Circuit realization of chaotic systems with quadratic nonlinearity using AD633 based generic topology

Kriti Suneja
Dept. of Electronics and Communication Engineering
Delhi Technological University
Delhi, India
kritisuneja@dtu.ac.in

Neeta Pandey
Dept. of Electronics and Communication Engineering
Delhi Technological University
Delhi, India
n66pandey@rediffmail.com

Rajeshwari Pandey
Dept. of Electronics and Communication Engineering
Delhi Technological University
Delhi, India
rajeshwaripandey@gmail.com

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***Abstract*—This paper presents the systematic realization of chaotic systems, with cross product or quadratic type non-linearities, using only one type of active building block, namely AD633, which is a commercially available Integrated Circuit (IC) of Analog Multiplier (AM), and few off the shelf passive components, namely resistors and capacitors. The number of AD633s required in the proposed scheme is equal to the dimension of the system. This reduces the requirement of summation/ subtraction circuits involving voltage and current mode active building blocks, thus reducing the component count significantly and make the design of chaotic circuits simple and cost effective. The simulations have been performed in LTspice design environment, and the results in the form of phase space diagrams have been presented for two chaotic systems: Rabinovich chaotic system and Lorenz chaotic system. The shape of the phase space plots, also known as attractors confirms the feasibility of the proposed approach.**

Keywords— ***Chaotic systems; quadratic non-linearity; analog multiplier; phase space diagrams***

I. INTRODUCTION

The chaotic systems have attracted significant research interest due to their potential applications in the field of encryption [1-5], secure communication [6-8], weather and financial studies [9,10], etc. Classically, chaotic systems are three-dimensional nonlinear systems and show different trajectories in phase space. The dynamics of chaotic systems is explained by three equations which primarily use time differentiation, addition, subtraction, scaling and nonlinear term generation operation. The representative equations of three-dimensional chaotic systems differ from each other in type and number of non-linear terms. The chaotic systems with quadratic non-linearity [11-15,16-29], exponential non-linearity [30], absolute non-linearity [31], etc. exist in literature.

The hardware realization of chaotic systems uses active blocks for performing addition/ subtraction/ scaling and time differentiation operation and Analog Multiplier (AM) to include quadratic nonlinearity. The realizations presented in [13,16-17,27,29] use Operational Amplifiers (OpAmps) whereas those presented in [15,18,19] are based on Current Feedback Operational Amplifier (CFOA). The OpAmp based realizations are bulkier than CFOA based ones due to flexibility of processing both current and voltage signals. Besides OpAmp/ CFOA, analog multiplier, in particular Analog Devices IC AD633, is an integral component of these realizations [13,15,16-19,27,29]. The component count is smaller than OpAmp based design. Thus, the realizations [13,15,16-19,27,29] use two different type of active building blocks to realize chaotic systems. The AD633 [32] is a versatile active block that has capability of performing addition, subtraction, scaling and generating quadratic nonlinear terms. Considering this, a methodology has been presented recently in [33] wherein minor modifications are suggested in governing equations for simplification of the governing equations and Operational Amplifier is used if AD633 is overloaded.

This work presents a generic AD633 based topology that may be customized to realize governing equations of chaotic systems having quadratic nonlinearity(ies). The distinctive features of the proposed work are as follows:

1. The total number of AD633s are equal to number of the state variables, i.e. three.

2. It doesn't require the need of introducing extra terms or any modification in the characteristic equations for the simplification, as reported in [33].

3. The capacitor used is grounded.

The rest of the manuscript comprises of sections detailing the generic structure, detailed discussion on customization for Rabinovich chaotic system followed by some more examples. Afterwards, simulation results are included for some representative systems, such as Rabinovich [20,21] and Lorenz [11] chaotic system. In last conclusions are given.

II. GENERIC AD633 BASED TOPOLOGY

A. The AD633

The AD633 is a versatile analog building block which has capability of performing subtraction and addition of voltages besides its primary function of multiplication. The AD633 is an eight pin IC, and its block diagram is shown in Fig.1.

