# Determining the Loss Factor by the Power Input Method (PIM), Part 1: Numerical Investigation

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*Damping must be accurately determined in the design and/or optimization of vehicle and aircraft trim. Yet, owing to the complexity of the dynamic interaction among the components in trimmed panel systems, until now it has been difficult to obtain reliable damping estimates. In this work, the power input method (PIM), which compares dissipated energy to the structure's strain energy, was evaluated as a damping evaluation tool. Numerical simulations were used to analyze the lumped mass system (with custom software) and the plates (with commercial finite element software) and consequently to evaluate the assumptions required to apply the PIM. It was thus possible to find a way of minimizing the effect of the assumptions on the results, whose importance would be fundamental in the successive phase involving the experimental application of the method.*

## Introduction

A common practice of manufacturers is to insert viscoelastic, porous materials, and heavy septum layers to reduce vibration and noise level in vehicle and aircraft nude bodies. Since damping is the main factor in reducing the resonance responses and increasing attenuation as the distance from the excitation area increases, damping measurements are essential elements in the design and/or optimization of trim in terms of damping effects. Yet, owing to the complexity of the dynamic interaction among the components in trimmed panel systems, obtaining reliable damping estimates has, up to now, been a problem (Plunt, 1985). The experimental methods currently used to determine structural damping include:

1. *Modal analysis curve fitting methods.* These methods determine the loss factors for individual vibration modes on measured frequency response function data. However, it is even harder to obtain accurate predictions with the high modal densities as the frequencies and damping levels increase.
2. *Decay- or reverberation-time method.* This simple method determines the loss factors in structural acoustics by plotting the decay of the logarithm of the RMS-magnitude of the free response. Complications arise when the modes exhibit diverse loss factors and/or when the analysis frequency band contains a large number of modes. A straight line is applied to fit to the decay curve. However, if the curve oscillates or splits into several straight line segments, the fit is not simple and the operator has to select the suitable decay rate or reverberation time. Bias error may occur even when the decay curve is smooth and linear or may result from a narrow analysis bandwidth in well-damped systems owing to filter decay.
3. *Power input method (PIM).* This method compares the dissipated energy to the structure's maximum strain energy. No serious bias errors are introduced, provided the response measurements are correctly weighted by each measurement point's corresponding partial mass. The result of the measurement is fundamentally unbiased at the natural frequencies of the single, well separated modes or with frequency-band averaged loss factors over many modes. Some bias and random errors may, however, be introduced by the measurement method.

This paper presents a numerical method for evaluating the PIM approximations. The development of an experimental procedure will be illustrated in a forthcoming work.

## Power Input Method (PIM)

The loss factor used to describe structural damping in the literature (Heylen et al., 1994; Jacobsen, 1991; and Ungar et al., 1992) is defined as follows

$$\eta(\omega) = \frac{\Delta E}{E_{pot}} \tag{1}$$

With a stationary force excitation at one of a structure's points, there is a stationary vibration response field, meaning that an equal amount of energy must be input from the excitation source to compensate for the dissipated energy from damping. The result is

$$\Delta E = E_{in} \tag{2}$$

Although the loss factor can be computed on the basis of (1) if we know $E_{in}$ and $E_{pot}$, neither energy can be measured directly. However, the energy input from an excitation source acting on the structure can be computed from simultaneous measurement of force and velocity at the excitation point as (Plunt, 1985)

$$E_{in}(\omega) = \frac{1}{2\omega}\,\mathrm{Re}[h_{FF}(\omega)]S_F(\omega) \tag{3}$$

The estimation of the denominator of Eq. (1) requires three approximations that will be explained in detail later on:

(1) replacement of the strain energy with kinetic energy
(2) evaluation of the kinetic energy with a reduced number of measurements
(3) assumption of a linear system.

