8 THE SGP8 MODEL

The NORAD mean element sets can be used for prediction with SGP8. All symbols nc
below are defined in the list of symbols in Section Twelve. The original mean motion
semimajor axis (a''_o) are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o}\right)^{\frac{2}{3}}$$

$$\delta_1 = \frac{3}{2}\frac{k_2}{a_1{}^2}\frac{(3\cos^2 i_o - 1)}{(1 - e_o{}^2)^{\frac{3}{2}}}$$

$$a_o = a_1\left(1 - \frac{1}{3}\delta_1 - \delta_1{}^2 - \frac{134}{81}\delta_1{}^3\right)$$

$$\delta_o = \frac{3}{2}\frac{k_2}{a_o{}^2}\frac{(3\cos^2 i_o - 1)}{(1 - e_o{}^2)^{\frac{3}{2}}}$$

$$n''_o = \frac{n_o}{1 + \delta_o}$$

$$a''_o = \frac{a_o}{1 - \delta_o}.$$

The ballistic coefficient (B term) is then calculated from the B^* drag term by

$$B = 2B^*/\rho_o$$

where

$$\rho_o = (2.461 \times 10^{-5})\ \mathrm{XKMPER\ kg/m^2/Earth\ radii}$$

is a reference value of atmospheric density.

Then calculate the constants

$$\beta^2 = 1 - e^2$$

$$\dot{M}_1 = -\frac{3}{2}\frac{n''k_2}{a''^2\beta^3}(1 - 3\theta^2)$$

$$\dot{\omega}_1 = -\frac{3}{2}\frac{n''k_2}{a''^2\beta^4}(1 - 5\theta^2)$$

$$\dot{\Omega}_1 = -3\frac{n''k_2}{a''^2\beta^4}\theta$$

$$\dot{M}_2 = \frac{3}{16}\frac{n''{k_2}^2}{a''^4\beta^7}(13 - 78\theta^2 + 137\theta^4)$$

$$\dot{\omega}_2 = \frac{3}{16}\frac{n''k_22}{a''^4\beta^8}(7 - 114\theta^2 + 395\theta^4) + \frac{5}{4}\frac{n''k_4}{a''^4\beta^8}(3 - 36\theta^2 + 49\theta^4)$$

$$\dot{\Omega}_2 = \frac{3}{2}\frac{n''{k_2}^2}{a''^4\beta^8}\theta(4 - 19\theta^2) + \frac{5}{2}\frac{n''k_4}{a''^4\beta^8}\theta(3 - 7\theta^2)$$

$$\dot{\ell} = n'' + \dot{M}_1 + \dot{M}_2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$\dot{\Omega} = \dot{\Omega}_1 + \dot{\Omega}_2$$

$$\xi = \frac{1}{a''\beta^2 - s}$$

$$\eta = es\xi$$

$$\psi = \sqrt{1 - \eta^2}$$

$$\alpha^2 = 1 + e^2$$

$$C_o = \frac{1}{2}B\rho_o(q_o - s)^4 n'' a'' \xi^4 \alpha^{-1}\psi^{-7}$$

$$D_1 = \xi\psi^{-2}/a''\beta^2$$

$$D_2 = 12 + 36\eta^2 + \frac{9}{2}\eta^4$$

$$D_3 = 15\eta^2 + \frac{5}{2}\eta^4$$

$$D_4 = 5\eta + \frac{15}{4}\eta^3$$

$$D_5 = \xi\psi^{-2}$$

$$B_1 = -k_2(1 - 3\theta^2)$$

$$B_2 = -k_2(1 - \theta^2)$$

$$B_3 = \frac{A_{3,0}}{k_2}\sin i$$

$$C_2 = D_1 D_3 B_2$$

$$C_3 = D_4 D_5 B_3$$

$$\dot{n}_o = C_1\left(2 + 3\eta^2 + 20e\eta + 5e\eta^3 + \frac{17}{2}e^2 + 34e^2\eta^2 + D_1 D_2 B_1 + C_2\cos 2\omega + C_3\sin\omega\right)$$

$$C_4 = D_1 D_7 B_2$$

$$C_5 = D_5 D_8 B_3$$

$$D_6 = 30\eta + \frac{45}{2}\eta^3$$

$$D_7 = 5\eta + \frac{25}{2}\eta^3$$

$$\dot{e}_o = -C_o \left(4\eta + \eta^3 + 5e + 15e\eta^2 + \frac{31}{2}e^2\eta + 7e^2\eta^3 + D_1 D_6 B_1 + C_4 \cos 2\omega + C_5 \sin \omega \right.$$