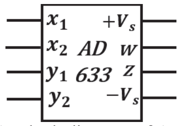


Fig.1. Block diagram of AD633

The voltage output at W terminal (V_W) is related to the voltage inputs applied at x_1, x_2, y_1, y_2 and Z terminals by (1).

$$V_W = \frac{(V_{x_1}-V_{x_2})(V_{y_1}-V_{y_2})}{10} + V_Z \tag{1}$$

where V_{x1}, V_{x2}, V_{y1}, V_{y2} and V_Z correspond to volage inputs at x_1, x_2, y_1, y_2 and Z terminals respectively.

It may be noted that AD633 can perform:

(1) Simple multiplication by connecting V_Z to ground and applying appropriate inputs to x_1, x_2, y_1 and y_2.

(2) Perform addition of voltages by applying input to 'z' terminal.

Therefore, different algebraic expressions may be obtained from (1) by applying suitable inputs to different terminals of AD633. Further, the component count may be reduced as no additional active block is required for addition/ subtraction operation.

B. *Proposed generic AD633 based topology*

A generic AD633 based topology is proposed in this subsection and the same is depicted in Fig.2. This topology can implement differential equations having linear/ non-linear terms and constants. Applying Kirchoff's Voltage Law (KVL) and Kirchoff's Current Law (KCL) in Fig.2, we obtain

$$C_{li}.\dot{V}_{li} = \frac{(V_{x1i}-V_{x2i})(V_{y1i}-V_{y2i})}{10R_{1i}} - \frac{V_{li}}{R_{2i}} - \frac{V_{li}-V_{mi}}{R_{3i}} \tag{2}$$

Here subscript i = 1,2,3 and refer to three governing equations of chaotic system wherein the input voltages (V_{x1i}, V_{x2i}, V_{y1i}, V_{y2i}, V_{mi}), resistors (R_{1i}, R_{2i} and R_{3i}) and capacitor C_{li} may be set according to the requirements.

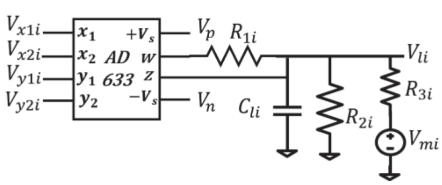


Fig.2. Generalized circuit to implement a differential equation

III. Customization of Proposed Generic Topology for Rabinovich Chaotic System

As a chaotic system is represented by three state variables, the complete design will require three instances of proposed generic topology. In this section, the customization is elucidated in detail for Rabinovich chaotic system. Seven other chaotic systems have also been worked upon and findings are comprehended.

The governing equations of Rabinovich chaotic system are given by

$$\dot{x} = hy - ax + yz \tag{3a}$$
$$\dot{y} = hx - by - xz \tag{3b}$$
$$\dot{z} = -dz + xy \tag{3c}$$

where x, y and z are the three state variables, whose time differentiations are being represented by $\dot{x}, \dot{y}$ and $\dot{z}$ respectively.

In circuit representation, the voltage across capacitor is considered as state variable. Let capacitor voltages V_{l1}, V_{l2} and V_{l3} represent state variables x, y and z respectively.

Setting V_{x2i} and V_{mi} to ground and R_{3i} to open circuit, (2) reduces to

$$C_{l1}.\dot{V}_{l1} = \frac{(V_{x11})(V_{y11}-V_{y21})}{10R_{11}} - \frac{V_{l1}}{R_{21}} \tag{4}$$

By simply setting $10R_{11}C_{l1} = 1$, $R_{21}C_{l1} = a^{-1}$ and $V_{y21} = -h$, (4) reduces to

$$\dot{V}_{l1} = (V_{x11})(V_{y11} + V_h) - aV_{l1} \tag{5}$$

The correspondence between (5) and (3a) is established by simply connecting V_{x1i}, V_{y1i} and V_{li} to V_{l2}, V_{l3} and V_{l1} respectively. The final equation is given by (6).