### Approximation 1: Replacement of Strain Energy with Kinetic Energy

Since strain energy cannot be calculated from experimental acceleration and force measurements, we replaced it with kinetic energy $E_{kin}$ that can be expressed as

$$E_{kin}(\omega) = \tfrac{1}{2}\int_V \rho S_V(\omega)\,dV \tag{4}$$

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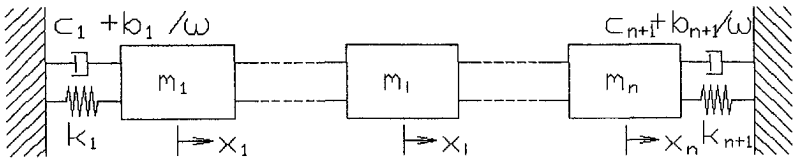


**Fig. 1 System with *n* degrees of freedom with a viscohysteretic dashpot**

**Approximation 2: Evaluation of the Kinetic Energy with a Limited Number of Measurements**

To evaluate the kinetic energy, the volume integral is estimated with a limited number $N$ of velocity measurements distributed over the whole structure. Each measuring point represents a portion of the structure which hereon shall be termed "portion." Equation (4) thus becomes

$$E_{kin}(\omega) = \tfrac{1}{2} \sum_{i=1}^{N} mp_i S_{Vi}(\omega) \tag{5}$$

**Approximation 3: Assumption of a Linear System**

With the assumption of a linear system

$$|h_{iF}(\omega)|^2 = \frac{S_{Vi}(\omega)}{S_F(\omega)} \tag{6}$$

Combining (1), (2), (3), (5), and (6) yields

$$\eta(\omega) = \frac{\mathrm{Re}[h_{FF}(\omega)]}{\omega \sum_{i=1}^{N} mp_i |h_{iF}(\omega)|^2} \tag{7}$$

The measuring points must be uniformly distributed to minimize the effects of approximating the volume integral with a summation. By assuming optimum distribution, i.e., one that allows an equal mass to be associated to all the measuring points, we can treat factor $mp_i$ as a constant. Equation (7) then becomes the loss factor estimation formula

$$\eta(\omega) = \frac{\mathrm{Re}[h_{FF}(\omega)]}{m\omega \sum_{i=1}^{N} |h_{iF}(\omega)|^2} \tag{8}$$

in which all of the necessary terms can be experimentally determined. Once the loss factor formula has been derived, we can proceed to analyze the approximations and attempt to find a way of minimizing their effects.

## Numerical Simulation with Lumped Mass Systems

To evaluate the error derived from the substitution of kinetic energy for strain energy, let us consider a system with $n$ degrees of freedom exhibiting the viscohysteretic dashpot illustrated in Fig. 1.

The system is characterized by the three matrixes: mass [**M**], stiffness [**K**], and damping [**C**] (Rao, 1990; Ewins, 1994):

$$[\mathbf{M}] = \begin{bmatrix} m_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & m_3 & \cdots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdots & 0 & m_n \end{bmatrix} \quad [\mathbf{K}] = \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 & \cdots & 0 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 & \cdots & 0 & 0 \\ 0 & -k_3 & (k_3 + k_4) & \cdots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdots & -k_n & (k_n + k_{n+1}) \end{bmatrix}$$

## Nomenclature

$b_i$ = hysteretic damping constant of the lumped mass system's $i$th damper (Fig. 1)
[**C**] = damping matrix
$c_i$ = viscous damping constant of the lumped mass system's $i$th damper (Fig. 1)
$E_{in}$ = input energy
$E_{kin}$ = kinetic energy
$E_{pot}$ = strain energy (also "elastic potential energy")
{**F**} = force vector
$h_{FF}(\omega)$ = point mobility spectrum (velocity/force) at the excitation point
$h_{iF}(\omega)$ = transfer mobility spectrum (velocity/force) between the excitation and $i$ points
[**K**] = stiffness matrix
$k_i$ = spring constant of the $i$th spring of the lumped mass system (Fig. 1)
[**M**] = mass matrix
$m_i$ = $i$th mass of the lumped mass system (Fig. 1)
$mp_i$ = $i$th partial mass of the portion of the structure associated with measuring point $i$
$Q_i$ = generalized force of the $i$th mode
{**q**} = generalized coordinates (Rao, 1990)
$S_F(\omega)$ = power spectral density of the excitation force
$S_v(\omega)$ = power spectral density of the velocity vector
$S_{Vi}(\omega)$ = power spectral density of the velocity vector at point $i$
{**x**} = physical coordinates

