the 1990s in both AI and theoretical computer science (Mahanti and Daniels, 1993; Grama and Kumar, 1995; Crauser *et al.*, 1998) and is making a comeback in the era of new multicore and cluster architectures (Ralphs *et al.*, 2004; Korf and Schultze, 2005). Also of increasing importance are search algorithms for very large graphs that require disk storage (Korf, 2008).

EXERCISES

3.1 Explain why problem formulation must follow goal formulation.

3.2 Your goal is to navigate a robot out of a maze. The robot starts in the center of the maze

facing north. You can turn the robot to face north, east, south, or west. You can direct the robot to move forward a certain distance, although it will stop before hitting a wall.

a. Formulate this problem. How large is the state space?

b. In navigating a maze, the only place we need to turn is at the intersection of two or more corridors. Reformulate this problem using this observation. How large is the state space now?

c. From each point in the maze, we can move in any of the four directions until we reach a turning point, and this is the only action we need to do. Reformulate the problem using these actions. Do we need to keep track of the robot's orientation now?

d. In our initial description of the problem we already abstracted from the real world, restricting actions and removing details. List three such simplifications we made.

3.3 Suppose two friends live in different cities on a map, such as the Romania map shown in Figure 3.2. On every turn, we can simultaneously move each friend to a neighboring city on the map. The amount of time needed to move from city i to neighbor j is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible.

a. Write a detailed formulation for this search problem. (You will find it helpful to define some formal notation here.)

b. Let $D(i, j)$ be the straight-line distance between cities i and j. Which of the following heuristic functions are admissible? (i) $D(i, j)$; (ii) $2 \cdot D(i, j)$; (iii) $D(i, j)/2$.

c. Are there completely connected maps for which no solution exists?

d. Are there maps in which all solutions require one friend to visit the same city twice?

3.4 Show that the 8-puzzle states are divided into two disjoint sets, such that any state is reachable from any other state in the same set, while no state is reachable from any state in the other set. (*Hint:* See Berlekamp *et al.* (1982).) Devise a procedure to decide which set a given state is in, and explain why this is useful for generating random states.

3.5 Consider the n-queens problem using the "efficient" incremental formulation given on page 72. Explain why the state space has at least $\sqrt[3]{n!}$ states and estimate the largest n for which exhaustive exploration is feasible. (*Hint*: Derive a lower bound on the branching factor by considering the maximum number of squares that a queen can attack in any column.)

3.6 Give a complete problem formulation for each of the following. Choose a formulation that is precise enough to be implemented.

a. Using only four colors, you have to color a planar map in such a way that no two adjacent regions have the same color.

b. A 3-foot-tall monkey is in a room where some bananas are suspended from the 8-foot ceiling. He would like to get the bananas. The room contains two stackable, movable, climbable 3-foot-high crates.

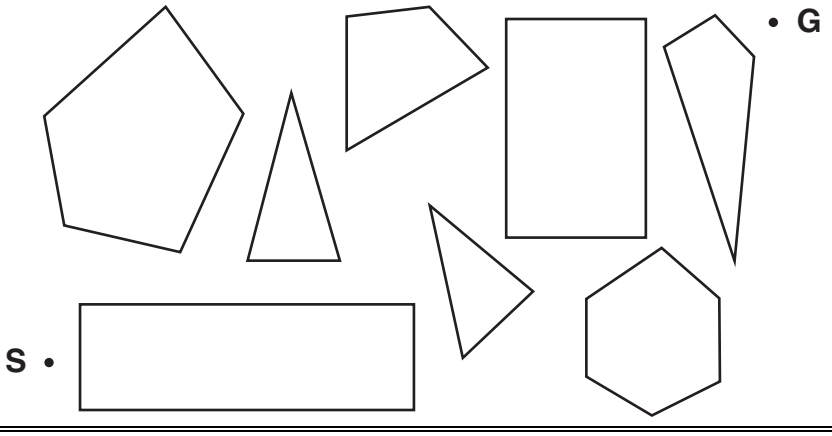
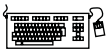


Figure 3.31 A scene with polygonal obstacles. S and G are the start and goal states.

c. You have a program that outputs the message "illegal input record" when fed a certain file of input records. You know that processing of each record is independent of the other records. You want to discover what record is illegal.

d. You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet. You can fill the jugs up or empty them out from one to another or onto the ground. You need to measure out exactly one gallon.