$$\dot{V}_{l1} = V_{l2}(V_{l3} + V_h) - aV_{l1} \tag{6}$$

Again, setting V_{x2i} and V_{mi} to ground and R_{3i} to open circuit, (2) reduces to

$$C_{l2}.\dot{V}_{l2} = \frac{(V_{x1})(V_{y1}-V_{y2})}{10R_{12}} - \frac{V_{l2}}{R_{22}} \tag{7}$$

Now, by setting $10R_{12}C_{l2} = 1$, $R_{22}C_{l2} = b^{-1}$ and $V_{y12} = h$, and connecting V_{x1i}, V_{y2i} and V_{li} to V_{l1}, V_{l3} and V_{l2} respectively, (7) reduces to

$$\dot{V}_{l2} = V_{l1}(V_h - V_{l3}) - bV_{l2} \tag{8}$$

The same procedure was repeated for (3c), where V_{x2i}, V_{y2i} and V_{mi} were set to ground, making (2) look like (9).

$$C_{l3}.\dot{V}_{l3} = \frac{(V_{x1})(V_{y1})}{10R_{13}} - \frac{V_{l3}}{R_{23}} \tag{9}$$

Putting $10R_{13}C_{l3} = 1$, $R_{23}C_{l3} = d^{-1}$, and connecting V_{x1i}, V_{y1i} and V_{li} to V_{l1}, V_{l2} and V_{l3} respectively, (9) reduces to

$$\dot{V}_{l3} = V_{l1}.V_{l2} - dV_{l2} \tag{10}$$

The complete circuit design to implement (3) is presented in Fig.3.

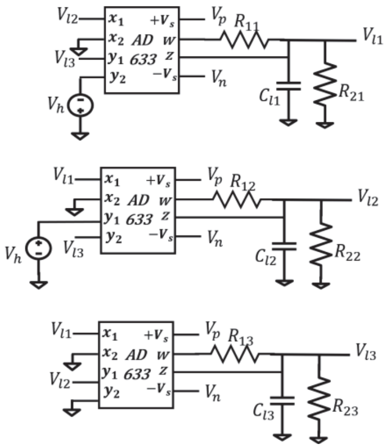


Fig.3. AD633 based complete circuit design of Rabinovich chaotic system

Further, the usefulness of proposed generic topology is examined for seven more chaotic systems. The state variables x, y and z are represented by V_{l1}, V_{l2} and V_{l3} for circuit implementation for all chaotic systems and all capacitor is considered as 1 unit and R_{1i} is taken as 0.1 unit. The input voltages to AD633, the conditions on values of R_{2i} and R_{3i} are summarized in Table 1.

IV. SIMULATION RESULTS

This section deals with functional verification of proposed AD633 based chaotic systems realization in LTspice design environment. The simulations have been carried out using Analog Devices micromodel of AD633 [32] and the resistors and capacitors are taken from built in library of LTspice. The power supply voltage for both the systems is ± 15V. These systems, tabulated in Table 1, have been simulated, and two most common of them have been shown as follows:

A. Rabinovich chaotic system (RCS)

The values of resistors R_{1i}, R_{2i} and R_{3i} are computed corresponding to parameters a = 4, b =1, d = 1 and h = 6.75. The V_{mi} is not required for all governing equations and same is true for R_{3i}. The value of resistors $R_{1i\,(i=1,2,3)}$ is kept at 1kΩ while R_{21}, R_{22} and R_{23} at 4kΩ, 10kΩ and 10kΩ respectively. Figure 4 shows simulated phase plots in x-y, x-z and y-z planes which confirm to the functionality of RCS by visual observations. The attractors generated in all the three planes are symmetric and double scroll in nature.