**Greek**

$\Delta E$ = energy dissipated during a harmonic displacement cycle
$\eta_{kin}$ = approximate loss factor using kinetic energy
$\eta_{pot}$ = exact loss factor using strain energy
$\rho$ = density of structure at $dV$
[$\Phi$] = matrix of the modal mass scaled mode shapes
$\omega$ = angular frequency
[$\omega^2$] = diagonal matrix of the natural frequencies squared

$$[\mathbf{C}] = \begin{bmatrix} \left(c_1 + \frac{b_1}{\omega} + c_2 + \frac{b_2}{\omega}\right) & -\left(c_2 + \frac{b_2}{\omega}\right) & 0 & \cdots & 0 & 0 \\ -\left(c_2 + \frac{b_2}{\omega}\right) & \left(c_2 + \frac{b_2}{\omega} + c_3 + \frac{b_3}{\omega}\right) & -\left(c_3 + \frac{b_3}{\omega}\right) & \cdots & 0 & 0 \\ 0 & -\left(c_3 + \frac{b_3}{\omega}\right) & \left(c_3 + \frac{b_3}{\omega} + c_4 + \frac{b_4}{\omega}\right) & \cdots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdots & -\left(c_n + \frac{b_n}{\omega}\right) & \left(c_n + \frac{b_n}{\omega} + c_{n+1} + \frac{b_{n+1}}{\omega}\right) \end{bmatrix} \quad (9)$$

Since the dampers and springs were positioned analogously in the lumped mass system, we assumed that [**C**] was the same shape as [**K**] (Rao, 1990) Supposing the displacement and velocity vectors as

$$\{\mathbf{x}\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \quad \{\dot{\mathbf{x}}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{Bmatrix} \quad (10)$$

the dissipated, kinetic, and strain energies can be expressed as follows (Rao, 1990)

$$2\Delta E = \{\mathbf{x}\}^T[\mathbf{C}]\{\dot{\mathbf{x}}\} \quad (11)$$

$$2E_{kin} = \{\dot{\mathbf{x}}\}^T[\mathbf{M}]\{\dot{\mathbf{x}}\} \quad (12)$$

$$2E_{pot} = \{\mathbf{x}\}^T[\mathbf{K}]\{\mathbf{x}\} \quad (13)$$

It is thus possible to evaluate the exact loss factor using the strain energy, $\eta_{pot}$, and the approximate loss factor using the kinetic energy, $\eta_{kin}$,

$$\eta_{pot}(\omega) = \frac{\Delta E}{E_{pot}} \quad (14)$$

$$\eta_{kin}(\omega) = \frac{\Delta E}{E_{kin}} \quad (15)$$

With the three system matrixes ([**M**], [**K**], and [**C**]) and the force vector {**F**}

$$\{\mathbf{F}\} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_v \end{Bmatrix} \quad (16)$$

a simple Matlab program could calculate the displacement vector {**x**}. First, to evaluate the generalized coordinates {**q**} (Rao, 1990) we used the equation of motion

$$\{\ddot{\mathbf{q}}\} + \lfloor \boldsymbol{\omega}^2 \rfloor \{\mathbf{q}\} = [\boldsymbol{\Phi}]\{\mathbf{F}\} \quad (17)$$

obtained by means of the modal transformation

$$\{\mathbf{x}\} = [\boldsymbol{\Phi}]\{\mathbf{q}\} \quad (18)$$

Equation (17) denotes a set of $n$ uncoupled differential equations of the second order. Assuming a solution of the form

$$\{\mathbf{q}(t)\} = \{\mathbf{q}\}e^{j\omega t} \quad (19)$$

Equation (17) becomes, for each $i$th mode,

$$q_i(\omega_i^2 - \omega^2) = Q_i \quad i = 1, 2, \ldots, n \quad (20)$$

where, if the external forces and the eigenvectors are known, the generalized force $Q_i$ of the $i$th mode can be evaluated. From (20), we can evaluate the generalized coordinates and thus, from (18), the physical coordinate {**x**}.

