Name: Key ID #:

Fill in all the gaps. Show your work.

1. (XX) For $A = \begin{pmatrix} -4 & 0 & -6 & -9 \\ 0 & 2 & 0 & 0 \\ -6 & -9 & -13 & -18 \\ 6 & 6 & 12 & 17 \end{pmatrix}$

1.1. $\lambda = -1$ is an eigenvalue. Compute a basis for the eigenspace.

$$\begin{pmatrix} -3 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$A + I = \begin{bmatrix} -3 & 0 & -6 & -9 \\ 0 & 3 & 0 & 0 \\ -6 & -9 & -12 & -18 \\ 6 & 6 & 12 & 18 \end{bmatrix} \sim \begin{bmatrix} -3 & 0 & -6 & -9 \\ 0 & 1 & 0 & 0 \\ -6 & 0 & -12 & -18 \\ 6 & 0 & 12 & 18 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -2x_3 - 3x_4$

$x_2 = 0$

1.2. $\lambda = 2$ is an eigenvalue. Compute a basis for the eigenspace.

$$\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3/2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$A - 2I = \begin{bmatrix} -6 & 0 & -6 & -9 \\ 0 & 0 & 0 & 0 \\ -6 & -9 & -15 & -18 \\ 6 & 6 & 12 & 15 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 5 & 6 \\ 2 & 2 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 \\ 0 & 2 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$2x_1 = -2x_3 - 3x_4$

$x_2 = -x_3 - x_4$

1.3. $A = P.D.P^{-1}$ with $P = \begin{pmatrix} -3 & -2 & -1 & -3/2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

2. (XX) For $A = \begin{pmatrix} 4 & 1 & -9 & 13 \\ -4 & -1 & 23 & -39 \\ -4 & -4 & 13 & -26 \\ -2 & -2 & 5 & -10 \end{pmatrix}$

$$Av = \begin{pmatrix} -4-4i \\ 4+8i \\ -8 \\ -4 \end{pmatrix} = 2i \begin{pmatrix} -2+2i \\ 4-2i \\ 4i \\ 2i \end{pmatrix}$$

2.1. Is $v = \begin{pmatrix} -2+2\,i \\ 4-2\,i \\ 4\,i \\ 2\,i \end{pmatrix}$ an evec of A? Yes or No $\boxed{\text{yes}}$

If it is $\lambda = \boxed{0+2i}$

$$A - 3I = \begin{bmatrix} 1 & 1 & -9 & 13 \\ -4 & -4 & 23 & -39 \\ -4 & -4 & 10 & -26 \\ -2 & -2 & 5 & -13 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -9 & 13 \\ 0 & 0 & -13 & 13 \\ 0 & 0 & -26 & 26 \\ 0 & 0 & -13 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -9 & 13 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2.2. $\lambda = 3$ is an evec of A. Compute a basis for the eigenspace.

$$\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

$x_1 = -x_2 - 4x_4$

$x_3 = x_4$

2.3. $A = P.\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & a & -b \\ 0 & 0 & b & a \end{pmatrix}.P^{-1}$ where

$a = \boxed{0}$, $b = \boxed{2}$ is $P = \begin{pmatrix} -1 & -4 & -2 & 2 \\ 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$

$$A\cdot\begin{pmatrix}-2\\1\\1\\-1\end{pmatrix}=\begin{pmatrix}6\\-3\\-3\\3\end{pmatrix}$$

$$A-2I=\begin{bmatrix}-20&20&-80&-30\\10&-10&40&15\\10&-10&40&15\\-10&10&-40&-15\end{bmatrix}\sim\begin{bmatrix}-1&1&-4&-3/2\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$\sim\begin{bmatrix}2&-2&8&3\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$2x_1 = 2x_2-8x_3-3x_4$$

3. (XX) For $A = \begin{pmatrix}-18&20&-80&-30\\10&-8&40&15\\10&-10&42&15\\-10&10&-40&-13\end{pmatrix}$

3.1. Is $v=\begin{pmatrix}-2\\1\\1\\-1\end{pmatrix}$ an evec of A? Yes or No $\boxed{\text{yes}}$

If it is $\lambda = \boxed{-3}$

3.2. $\lambda = 2$ is an evec of A. Compute a basis for the eigenspace.

$$\begin{pmatrix}1\\1\\0\\0\end{pmatrix},\begin{pmatrix}-4\\0\\1\\0\end{pmatrix},\begin{pmatrix}-3/2\\0\\0\\1\end{pmatrix},\begin{pmatrix}\ \\ \\ \\ \ \end{pmatrix}$$

3.3. $A = P.D.P^{-1}$ with P = $\begin{pmatrix}1&-4&-3/2&-2\\1&0&0&1\\0&1&0&1\\0&0&1&-1\end{pmatrix}$ and $D=\begin{pmatrix}2&0&0&0\\0&2&0&0\\0&0&2&0\\0&0&0&-3\end{pmatrix}$