Sept 26th

Name: ____________________ ID #:______________________

As always you need to show your work. Fill in the appropriate blanks

1. A pair (λ, v) is an eigen pair if $\boxed{Av = \lambda v}$ and $\boxed{v \neq 0}$

2. A matrix A is $\boxed{\text{Diagonal..}}$ iff it has n LI eigenvectors and $A =$ $\boxed{PDP^{-1}}$

3. For $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

$(1-\lambda)^2 - 2^2 = 0 \Rightarrow 1-\lambda = \pm 2$

$\lambda = 1 \pm 2 = -1 \text{ or } 3$

3.1. Compute the eigenvalues of A $\boxed{-1, 3}$

3.2. Compute the eigenvectors of A $\boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

$D = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$

3.3. Write down the diagonalization of A $\boxed{PDP^{-1}}$

$P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$\lambda_1 = 3$

$A - 3I = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$ $\quad v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x_1 - x_2 = 0$

$\lambda_2 = -1$

$A + I = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$(1-\lambda)^2 + 2^2 = 0 \Rightarrow (1-\lambda)^2 = -4$

$\Rightarrow 1-\lambda = \pm\sqrt{-4} = \pm 2I$

$\lambda = 1 \pm 2I$

4. For $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

4.1. Compute the eigenvalues of A $\boxed{1 \pm 2I}$

$v_1 = \begin{pmatrix} 1 \\ I \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + I\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$I x_1 = x_2$

4.2. Compute the eigenvectors of A $\boxed{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \pm \begin{pmatrix} 0 \\ 1 \end{pmatrix} I}$

$-I x_1 + x_2 = 0$

$\lambda_1 = 1 + 2I$

$A - \lambda_1 Id = \begin{pmatrix} 1-(1+2I) & 2 \\ -2 & 1-(1+2I) \end{pmatrix} = \begin{pmatrix} -2I & 2 \\ -2 & -2I \end{pmatrix} \sim \begin{pmatrix} -I & 1 \\ 0 & 0 \end{pmatrix}$

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1. The $\boxed{\text{Diag.}}$ of A is $\boxed{PDP^{-1}}$ where $P = \begin{pmatrix} v_1 & \vdots & \cdots & v_n \end{pmatrix}$ and

$D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$

2. The matrix $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is a scaling by $\boxed{r = \sqrt{a^2+b^2}}$ and a rotation by $\boxed{\operatorname{atan}(b/a)}$

3. A scaling by r and a rotation by angle ϕ has matrix $A = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix}\begin{pmatrix} \cos(\phi) & \\ & \end{pmatrix}$ i

4. The matrix $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ is a scaling by $\boxed{\sqrt{1+4}}$ and a rotation by $\boxed{\operatorname{atan}(2)}$

5. For $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$

$\det(A - \lambda I) = (6-\lambda)\begin{vmatrix} 2-\lambda & 4 \\ 4 & 2-\lambda \end{vmatrix} = (6-\lambda)\left((2-\lambda)^2 - 4^2\right)$

$\quad '' \quad = 0 \Rightarrow 6-\lambda = 0$ or $(2-\lambda)^2 = 4^2$

$\lambda = 6 \qquad 2-\lambda = \pm 4$

$\lambda = 2 \pm 4 = 6, -2$

5.1. Compute the eigenvalues of A $\boxed{6, 6, -2}$

5.2. compute the eigenvectors of A $\boxed{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}$

$(A - 6I) = \begin{pmatrix} -4 & 0 & 4 \\ 0 & 0 & 0 \\ 4 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $x_1 = x_3$, x_2, x_3 free $\quad v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$A + 2I = \begin{pmatrix} 4 & 0 & 4 \\ 0 & 8 & 0 \\ 4 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$x_1 = -x_3$

$x_2 = 0$

6. The matrix $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ is a scaling by $\boxed{\sqrt{1^2+2^2}}$ and a rotation by $\boxed{\operatorname{atan}(2)}$