3.7 Consider the problem of finding the shortest path between two points on a plane that has convex polygonal obstacles as shown in Figure 3.31. This is an idealization of the problem that a robot has to solve to navigate in a crowded environment.

a. Suppose the state space consists of all positions (x, y) in the plane. How many states are there? How many paths are there to the goal?

b. Explain briefly why the shortest path from one polygon vertex to any other in the scene must consist of straight-line segments joining some of the vertices of the polygons. Define a good state space now. How large is this state space?

c. Define the necessary functions to implement the search problem, including an ACTIONS function that takes a vertex as input and returns a set of vectors, each of which maps the current vertex to one of the vertices that can be reached in a straight line. (Do not forget the neighbors on the same polygon.) Use the straight-line distance for the heuristic function.

d. Apply one or more of the algorithms in this chapter to solve a range of problems in the domain, and comment on their performance.

3.8 On page 68, we said that we would not consider problems with negative path costs. In this exercise, we explore this decision in more depth.

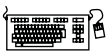
a. Suppose that actions can have arbitrarily large negative costs; explain why this possibility would force any optimal algorithm to explore the entire state space.

b. Does it help if we insist that step costs must be greater than or equal to some negative constant c? Consider both trees and graphs.

c. Suppose that a set of actions forms a loop in the state space such that executing the set in some order results in no net change to the state. If all of these actions have negative cost, what does this imply about the optimal behavior for an agent in such an environment?

d. One can easily imagine actions with high negative cost, even in domains such as route finding. For example, some stretches of road might have such beautiful scenery as to far outweigh the normal costs in terms of time and fuel. Explain, in precise terms, within the context of state-space search, why humans do not drive around scenic loops indefinitely, and explain how to define the state space and actions for route finding so that artificial agents can also avoid looping.

e. Can you think of a real domain in which step costs are such as to cause looping?



3.9 The **missionaries and cannibals** problem is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place. This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint (Amarel, 1968).

a. Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.

b. Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?

c. Why do you think people have a hard time solving this puzzle, given that the state space is so simple?

3.10 Define in your own words the following terms: state, state space, search tree, search node, goal, action, transition model, and branching factor.

3.11 What's the difference between a world state, a state description, and a search node? Why is this distinction useful?

3.12 An action such as *Go(Sibiu)* really consists of a long sequence of finer-grained actions: turn on the car, release the brake, accelerate forward, etc. Having composite actions of this kind reduces the number of steps in a solution sequence, thereby reducing the search time. Suppose we take this to the logical extreme, by making super-composite actions out of every possible sequence of *Go* actions. Then every problem instance is solved by a single super-composite action, such as *Go(Sibiu)Go(Rimnicu Vilcea)Go(Pitesti)Go(Bucharest)*. Explain how search would work in this formulation. Is this a practical approach for speeding up problem solving?

3.13 Prove that GRAPH-SEARCH satisfies the graph separation property illustrated in Figure 3.9. (*Hint*: Begin by showing that the property holds at the start, then show that if it holds before an iteration of the algorithm, it holds afterwards.) Describe a search algorithm that violates the property.