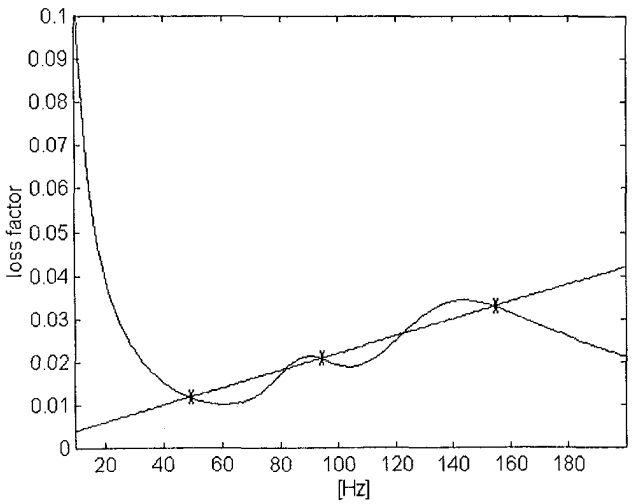


**Fig. 2 Exact (straight line) and approximate (curved line) loss factors for a system with three lumped masses**

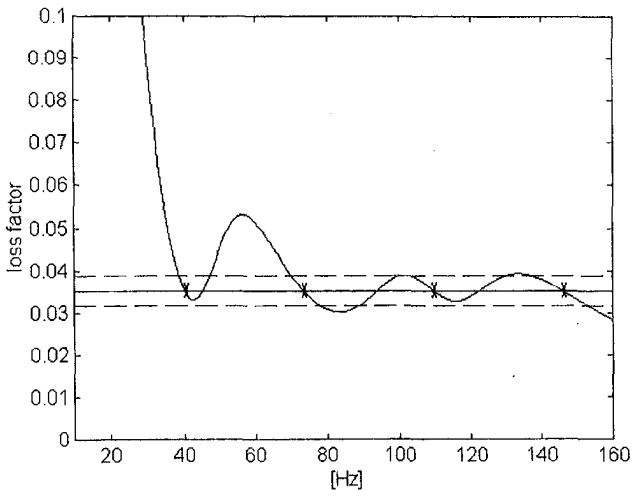


**Fig. 3 Exact (straight line) and approximate (curved line) loss factors for a system with equidistant natural frequencies; exact loss factor error ±10 percent (dashed line)**

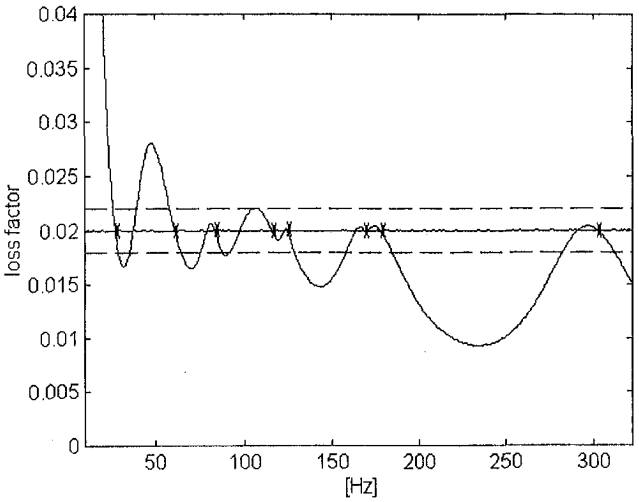


**Fig. 4 Exact (straight line) and approximate (curved line) loss factors for a system with eight lumped masses; exact loss factor error ±10 percent (dashed line)**

Finally, using the physical coordinates, the program can calculate the exact and approximate loss factors in relation to the frequency. The exact and approximate loss factors for a system with three lumped masses are plotted in Fig. 2.

Since the kinetic and potential energies are equal at the resonance frequencies, the exact and approximate loss factors from Eqs. (14) and (15) coincide at these frequencies. A characteristic wave pattern—which has been confirmed by repeated lumped mass simulations—has been observed between natural frequencies.