$$\dot{\alpha}/\alpha = e\dot{e}\alpha^{-2}$$

$$C_6 = \frac{1}{3}\frac{\dot{n}}{n''}$$

$$\dot{\xi}/\xi = 2a''\xi(C_6\beta^2 + e\dot{e})$$

$$\dot{\eta} = (\dot{e} + e\dot{\xi}/\xi)s\xi$$

$$\dot{\psi}/\psi = -\eta\dot{\eta}\psi^{-2}$$

$$\dot{C}_o/C_o = C_6 + 4\dot{\xi}/\xi - \dot{\alpha}/\alpha - 7\dot{\psi}/\psi$$

$$\dot{C}_1/C_1 = \dot{n}/n'' + 4\dot{\alpha}/\alpha + \dot{C}_o/C_o$$

$$D_9 = 6\eta + 20e + 15e\eta^2 + 68e^2\eta$$

$$D_{10} = 20\eta + 5\eta^3 + 17e + 68e\eta^2$$

$$D_{11} = 72\eta + 18\eta^3$$

$$D_{12} = 30\eta + 10\eta^3$$

$$D_{13} = 5 + \frac{45}{4}\eta^2$$

$$D_{14} = \dot{\xi}/\xi - 2\dot{\psi}/\psi$$

$$D_{15} = 2(C_6 + e\dot{e}\beta^{-2})$$

$$\dot{D}_1 = D_1(D_{14} + D_{15})$$

$$\dot{D}_3 = \dot{\eta} D_{12}$$

$$\dot{D}_4 = \dot{\eta} D_{13}$$

$$\dot{D}_5 = D_5 D_{14}$$

$$\dot{C}_2 = B_2(\dot{D}_1 D_3 + D_1 \dot{D}_3)$$

$$\dot{C}_3 = B_3(\dot{D}_5 D_4 + D_5 \dot{D}_4)$$

$$\dot{\omega} = -\frac{3}{2}\frac{n'' k_2}{a''^2 \beta^4}(1 - 5\theta^2)$$

$$D_{16} = D_9\dot{\eta} + D_{10}\dot{e} + B_1(\dot{D}_1 D_2 + D_1\dot{D}_2) + \dot{C}_2 \cos 2\omega + \dot{C}_3 \sin\omega + \dot{\omega}(C_3\cos\omega - 2C_2 \ldots$$

$$\ddot{n}_o = \dot{n}\dot{C}_1/C_1 + C_1 D_{16}$$

$$\begin{aligned}\ddot{e}_o = \;& \dot{e}\dot{C}_o/C_o - C_o\left\{\left(4 + 3\eta^2 + 30e\eta + \frac{31}{2}e^2 + 21e^2\eta^2\right)\dot{\eta} + (5 + 15\eta^2 + 31e\eta + 14e \ldots\right.\\ & +B_1\left[\dot{D}_1 D_6 + D_1\dot{\eta}\left(30 + \frac{135}{2}\eta^2\right)\right] + B_2\left[\dot{D}_1 D_7 + D_1\dot{\eta}\left(5 + \frac{75}{2}\eta^2\right)\right]\cos\omega\\ & \left. +B_3\left[\dot{D}_5 D_8 + D_5\eta\dot{\eta}\left(\frac{27}{2} + 4\eta^2\right)\right]\sin\omega + \dot{\omega}(C_5\cos\omega - 2C_4\sin 2\omega)\right\}\end{aligned}$$

$$D_{17} = \ddot{n}/n'' - (\dot{n}/n'')^2$$

$$\ddot{\xi}/\xi = 2(\dot{\xi}/\xi - C_6)\dot{\xi}/\xi + 2a''\xi\left(\frac{1}{3}D_{17}\beta^2 - 2C_6 e\dot{e} + \dot{e}^2 + e\ddot{e}\right)$$

