MA2330 Exam 1 Name: KEY

1. For $A\,x = b$ the reduced augmented matrix is

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 0 & 0 & -5-\alpha \\ 0 & 0 & 0 & 0 & 1 & 0 & 6+\alpha \\ 0 & 0 & 0 & 0 & 0 & 1 & 6+3\alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & 6-\alpha \end{pmatrix} \begin{matrix} 3 \\ -11 \\ 12 \\ 24 \\ 0 \end{matrix}$$

Fill in the gaps. The system is consistent if $\alpha = \boxed{6}$.

The pivot variables are $\boxed{1,3,5,6}$. The free variables are

$\boxed{2,4}$.

The solution is $x = \begin{pmatrix} 3 \\ 0 \\ -11 \\ 0 \\ 12 \\ 24 \end{pmatrix} + \boxed{x_2} \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \boxed{x_4} \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

2. Fill in the gaps in the row reduction

$$\begin{pmatrix} 1 & -2 & -1 & 5 & -1 & -1 & 5 & 3 \\ 0 & 3 & -2 & 2 & 4 & 5 & -4 & 3 \\ 0 & 0 & -5 & 4 & -3 & -3 & -3 & 4 \\ 0 & 0 & -25 & -1 & -5 & -4 & 5 & -2 \\ 0 & 0 & 10 & 4 & -2 & 5 & 3 & 4 \\ 0 & 0 & 25 & -2 & 2 & -1 & -3 & 2 \\ 0 & 0 & -25 & -4 & 3 & 4 & -5 & 3 \\ 0 & 0 & 15 & 2 & -1 & 3 & 4 & -4 \end{pmatrix} \longrightarrow$$

$$\begin{array}{cl} \begin{pmatrix} 1 & -2 & -1 & 5 & -1 & -1 & 5 & 3 \\ 0 & 3 & -2 & 2 & 4 & 5 & -4 & 3 \\ 0 & 0 & -5 & 4 & -3 & -3 & -3 & 4 \\ 0 & 0 & 0 & -21 & 10 & 11 & 20 & -22 \\ 0 & 0 & 0 & 12 & -8 & -1 & -3 & 12 \\ 0 & 0 & 0 & 18 & -13 & -16 & -18 & 22 \\ 0 & 0 & 0 & -24 & 18 & 19 & 10 & -17 \\ 0 & 0 & 0 & 14 & -10 & -6 & -5 & 8 \end{pmatrix} & \begin{array}{l} \text{row}_1 \\ \text{row}_2 \\ \text{row}_3 \\ \text{row}_4 \to \text{row}_4 + (-5)\,\text{row}_3 \\ \text{row}_5 \to \text{row}_5 + (2)\,\text{row}_3 \\ \text{row}_6 \to \text{row}_6 + (5)\,\text{row}_3 \\ \text{row}_7 \to \text{row}_7 + (-5)\,\text{row}_3 \\ \text{row}_8 \to \text{row}_8 + (3)\,\text{row}_3 \end{array} \end{array}$$

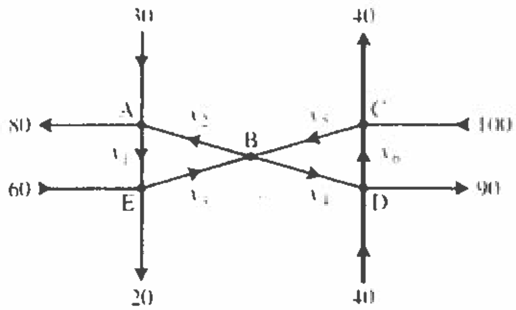
3. Write down the matrix you would row reduce to balance the chemical equation.

$x_1\, NaHCO_3 + x_2\, H_3C_6H_5O_7 \to x_3\, Na_3C_6H_5O_7 + x_4\, H_2O + x_5\, CO_2$

Name your coefficients in the order the terms appear in the reaction above. The rows should express balance in the order Sodium (Na), Hydrogen (H), Carbon (C), and Oxygen (O). Careful about the Hs in the second term

$$\begin{array}{c c}
\begin{array}{c} Na \\ H \\ C \\ O \end{array} & \left(\begin{array}{c|c|c|c|c} 1 & 0 & -3 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 \\ 1 & 6 & -6 & 0 & -1 \\ 3 & 7 & -7 & -1 & -2 \end{array}\right) \\
 & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}
\end{array}$$

