§1.5 Sept 3rd 2024

Ex 1) Does the linear system $A\,x = 0$ have non-trivial solutions when

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -3 & -6 \end{pmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_3$
$x_1 = -2x_3$
$x_3 = \text{free}$

Find a formula for all solutions.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

There are non-trivial solutions.

Ex 2) Fill in the missing value so that the linear system $A\,x = b$ with

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \text{ and } b = \begin{pmatrix} -7 \\ 5 \\ K \end{pmatrix} \quad (A|b) \sim \begin{pmatrix} 1 & 2 & 3 & -7 \\ 0 & -3 & -6 & 33 \\ 0 & -3 & -6 & 35+K \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & -7 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 0 & K+2 \end{pmatrix}$$

is consistent. Find a formula for all solutions in this case .

consistent iff $K+2=0$ i.e. $K=-2$

$$\begin{pmatrix} 1 & 2 & 3 & -7 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 15 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 15 \\ -11 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$x_1 = 15 + x_3$
$x_2 = -11 - 2x_3$
x_3 free

Ex 3) Explain what it means about the vectors a_1, a_2, and a_3 if

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$$

has a non-trivial solution for the weights x_i.

It means there are not really 3 different vectors. You do not need all 3 vectors!