Sept 30

Name: Scln ID #:

Fill in all the gaps. Show your work.

1. Matrices A and B have the same eigenvalues if they are similar.

Matrices A and B are similar if $\boxed{A = PBP^{-1}}$ for some $\boxed{\text{invertible } P}$

2. The $\boxed{\text{diagonalization}}$ of A is $\boxed{PDP^{-1}}$ where

$P = \begin{pmatrix} v_1 & \cdots & v_n \end{pmatrix}$ and $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{pmatrix}$

3. The matrix $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is a scaling by $\boxed{r = \sqrt{a^2 + b^2}}$ and a rotation by $\boxed{\operatorname{atan}(b/a)}$

4. A scaling by r and a rotation by angle ϕ has matrix

$$A = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix}$$

5. The matrix $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ is a scaling by $\boxed{\sqrt{1+4}}$ and a rotation by $\boxed{\operatorname{atan}(2/1)}$

6 The characteristic equation of a matrix A is

$\boxed{\det(A - \lambda I) \quad = 0}$.

7. Eigenvalues are roots of $\boxed{\det(A - \lambda I) \quad = 0}$.

8. A pair (λ, v) is an eigen pair if $\boxed{Av = \lambda v}$ and

$\boxed{v \neq 0}$

9. A matrix A is $\boxed{\text{diagonalizable}}$ iff it has n LI eigenvectors ~~in~~ and

$A = \boxed{P D P^{-1}}$

10. For $A = \begin{pmatrix} 9 & -10 \\ 5 & -5 \end{pmatrix}$ $\det\begin{pmatrix} 9-\lambda & -10 \\ 5 & -5-\lambda \end{pmatrix} = -(9-\lambda)(5+\lambda)+50$
$= \lambda^2 - 4\lambda + 5 = (\lambda-2)^2 + 1$
$\lambda = 2 \pm i$

10.1. Eigenvalues of A are $\boxed{2 \pm i}$

10.2. Eigenvectors of A are $\begin{pmatrix} 10 \\ 7-i \end{pmatrix}$, $\begin{pmatrix} 10 \\ 7+i \end{pmatrix}$

$A - (2+i)I = \begin{bmatrix} 7-i & -10 \\ 5 & -7-i \end{bmatrix} \sim \begin{bmatrix} 7-i & -10 \\ 0 & 0 \end{bmatrix}$ $x_1 = 10$, $x_2 = 7-i$

11. For A = $\begin{pmatrix} -8 & 10 \\ -5 & 7 \end{pmatrix}$ $\det\begin{pmatrix} -8-\lambda & 10 \\ -5 & 7-\lambda \end{pmatrix} = -(8+\lambda)(7-\lambda)+50$
$= \lambda^2+\lambda-6 = (\lambda+3)(\lambda-2)$

11.1. Eigenvalues of A are $\boxed{2\ ,\ -3}$

$\lambda_1 = 2$ $\quad A-2I = \begin{pmatrix} -10 & 10 \\ -5 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$ $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -3$ $\quad A+3I = \begin{pmatrix} -5 & 10 \\ -5 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$ $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

11.2. Eigenvectors of A are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

12. For $A = \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}$. $\quad \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -28 \\ 12 \end{pmatrix}$

12.1. Is $v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ an evec of A? Yes No $\lambda =$ ///.

12.2. Is $v = \begin{pmatrix} 2+i \\ 3+i \end{pmatrix}$ an evec of A? Yes No $\lambda =$ //.

$A \cdot \begin{pmatrix} 2+i \\ 3+i \end{pmatrix} = \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} 2+i \\ 3+i \end{pmatrix} = \begin{pmatrix} 44+12i \\ 16+4i \end{pmatrix}$ not a multiple of v

$$\det\begin{pmatrix} -8-\lambda & 20 \\ -4 & 8-\lambda \end{pmatrix} = -(8+\lambda)(8-\lambda)+80$$
$$= -(64-\lambda^2)+80 = \lambda^2+16=0$$
$$\lambda = \pm 4i$$

For $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ we have $A = P.C.P^{-1}$ where

$a = \boxed{0}$, $b = \boxed{4}$ and P = $\begin{pmatrix} 20 & 0 \\ 8 & 4 \end{pmatrix}$

$\lambda_1 = 4i$

$$A - 4iI = \begin{bmatrix} -8-4i & 20 \\ -4 & 8-4i \end{bmatrix} \sim \begin{bmatrix} -(8+4i) & 20 \\ 0 & 0 \end{bmatrix} \quad v_1 = \begin{pmatrix} 20 \\ 8+4i \end{pmatrix}$$

12.4. A is a rotation through an angle $\boxed{\pi/2}$ followed by a uniform scaling by $\boxed{4}$

A = $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

13. For $A = \begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix}$

$$\begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

13.1. Is $v = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{2}$.

13.2. Is $v = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{}$.

13.3. Is $v = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{0}$.

13.4. $\lambda = 1$ is an eval of A compute a basis for the espace

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

13.5. $A = P.D.P^{-1}$ with P = $\begin{pmatrix} 1 & 1 & 0 \\ -1 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ 12 \end{pmatrix}$$

$\lambda = 1$

$$A - \lambda I = \begin{bmatrix} 3 & 2 & 0 \\ -4 & -3 & 0 \\ 6 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 0 \\ -1 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_2 = 0$ $\quad$ x_3 free