4. Write down the Augmented matrix you would row reduce to find the general flow pattern for the network



A $\quad -50 - x_1 + x_2 = 0$

B $\quad -x_2 + x_3 - x_4 + x_5 = 0$

C $\quad 60 + x_6 - x_5 = 0$

$$\begin{array}{cc} \left(\begin{array}{cccccc|c} -1 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \end{array}\right) & \begin{array}{c} A \\ B \\ C \\ D \\ E \\ \end{array} \end{array}$$

D $\quad -90 + 40 - x_6 + x_4 = 0$

E $\quad 60 - 20 + x_1 - x_3 = 0$

5. A car rental company has 4 locations around Pittsburgh: N, S, E, and W.

- 90% of the cars rented at N are returned to N. The remaining 10% get returned to South.
- 80% of the cars rented at S are returned to S. Of the remaining 20% half get returned to N and the rest get returned to E.
- 70% of the cars rented at E are returned to E. The remaining 30% get returned to N.
- 80% of the cars rented at W are returned to W. The remaining 20% get returned to N.

5.1. Using the order N, S, E,W write down the transformation matrix for the cars location from one day to the next.

$$A = \begin{pmatrix} 0.9 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.8 & 0 & 0 \\ 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.8 \end{pmatrix}$$

5.2. There are originally 100 cars at N, 85 cars at S, 120 cars at E, and 123 cars at W. The initial car distribution is $x_0 = \begin{pmatrix} 100 \\ 85 \\ 120 \\ 123 \end{pmatrix}$

5.3. How are the cars distributed after n days?

$x_n = A^n x_0$ or $x_n = A x_{n-1}$

6. Row Reduce the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ -1 & 4 & 2 \\ 0 & 3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 6 & 6 \\ 0 & 3 & 3 \end{pmatrix}$ enough to determine if the columns are LI or LD

6.1. Columns are $\boxed{\text{LD}}$

6.2. Explain why. There are free variables!

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ -1 & 4 & 2 \\ 0 & 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 6 & 6 \\ 0 & 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

7. Fill in the gap of a definition.

7.1. An indexed set of vectors $\{v_1, v_2, \ldots, v_p\}$ is

$\boxed{\text{L I}}$ if the vector equation

$x_1 v_1 + x_2 v_2 + \ldots + x_p v_p = 0$

has only the trivial solution.

7.2. A transformation T is $\boxed{\text{LINEAR}}$ if

$T(U + V) = T(U) + T(V)$ and $T(c\,U) = c\,T(U)$.

7.3. Columns of a reduced echelon form with leading 1s are

$\boxed{\text{PIVOT}}$ columns.

7.4. Basic variables correspond to pivot columns. The remaining variables are $\boxed{\text{FREE}}$ variables.

7.5. The set of all linear combinations of the vectors $v_1, v_2, \ldots v_p$ is the $\boxed{\text{SPAN}}$ of the vectors.

8. Write T or F in the box.

8.1. The echelon form of a matrix is unique. F

8.2. Every matrix is row equivalent to a unique matrix in echelon form. F

8.3. If a system $A\,x = b$ has more than one solution then so does $A\,x = 0$. T

8.4. The equation $A\,x = 0$ has non-trivial solutions if there are free variables. T

8.5. If $A\,B = C$ then $C = A\,B$ T

8.6. If A and B are square and $A\,B$ is an identity matrix then $B = A^{-1}$ T

8.7. If $A\,B = 0$ then either A or B is the zero matrix F

8.8. If $A\,B$ is defined then so is $B\,A$ F

8.9. If A and B are invertible $n \times n$ matrices then $(A\,B)^{-1} = A^{-1}\,B^{-1}$ F

8.10. If A and B are invertible $n \times n$ matrices then $(A\,B)^{-1} = B^{-1}\,A^{-1}$ T

9. Fill in the blanks in matrix-matrix, matrix vector and block matrix products.

9.1. $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 6 \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 13 & 22 \\ 9 & 14 & 10 \end{pmatrix}$

9.2. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

9.3. $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} (1, 3) = \begin{pmatrix} 3 & 9 \\ 2 & 6 \\ 1 & 3 \end{pmatrix}$

9.4. Assume submatrices conform and $M = \begin{pmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{pmatrix}$ and

$B = M^2 = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$ then

$B_{11} = M_{11} M_{11}$

$B_{12} = M_{11} M_{12} + M_{12} M_{22}$

10. Compute a matrix Inverse

10.1. Write down the augmented matrix you would use to compute the inverse of $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

10.2. Compute the inverse below and record the answer

$$A^{-1} = \begin{pmatrix} 1 & 2 & -4 \\ 0 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 1 & 2 & -4 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix}$$