$$\ddot{\eta} = (\ddot{e} + 2\dot{e}\dot{\xi}/\xi)s\xi + \eta\ddot{\xi}/\xi$$

$$D_{18} = \ddot{\xi}/\xi - (\dot{\xi}/\xi)^2$$

$$D_{19} = -(\dot{\psi}/\psi)^2(1 + \eta^{-2}) - \eta\ddot{\eta}\psi^{-2}$$

$$
\begin{aligned}
\dddot{n}_o = \; & \dot{n}\left[\frac{4}{3}D_{17} + 3\dot{e}^2\alpha^{-2} + 3e\ddot{e}\alpha^{-2} - 6(\dot{\alpha}/\alpha)^2 + 4D_{18} - 7D_{19}\right] \\
& + \ddot{n}\dot{C}_1/C_1 + C_1\Big\{ D_{16}\dot{C}_1/C_1 + D_9\ddot{\eta} + D_{10}\ddot{e} + \dot{\eta}^2(6 + 30e\eta + 68e^2) \\
& + \dot{\eta}\dot{e}(40 + 30\eta^2 + 272e\eta) + \dot{e}^2(17 + 68\eta^2) \\
& + B_1[\ddot{D}_1 D_2 + 2\dot{D}_1\dot{D}_2 + D_1(\ddot{\eta}D_{11} + \dot{\eta}^2(72 + 54\eta^2))] \\
& + B_2[\ddot{D}_1 D_3 + 2\dot{D}_1\dot{D}_3 + D_1(\ddot{\eta}D_{12} + \dot{\eta}^2(30 + 30\eta^2))]\cos 2\omega \\
& + B_3\left[(\dot{D}_5 D_{14} + D_5(D_{18} - 2D_{19}))D_4 + 2\dot{D}_4\dot{D}_5 + D_5\left(\ddot{\eta}D_{13} + \frac{45}{2}\eta\dot{\eta}^2\right)\right]\sin\omega \\
& + \dot{\omega}[(7C_6 + 4e\dot{e}\beta^{-2})(C_3\cos\omega - 2C_2\sin 2\omega) + 2C_3\cos\omega \\
& - 4C_2\sin 2\omega - \dot{\omega}(C_3\sin\omega + 4C_2\cos 2\omega)]\Big\}
\end{aligned}
$$

$$
p = \frac{2\ddot{n}_o{}^2 - \dot{n}_o\,\dddot{n}_o}{\ddot{n}_o{}^2 - \dot{n}_o\,\dddot{n}_o}
$$

$$
\gamma = -\frac{\dddot{n}_o}{\ddot{n}_o}\frac{1}{(p-2)}
$$

$$
n_D = \frac{\dot{n}_o}{p\gamma}
$$

$$
q = 1 - \frac{\ddot{e}_o}{\dot{e}_o\gamma}
$$

$$
e_D = \frac{\dot{e}_o}{q\gamma}
$$

where all quantities are epoch values.

The secular effects of atmospheric drag and gravitation are included by

$$
n = n_o'' + n_D[1 - (1 - \gamma(t - t_o))^p]
$$

$$
e = e_o + e_D[1 - (1 - \gamma(t - t_o))^q]
$$

$$\Omega = \Omega_o'' + \dot{\Omega}_1 \left[(t - t_o) + \frac{7}{3}\frac{1}{n_o''}Z_1\right] + \dot{\Omega}_2(t - t_o)$$

$$M = M_o + n_o''(t - t_o) + Z_1 + \dot{M}_1 \left[(t - t_o) + \frac{7}{3}\frac{1}{n_o''}Z_1\right] + \dot{M}_2(t - t_o)$$

where

$$Z_1 = \frac{\dot{n}_o}{p\gamma}\left\{(t - t_o) + \frac{1}{\gamma(p+1)}[(1 - \gamma(t - t_o))^{p+1} - 1]\right\}.$$

If drag is very small ($\frac{\dot{n}}{n_o''}$ less than 1.5×10^{-6}/min) then the secular equations for n, e
should be replaced by

$$n = n_o'' + \dot{n}(t - t_o)$$

$$e = e_o'' + \dot{e}(t - t_o)$$

$$Z_1 = \frac{1}{2}\dot{n}_o(t - t_o)^2$$

where $(t - t_o)$ is time since epoch and where

$$\dot{e} = -\frac{2}{3}\frac{\dot{n}_o}{n_o''}(1 - e_o).$$

Solve Kepler's equation for E by using the iteration equation

$$E_{i+1} = E_i + \Delta E_i$$

with

$$\Delta E_i = \frac{M + e\sin E_i - E_i}{1 - e\cos E_i}$$

and

$$E_1 = M + e\sin M + \frac{1}{2}e^2\sin 2M.$$

The following equations are used to calculate preliminary quantities needed for the sho
periodics.

$$a = \left(\frac{k_e}{n}\right)^{\frac{2}{3}}$$

$$\beta = (1 - e^2)^{\frac{1}{2}}$$

$$\sin f = \frac{\beta \sin E}{1 - e\cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e\cos E}$$

$$u = f + \omega$$

$$r'' = \frac{a\beta^2}{1 + e\cos f}$$

$$\dot{r}'' = \frac{nae}{\beta}\sin f$$

$$(r\dot{f})'' = \frac{na^2\beta}{r}$$

$$\delta r = \frac{1}{2}\frac{k_2}{a\beta^2}[(1-\theta^2)\cos 2u + 3(1-3\theta^2)] - \frac{1}{4}\frac{A_{3,0}}{k_2}\sin i_o \sin u$$

$$\delta \dot{r} = -n\left(\frac{a}{r}\right)^2\left[\frac{k_2}{a\beta^2}(1-\theta^2)\sin 2u + \frac{1}{4}\frac{A_{3,0}}{k_2}\sin i_o\cos u\right]$$

