Name: Key ID #: ____________

Fill in all the gaps. Show your work.

1. (12) For $A = \begin{pmatrix} 4 & 3 \\ -6 & -5 \end{pmatrix}$

$(4-\lambda)(-5-\lambda)+18=0$

$\lambda^2+\lambda-2=0 \quad (\lambda+2)(\lambda-1)=0$

(6 pts) **1.1.** Eigenvalues of A are $\boxed{-2 \text{ and } 1}$

(6 pts) **1.2.** Eigenvectors of A are $\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$A+2I = \begin{bmatrix} 6 & 3 \\ -6 & -3 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \quad 2x_1 + x_2 = 0$

$x_2 = -2x_1$

$A-I = \begin{bmatrix} 3 & 3 \\ -6 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad x_1 + x_2 = 0$

2. (12) For A = $\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$

$(3-\lambda)(1-\lambda)+2=0$

$3+\lambda^2-4\lambda+2=0=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

$(\lambda-2)^2=-1$

$\lambda=2\pm i$

2.1. Eigenvalues of *A* are $\boxed{2 \pm i}$

2.2. Eigenvectors of *A* are $\begin{pmatrix} -2 \\ 1-i \end{pmatrix}, \begin{pmatrix} -2 \\ 1+i \end{pmatrix}$

$\lambda_1 = 2+i$

$[A-\lambda_1 I] = \begin{pmatrix} 1-i & 2 \\ -1 & -1-i \end{pmatrix} \sim \begin{pmatrix} 1-i & 2 \\ 0 & 0 \end{pmatrix}$

$(1-i)x_1 + 2x_2 = 0$

$v_1 = \begin{bmatrix} -2 \\ 1-i \end{bmatrix}$

$\lambda_2 = 2-i \qquad v_2 = \begin{bmatrix} -2 \\ 1+i \end{bmatrix}$

$$\begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix} \begin{pmatrix} 2+i \\ 1 \end{pmatrix} = \begin{pmatrix} -16-8i+20 \\ -8-4i+8 \end{pmatrix} = \begin{pmatrix} 4-8i \\ -4i \end{pmatrix} = -4i \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$$

3. (12 points) For $A = \begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix}$.

3.1. Is $v = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{-4i}$.

3.2. Is $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{}$.

3.3. For $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ we have $A = P.C.P^{-1}$ where

$a = \boxed{0}$, $b = \boxed{-4}$ and $P = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} -8 & 20 \\ -4 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -8+46 \\ -4+16 \end{pmatrix} = \begin{pmatrix} 32 \\ 12 \end{pmatrix} \neq \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\lambda = 0 - 4i$

$v = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -5 & 3 & 3 \\ -6 & 4 & 6 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -5+6+0 \\ -6+8+0 \\ 0+0+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

4. (14 points) For $A = \begin{pmatrix} -5 & 3 & 3 \\ -6 & 4 & 6 \\ 0 & 0 & -2 \end{pmatrix}$

4.1. Is $v = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ an evec of A? Yes No $\lambda = \boxed{1}$.

4.2. $\lambda = -2$ is an eval of A compute a basis for the espace is

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \, \\ \, \\ \, \end{pmatrix}$$

4.3. $A = P.D.P^{-1}$ with P $= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

$$(A+2I) = \begin{pmatrix} -3 & 3 & 3 \\ -6 & 6 & 6 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

two free variables
x_2 and x_3
$x_1 = x_2 + x_3$

$$(A+2I)v = 0 \Rightarrow v = x_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$