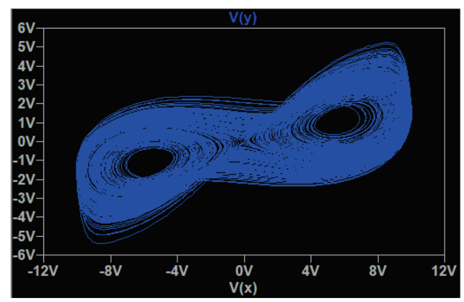
B. Lorenz chaotic system (LCS)

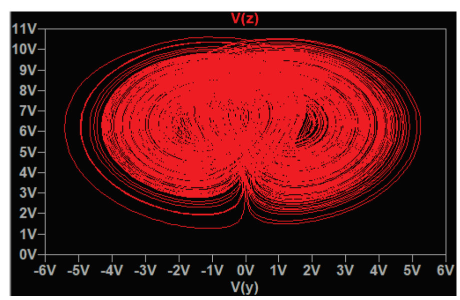
Similar to RCS, LCS also does not require R_{3i} and $V_{mi\,(i=1,2,3)}$. Using the parameters σ = 10, r =28 and ρ = 8/3, the values of resistors R_{11}, R_{12}, R_{13}, R_{22}, R_{23} are computed as 1kΩ, 100Ω, 1kΩ, 10kΩ and 3.75kΩ respectively. It is to be noted that in order to scale the value of ρ = 28V to 2.8 V, R_{l2} has been reduced to 100Ω. This design has consumed 65% lesser passive components and 70% lesser active components in comparison to the design in [16]. The phase space plots in x-y, x-z and y-z planes are depicted in Fig. 4, where all of them are two scroll in nature.

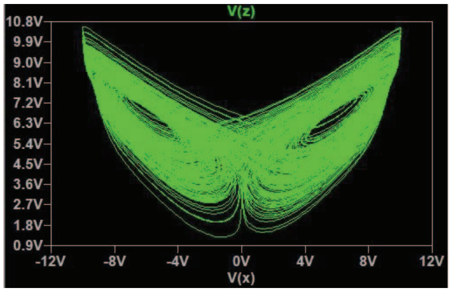
The simulation results in phase space have been placed in Fig.4. Also known as attractors, these phase space plots are same as obtained by numerical simulations in [11,21], which proves the authenticity of the circuit.

V. CONCLUSION

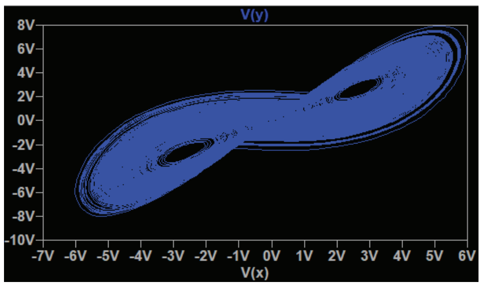
An optimized circuit design of chaotic systems has been proposed. To the best of our knowledge, this is the simplest circuit design of a chaotic system with non-linear quadratic terms. To support this argument, the active component count for LCS has been reduced by 70% in comparison to [16]. It utilizes AD633s equal to the dimension of the chaotic system, along with passive linear components, such as resistors and capacitors. LTspice simulations are in agreement with the numerical simulations existing in literature. Circuit simplification by reducing hardware complexity plays a vital role in the generation of chaotic attractors for various real time applications. This also makes the circuit highly accurate because of the reduced number of components, thus their inherent errors. Thus, it will be beneficial for chaos applications to engineering problems.

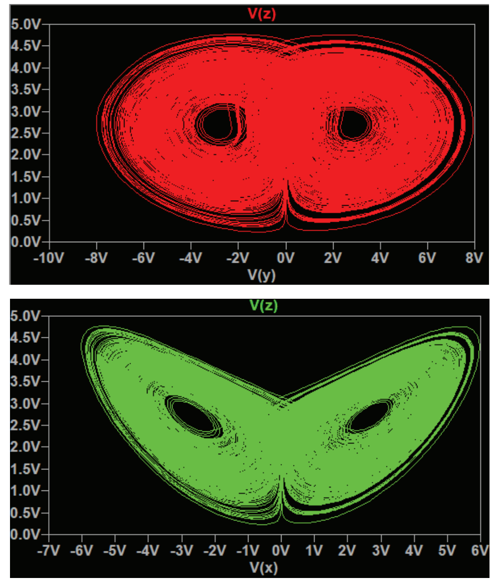






(a)





