5.1 Sept 19th

1. Example: Is $v = \{1, 1\}$ an eigenvector of $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$? If it is what is the associated eigenvalue λ?

$$A\vec{v} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda \vec{v}$$

yes and $\lambda = 4$

2. Is $v = \{1, 3, 2\}$ an eigenvector of $\begin{pmatrix} 616 & 149 & -529 \\ 234 & 61 & -201 \\ 780 & 190 & -670 \end{pmatrix}$? If it is what is the associated eigenvalue λ?

$$\begin{pmatrix} 616 & 149 & -529 \\ 234 & 61 & -201 \\ 780 & 190 & -670 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \\ 10 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

yes and $\lambda = 5$

3. Is $v = \{1, 2, 2\}$ an eigenvector of $\begin{pmatrix} 616 & 149 & -529 \\ 234 & 61 & -201 \\ 780 & 190 & -670 \end{pmatrix} = A$? If it is what is the associated eigenvalue λ?

$$A\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -144 \\ -46 \\ -180 \end{bmatrix} \neq \lambda \vec{v}$$

Not an evec

4. Example: Show that $\lambda = 2$ is an eigenvalue of $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} = A$ and compute a corresponding eigenvector v.

If $\lambda = 2$ is an eval then $A\vec{v} = \lambda \vec{v}$ for some $\vec{v}$

$A\vec{v} = \lambda\vec{v} \iff A\vec{v} = \lambda I \vec{v} \iff (A - \lambda I)\vec{v} = \vec{0}$

$\begin{pmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{pmatrix} \vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad \begin{bmatrix} 3-2 & 1 \\ 1 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$ is an evec. so is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. so is $\begin{bmatrix} 0.707 \\ -0.707 \end{bmatrix}$

5. Example: $\lambda = 2$ is an eigenvalue of $\begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$ compute a basis for the corresponding eigenspace.

$$\begin{bmatrix} 4-2 & -1 & 6 \\ 2 & 1-2 & 6 \\ 2 & -1 & 8-2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$2x_1 - x_2 + 6x_3 = 0$

$2x_1 = x_2 - 6x_3$

$x_1 = \frac{1}{2}x_2 - 3x_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

$\vec{v}_1 = \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$\lambda = 2$ $\qquad \lambda = 2$