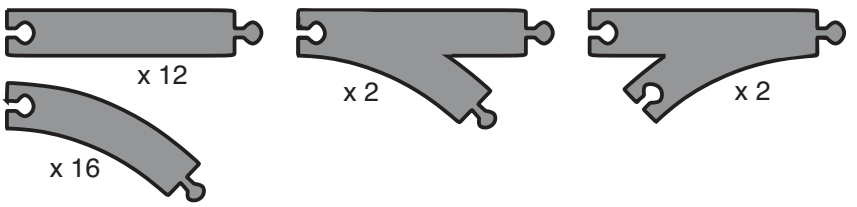


Figure 3.32 The track pieces in a wooden railway set; each is labeled with the number of copies in the set. Note that curved pieces and "fork" pieces ("switches" or "points") can be flipped over so they can curve in either direction. Each curve subtends 45 degrees.

3.14 Which of the following are true and which are false? Explain your answers.

a. Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.

b. $h(n) = 0$ is an admissible heuristic for the 8-puzzle.

c. A* is of no use in robotics because percepts, states, and actions are continuous.

d. Breadth-first search is complete even if zero step costs are allowed.

e. Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

3.15 Consider a state space where the start state is number 1 and each state k has two successors: numbers $2k$ and $2k + 1$.

a. Draw the portion of the state space for states 1 to 15.

b. Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search with limit 3, and iterative deepening search.

c. How well would bidirectional search work on this problem? What is the branching factor in each direction of the bidirectional search?

d. Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?

e. Call the action going from k to $2k$ Left, and the action going to $2k + 1$ Right. Can you find an algorithm that outputs the solution to this problem without any search at all?

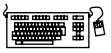
3.16 A basic wooden railway set contains the pieces shown in Figure 3.32. The task is to connect these pieces into a railway that has no overlapping tracks and no loose ends where a train could run off onto the floor.

a. Suppose that the pieces fit together *exactly* with no slack. Give a precise formulation of the task as a search problem.

b. Identify a suitable uninformed search algorithm for this task and explain your choice.

c. Explain why removing any one of the "fork" pieces makes the problem unsolvable.

d. Give an upper bound on the total size of the state space defined by your formulation. (*Hint*: think about the maximum branching factor for the construction process and the maximum depth, ignoring the problem of overlapping pieces and loose ends. Begin by pretending that every piece is unique.)



3.17 On page 90, we mentioned **iterative lengthening search**, an iterative analog of uniform cost search. The idea is to use increasing limits on path cost. If a node is generated whose path cost exceeds the current limit, it is immediately discarded. For each new iteration, the limit is set to the lowest path cost of any node discarded in the previous iteration.

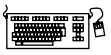
a. Show that this algorithm is optimal for general path costs.

b. Consider a uniform tree with branching factor b, solution depth d, and unit step costs. How many iterations will iterative lengthening require?

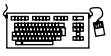
c. Now consider step costs drawn from the continuous range $[\epsilon, 1]$, where $0 < \epsilon < 1$. How many iterations are required in the worst case?

d. Implement the algorithm and apply it to instances of the 8-puzzle and traveling salesperson problems. Compare the algorithm's performance to that of uniform-cost search, and comment on your results.

3.18 Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, $O(n^2)$ vs. $O(n)$).



3.19 Write a program that will take as input two Web page URLs and find a path of links from one to the other. What is an appropriate search strategy? Is bidirectional search a good idea? Could a search engine be used to implement a predecessor function?



3.20 Consider the vacuum-world problem defined in Figure 2.2.

a. Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?

b. Apply your chosen algorithm to compute an optimal sequence of actions for a 3×3 world whose initial state has dirt in the three top squares and the agent in the center.

c. Construct a search agent for the vacuum world, and evaluate its performance in a set of 3×3 worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.

d. Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.

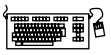
e. Consider what would happen if the world were enlarged to $n \times n$. How does the performance of the search agent and of the reflex agent vary with n?

3.21 Prove each of the following statements, or give a counterexample:

a. Breadth-first search is a special case of uniform-cost search.

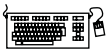
b. Depth-first search is a special case of best-first tree search.

c. Uniform-cost search is a special case of A* search.



3.22 Compare the performance of A* and RBFS on a set of randomly generated problems in the 8-puzzle (with Manhattan distance) and TSP (with MST—see Exercise 3.30) domains. Discuss your results. What happens to the performance of RBFS when a small random number is added to the heuristic values in the 8-puzzle domain?

