Sept 30th

Name: Soln   ID #:

Fill in all the gaps. Show your work.

1. Matrices A and B have the same eigenvalues if they are similar.

Matrices A and B are similar if $\boxed{A = PBP^{-1}}$ for

some $\boxed{\text{invertible } P}$

2. The $\boxed{\text{dia}}$ of A is $\boxed{A = PDP^{-1}}$ where

$P = \left(v_1 \mid \cdots \mid v_n\right)$ and $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$

3. The matrix $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is a scaling by $\boxed{r = \sqrt{a^2+b^2}}$ and a

rotation by $\boxed{\operatorname{atan}(b/a)}$

4. A scaling by r and a rotation by angle ϕ has matrix

$$A = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

5. The matrix $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ is a scaling by $\boxed{\sqrt{1+4}}$ and a

rotation by $\boxed{\operatorname{atan}(2)}$

The characteristic equation of a matrix A is

$\boxed{\det(A - \lambda I) \quad = 0}$.

7. Eigenvalues are roots of $\boxed{\det(A - \lambda I) \quad = 0}$.

8. A pair (λ, v) is an eigen pair if $\boxed{A \cdot v = \lambda v}$ and

$\boxed{v \neq 0}$

9. A matrix A is $\boxed{\text{diagonl.}}$ iff it has n LI eigenvectors in and

$A = \boxed{P D P^{-1}}$

10. For $A = \begin{pmatrix} 9 & -10 \\ 5 & -5 \end{pmatrix}$ $\det\begin{pmatrix} 9-\lambda & -10 \\ 5 & -5-\lambda \end{pmatrix} = -(9-\lambda)(5+\lambda) + 50$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1 \quad \lambda = 2 \pm i$

10.1. Eigenvalues of A are $\boxed{2 \pm i}$

$A - (2+i)I = \begin{bmatrix} 7-i & -10 \\ 5 & -7-i \end{bmatrix}$

10.2. Eigenvectors of A are $\begin{pmatrix} 10 \\ 7-i \end{pmatrix}$, $\begin{pmatrix} 10 \\ 7+i \end{pmatrix}$

$(7-i)x_1 - 10x_2 = 0$

$10x_2 = (7-i)x_1$

$x_1 = 10$

$x_2 = 7-i$

11. For A = $\begin{pmatrix} -8 & 10 \\ -5 & 7 \end{pmatrix}$ $\det\begin{pmatrix} -8-\lambda & 10 \\ -5 & 7-\lambda \end{pmatrix} = -(8+\lambda)(7-\lambda)+50$
$= \lambda^2+\lambda-6 = (\lambda+3)(\lambda-2)$
$\lambda_1 = 2$ and $\lambda_2 = -3$

11.1. Eigenvalues of A are $\boxed{2 \text{ and } -3}$

$\lambda_1 = 2$
$(A-2I) = \begin{pmatrix} -10 & 10 \\ -5 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$ $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

11.2. Eigenvectors of A are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda_2 = -3$
$(A+3I) = \begin{pmatrix} -5 & 10 \\ -5 & 10 \end{pmatrix}$ $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
$\sim \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$

12. For $A = \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}$. $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -8-20 \\ -4-8 \end{pmatrix} = \begin{pmatrix} -28 \\ -12 \end{pmatrix}$

12.1. Is $v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ an evec of A? Yes No $\lambda =$ $\boxed{\quad}$.

12.2. Is $v = \begin{pmatrix} 2+i \\ 3+i \end{pmatrix}$ an evec of A? Yes No $\lambda =$ $\boxed{\quad}$.

$A \cdot \begin{pmatrix} 2+i \\ 3+i \end{pmatrix} = \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} 2+i \\ 3+i \end{pmatrix} = \begin{pmatrix} -16-8i+20i+60 \\ -8-4i+24+8i \end{pmatrix} = \begin{pmatrix} 44+12i \\ 16+4i \end{pmatrix}$

$\det \begin{pmatrix} -8-\lambda & 20 \\ -4 & 8-\lambda \end{pmatrix} = -(8+\lambda)(8-\lambda)+80$
$= -(64-\lambda^2)+80 = \lambda^2+16=0$
$\lambda = \pm 4i$
$\lambda = 0 \pm 4i$

For $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ we have $A = P.C.P^{-1}$ where

$a = \boxed{0}$, $b = \boxed{4}$ and P = $\begin{pmatrix} 10 & 0 \\ 8 & 4 \end{pmatrix}$

$A-(4i)I$

$= \begin{pmatrix} -8-(4i) & 10 \\ -5 & 7-4i \end{pmatrix} \sim \begin{pmatrix} 8+i4 & -10 \\ 0 & 0 \end{pmatrix}$ $(8+4i)x_1 = 10x_2$

$v = \begin{pmatrix} 10 \\ 8+4i \end{pmatrix}$

12.4. A is a rotation through an angle $\boxed{\text{atan}(4/0)}$ followed by a uniform

aka $\pi/2$

scaling by $\boxed{4}$

A = $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4-2+0 \\ -4+2+0 \\ 6-3+3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

13. For $A = \begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix}$ $\quad \begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ 12 \end{pmatrix}$

13.1. Is $v = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{2}$.

13.2. Is $v = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{}$.

13.3. Is $v = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{0}$. $\quad \begin{pmatrix} 4 & 2 & 0 \\ -4 & -2 & 0 \\ 6 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

13.4. $\lambda = 1$ is an eval of A compute a basis for the espace

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

13.5. $A = P.D.P^{-1}$ with P = $\begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 3 & 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\lambda_1 = 1$

$$A-(1)I = \begin{pmatrix} 3 & 2 & 0 \\ -4 & -3 & 0 \\ 6 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 2, & 1, & 0 \\ 4, & 3, & 0 \\ 3, & 2, & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$