The exact and approximate loss factors for a system with four lumped masses with equidistant natural frequencies are plotted in Fig. 3, in which the solid line indicates the exact value and the dashed lines indicate an error of ±10 percent. In view of the decrease in error with increasing frequency, experimental samples whose first natural frequencies were below the low limit of the frequency range were selected for the experimental phase to remove the frequencies where we had encountered the highest errors in previous testing.

The exact and approximate loss factors for a system with eight lumped masses are plotted in Fig. 4. Since it was observed that the error continued to decrease as the natural frequencies became closer, i.e., as the modal density increased, the structure's natural frequencies were kept as equidistant as possible in the successive experimental phase.

The exact and approximate loss factors for the systems with eight lumped masses are plotted in Fig. 5 in relation to the variations in the damping level obtained by varying $c_i$ in (9). Since the percentage error does not depend on the damping level, this error may be considered independent of the characteristics of the test panel material.

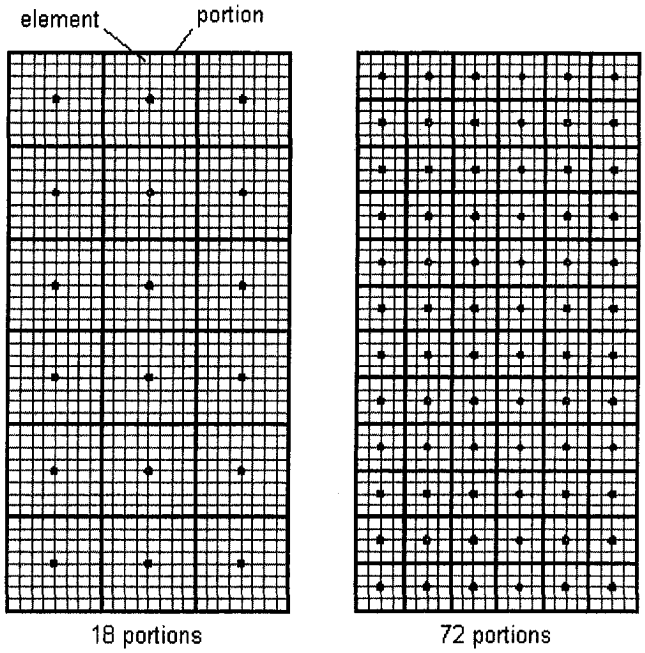


**Fig. 6 Flat plate with shell63 Ansys elements and portions**

## Finite Element Simulation on Plates

Further investigation of the second approximation (discretization) introduced by the PIM was conducted by carrying out a finite element (FE) simulation on flat plates with ANSYS Shell63 elements (Fig. 6). The results of a modal analysis, with the assumption of a constant loss factor of 0.02, were then used to evaluate the frequency response functions (FRFs) needed to apply Eq. (8).

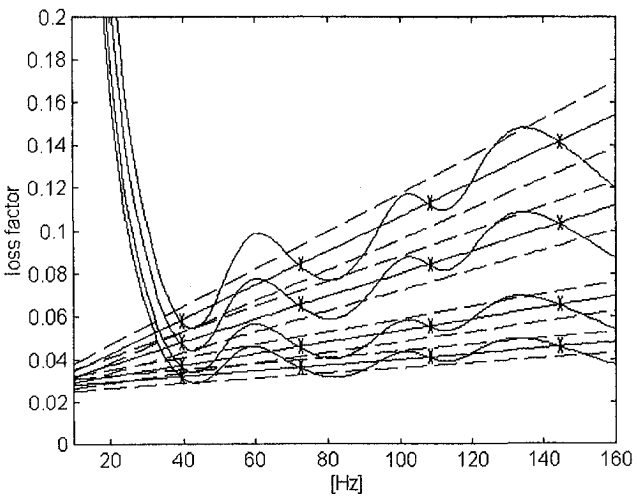


**Fig. 5 Exact (straight line) and approximate (curved line) loss factors in relation to variations in the damping level; exact loss factor error ±10 percent (dashed line)**