3.23 Trace the operation of A* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic. That is, show the sequence of nodes that the algorithm will consider and the f, g, and h score for each node.



3.24 Devise a state space in which A* using GRAPH-SEARCH returns a suboptimal solution with an $h(n)$ function that is admissible but inconsistent.

HEURISTIC PATH ALGORITHM

3.25 The **heuristic path algorithm** (Pohl, 1977) is a best-first search in which the evaluation function is $f(n) = (2 - w)g(n) + wh(n)$. For what values of w is this complete? For what values is it optimal, assuming that h is admissible? What kind of search does this perform for $w = 0$, $w = 1$, and $w = 2$?

3.26 Consider the unbounded version of the regular 2D grid shown in Figure 3.9. The start state is at the origin, (0,0), and the goal state is at (x, y).

a. What is the branching factor b in this state space?
b. How many distinct states are there at depth k (for $k > 0$)?
c. What is the maximum number of nodes expanded by breadth-first tree search?
d. What is the maximum number of nodes expanded by breadth-first graph search?
e. Is $h = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v)? Explain.
f. How many nodes are expanded by A* graph search using h?
g. Does h remain admissible if some links are removed?
h. Does h remain admissible if some links are added between nonadjacent states?

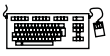
3.27 n vehicles occupy squares $(1, 1)$ through $(n, 1)$ (i.e., the bottom row) of an $n \times n$ grid. The vehicles must be moved to the top row but in reverse order; so the vehicle i that starts in $(i, 1)$ must end up in $(n - i + 1, n)$. On each time step, every one of the n vehicles can move one square up, down, left, or right, or stay put; but if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

a. Calculate the size of the state space as a function of n.
b. Calculate the branching factor as a function of n.
c. Suppose that vehicle i is at (x_i, y_i); write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n - i + 1, n)$, assuming no other vehicles are on the grid.
d. Which of the following heuristics are admissible for the problem of moving all n vehicles to their destinations? Explain.

(i) $\sum_{i=1}^{n} h_i$.
(ii) $\max\{h_1, \ldots, h_n\}$.
(iii) $\min\{h_1, \ldots, h_n\}$.

3.28 Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.) Prove that if h never overestimates by more than c, A* using h returns a solution whose cost exceeds that of the optimal solution by no more than c.

3.29 Prove that if a heuristic is consistent, it must be admissible. Construct an admissible heuristic that is not consistent.



3.30 The traveling salesperson problem (TSP) can be solved with the minimum-spanning-tree (MST) heuristic, which estimates the cost of completing a tour, given that a partial tour has already been constructed. The MST cost of a set of cities is the smallest sum of the link costs of any tree that connects all the cities.

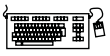
a. Show how this heuristic can be derived from a relaxed version of the TSP.

b. Show that the MST heuristic dominates straight-line distance.

c. Write a problem generator for instances of the TSP where cities are represented by random points in the unit square.

d. Find an efficient algorithm in the literature for constructing the MST, and use it with A* graph search to solve instances of the TSP.

3.31 On page 105, we defined the relaxation of the 8-puzzle in which a tile can move from square A to square B if B is blank. The exact solution of this problem defines **Gaschnig's heuristic** (Gaschnig, 1979). Explain why Gaschnig's heuristic is at least as accurate as h_1 (misplaced tiles), and show cases where it is more accurate than both h_1 and h_2 (Manhattan distance). Explain how to calculate Gaschnig's heuristic efficiently.



3.32 We gave two simple heuristics for the 8-puzzle: Manhattan distance and misplaced tiles. Several heuristics in the literature purport to improve on this—see, for example, Nilsson (1971), Mostow and Prieditis (1989), and Hansson *et al.* (1992). Test these claims by implementing the heuristics and comparing the performance of the resulting algorithms.