$$\delta I = \theta\left[\frac{3}{2}\frac{k_2}{a^2\beta^4}\sin i_o\cos 2u - \frac{1}{4}\frac{A_{3,0}}{k_2 a\beta^2}e\sin\omega\right]$$

$$\delta(r\dot{f}) = -n\left(\frac{a}{r}\right)^2\delta r + na\left(\frac{a}{r}\right)\frac{\sin i_o}{\theta}\delta I$$

$$\delta u = \frac{1}{2}\frac{k_2}{a^2\beta^4}\left[\frac{1}{2}(1-7\theta^2)\sin 2u - 3(1-5\theta^2)(f - M + e\sin f)\right]$$

$$\delta\lambda = \frac{1}{2}\frac{k_2}{a^2\beta^4}\left[\frac{1}{2}(1+6\theta-7\theta^2)\sin 2u - 3(1+2\theta-5\theta^2)(f-M+e\sin f)\right]$$
$$+\frac{1}{4}\frac{A_{3,0}}{k_2 a\beta^2}\sin i_o\left[\frac{e\theta}{1+\theta}\cos\omega - (2+e\cos f)\cos u\right]$$

The short-period periodics are added to give the osculating quantities

$$r = r'' + \delta r$$

$$\dot{r} = \dot{r}'' + \delta\dot{r}$$

$$r\dot{f} = (r\dot{f})'' + \delta(r\dot{f})$$

$$y_4 = \sin\frac{i_o}{2}\sin u + \cos u\sin\frac{i_o}{2}\delta u + \frac{1}{2}\sin u\cos\frac{i_o}{2}\delta I$$

$$y_5 = \sin\frac{i_o}{2}\cos u - \sin u\sin\frac{i_o}{2}\delta u + \frac{1}{2}\cos u\cos\frac{i_o}{2}\delta I$$

$$\lambda = u + \Omega + \delta\lambda.$$

Unit orientation vectors are calculated by

$$U_x = 2y_4(y_5\sin\lambda - y_4\cos\lambda) + cos\lambda$$

$$U_y = -2y_4(y_5\cos\lambda + y_4\sin\lambda) + sin\lambda$$

$$U_z = 2y_4\cos\frac{I}{2}$$

$$V_x = 2y_5(y_5\sin\lambda - y_4\cos\lambda) - sin\lambda$$

$$V_y = -2y_5(y_5\cos\lambda + y_4\sin\lambda) + cos\lambda$$

$$V_z = 2y_5\cos\frac{I}{2}$$

where

Position and velocity are given by

$$\mathbf{r} = r\mathbf{U}$$

$$\dot{\mathbf{r}} = \dot{r}\mathbf{U} + r\dot{f}\mathbf{V}.$$

A FORTRAN IV computer code listing of the subroutine SGP8 is given below.

```
*      SGP8                                                  14 NOV 80
      SUBROUTINE SGP8(IFLAG,TSINCE)
      COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT2O,
     1           XNDD6O,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
     1           XJ3,XKE,XKMPER,XMNPDA,AE
      DOUBLE PRECISION EPOCH, DS50
      DATA RHO/.15696615/

      IF (IFLAG .EQ. 0) GO TO 100

*      RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
*      FROM INPUT ELEMENTS --------- CALCULATE BALLISTIC COEFFICIENT
*      (B TERM) FROM INPUT B* DRAG TERM

      A1=(XKE/XNO)**TOTHRD
      COSI=COS(XINCL)
      THETA2=COSI*COSI
      TTHMUN=3.*THETA2-1.
      EOSQ=EO*EO
      BETAO2=1.-EOSQ
      BETAO=SQRT(BETAO2)
      DEL1=1.5*CK2*TTHMUN/(A1*A1*BETAO*BETAO2)
      AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
      DELO=1.5*CK2*TTHMUN/(AO*AO*BETAO*BETAO2)
      AODP=AO/(1.-DELO)
      XNODP=XNO/(1.+DELO)
      B=2.*BSTAR/RHO

*      INITIALIZATION

      ISIMP=0
      PO=AODP*BETAO2
      POM2=1./(PO*PO)
      SINI=SIN(XINCL)
      SING=SIN(OMEGAO)
      COSG=COS(OMEGAO)
      TEMP=.5*XINCL
      SINIO2=SIN(TEMP)
      COSIO2=COS(TEMP)
      THETA4=THETA2**2
      UNM5TH=1.-5.*THETA2
      UNMTH2=1.-THETA2
      A3COF=-XJ3/CK2*AE**3
      PARDT1=3.*CK2*POM2*XNODP
      PARDT2=PARDT1*CK2*POM2
      PARDT4=1.25*CK4*POM2*POM2*XNODP
```