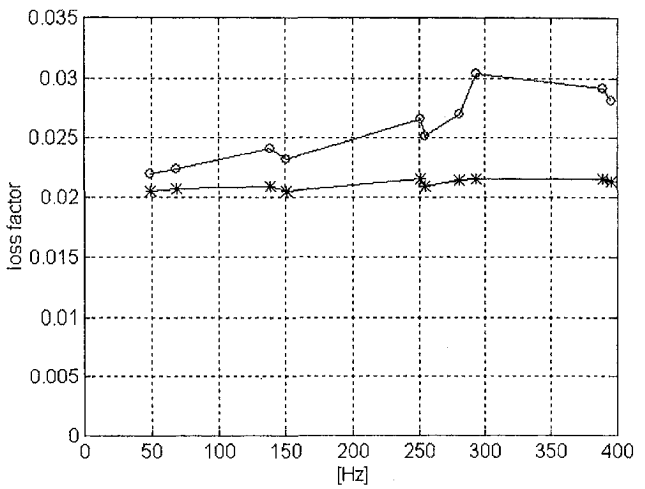


**Fig. 7 Estimated loss factors with 18 (○) and 72 (*) portions (exact loss factor = 0.02)**

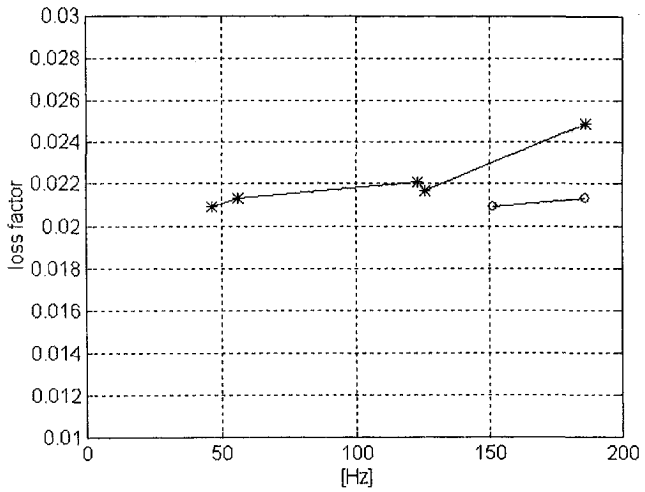


**Fig. 8 Loss factors of two different-size plates with two coincident natural frequencies (small plate = ○; large plate = *)**

The loss factors estimated at the resonance frequency with 18 and 72 portions (Fig. 6) are plotted in Fig. 7. We can observe that:

- With the number of portions being equal, the error increased as the frequency increased. This increase resulted from the increase in mode shape complexity with the frequency, which in turn caused an increase in the effects of the kinetic energy approximation, since the energy was measured at only a single point per portion.
- The error decreased as the discretization increased.
- The decrease in the discretization error was greater as the frequency increased. This decrease occurred, as in the first observation, because of the increase in mode shape complexity as the frequency increased.

The loss factors calculated for two different-size plates, which, however, have the same side-to-side ratios and thus share the same mode shape sequence, are shown in Fig. 8. The small plate's second frequency, $\omega_{22}$, coincides with the large plate's fifth frequency, $\omega_{51}$: At the fifth frequency, $\omega = \omega_{22} = \omega_{51}$, the large plate's discretization-generated error is so much greater because its mode shape is more complex than that associated to $\omega_{22}$.

## Conclusions

The assumptions required to apply the power input method (PIM) in the experimental measurement of the loss factor versus frequency have been analyzed in this work. The proposed numerical simulations also provide insight into how to significantly reduce the errors introduced by the assumptions. It was found that the approximation in the loss factor evaluation introduced by using kinetic energy in lieu of potential energy in the loss factor formula produced errors at frequencies other than resonance frequency. The error decreased as the natural frequencies became closer, with increasing modal density. Consequently, it was determined that test specimens should have natural frequencies as equidistant as possible, with their first natural frequencies being lower than the low limit of the frequency range being examined. In addition, a suitable number of measuring points should be provided to reduce the discretization error, i.e., the approximation due to the evaluation of the kinetic energy with a limited number of portions. The optimum number of portions is will be determined through finite element simulation and experimentally verified in the successive phase. The forthcoming experimental phase will be devoted to further investigating the approximations described in this work and evaluating the suggestions that have developed from it to develop a reliable experimental procedure for the loss factor evaluation.

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