(b)

Fig.4. Phase space plots of (a) Rabinovich and (b) Lorenz chaotic system in x-y, x-z and y-z planes

TABLE I CHAOTIC SYSTEMS MAPPED ON THE PROPOSED TOPOLOGY

Ref	Chaotic system and its Governing equations	V_{li}	V_{x1i}	V_{x2i}	V_{y1i}	V_{y2i}	R_{2i}	R_{3i}
[11]	$\dot{x} = \sigma(y-x)$	V_{l1}	σ	gnd	V_{l2}	V_{l1}	inf	inf
	$\dot{y} = \rho x - y - xz$	V_{l2}	V_{l1}	gnd	ρ'****	V_{l3}	1	inf
	$\dot{z} = xy - rz$	V_{l3}	V_{l1}	gnd	V_{l2}	gnd	1/r	inf
[22,23]	$\dot{x} = a(y-x)$	V_{l1}	a	gnd	V_{l2}	V_{l1}	inf	inf
	$\dot{y} = cx - xz$	V_{l2}	V_{l1}	gnd	c	V_{l3}	inf	inf
	$\dot{z} = -bz + xy$	V_{l3}	V_{l1}	gnd	V_{l2}	gnd	1/b	inf
[24]	$\dot{x} = a(y-x) + yz$	V_{l1}	V_{l2}	gnd	V_{l3}	-a	1/a	inf
	$\dot{y} = cx - y - xz$	V_{l2}	V_{l1}	gnd	c	V_{l3}	1	inf
	$\dot{z} = xy - bz$	V_{l3}	V_{l1}	gnd	V_{l2}	gnd	1/b	inf
[25]	$\dot{x} = -ax + by - yz$	V_{l1}	V_{l2}	gnd	b	V_{l3}	1/a	inf
	$\dot{y} = x + xz$	V_{l2}	V_{l1}	gnd	V_{l3}	-1V	inf	inf
	$\dot{z} = -cz + y^2$	V_{l3}	V_{l2}	gnd	V_{l2}	gnd	1/c	inf
[27]	$\dot{x} = a(y-x)$	V_{l1}	a	gnd	V_{l2}	V_{l1}	inf	inf
	$\dot{y} = xz - y$	V_{l2}	V_{l1}	gnd	V_{l3}	gnd	1	inf
	$\dot{z} = b - xy - cz$	V_{l3}	V_{l1}	gnd	gnd	V_{l2}	1/c	1#
[28]	$\dot{x} = a(y-x)$	V_{l1}	a	gnd	V_{l2}	V_{l1}	inf	inf
	$\dot{y} = (c-a)x - axz$	V_{l2}	V_{l1}	gnd	k**	$R_4/(R_4+R_2)$. V_{l3}*	inf	inf
	$\dot{z} = -bz + xy$	V_{l3}	V_{l1}	gnd	V_{l2}	gnd	inf	inf
[29]	$\dot{x} = -ax - byz$	V_{l1}	gnd	V_{l2}	V_{l3}	gnd	1/a	inf
	$\dot{y} = -x + cy$	V_{l2}	V_{l2}	V_{l1}	c'***	gnd	1	inf
	$\dot{z} = d - y^2 - z$	V_{l3}	V_{l2}	gnd	gnd	V_{l2}	1	1##

* a voltage divider circuit to take are of coefficient 'a'
** k is a constant voltage equal to 'c-a'
*** c'= $\left(c + \frac{1}{R_{22}}\right).(0.1R_{12})$
**** ρ'= $\frac{\rho . R_{22}}{0.1}$
\# V_{m3} = b, ## V_{m3} = d

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