Sept 25

1. Diagonalize $\overset{A}{\begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 3 & 3 \\ -3 & -5-\lambda & -3 \\ 3 & 3 & 1-\lambda \end{bmatrix}$$

$\lambda = -2$ gives

$$\begin{bmatrix} 3 & 3 & 3 \\ -3 & -3 & -3 \\ 3 & 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_2 + x_3 = 0$

$x_1 = -x_2 - x_3$

$\{A + 2I\} \vec{v} = \vec{0}$

$$\vec{v} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

two evecs matching $\lambda = -2, -2$

$$\det(A - \lambda I) = (1-\lambda) \begin{vmatrix} -5-\lambda & -3 \\ 3 & 1-\lambda \end{vmatrix} - (3) \begin{vmatrix} -3 & -3 \\ 3 & 1-\lambda \end{vmatrix} + 3 \begin{vmatrix} -3 & -5-\lambda \\ 3 & 3 \end{vmatrix}$$

$$= (1-\lambda)\left((-5-\lambda)(1-\lambda) + 9\right) - 3\left(-3(1-\lambda) + 9\right) + 3\left(-9 + 3(5+\lambda)\right)$$

$$= (1-\lambda)\left(\quad " \quad \right) - 3(-3 + 9 + 3\lambda) + 3(-9 + 15 + 3\lambda)$$

$$\det(A - \lambda I) = (1-\lambda)\left(\quad " \quad \right).$$

$\lambda = 1$ is an eval

$$\{A - (1)I\} = \begin{pmatrix} 0 & 3 & 3 \\ -3 & -6 & -3 \\ 3 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$(A - I)\vec{v} = 0 \qquad \vec{v} = x_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$$\boxed{P = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad A = P \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} P^{-1}}$$