Wed Oct 9th

Name: ____________________ ID #:________________________

Fill in all the gaps. Show your work.

$A^T A x = A^T b$

1. (XX) Remember we have had lots of fill in the blank Definitions and Theorems on Skills Tests

1.1. The least squares solution of $A\,x = b$ is $x =$ $\boxed{(A^T A)^{-1} A^T b}$

$A = QR$

1.2. The QR decomposition of A satisfies $A =$ $\boxed{QR}$ with Q

satisfying $\boxed{Q^T Q = I}$ and R $\boxed{\text{upper Triang.}}$.

1.3. The $\boxed{\text{Gram Schmidt}}$ process for A generates a $\boxed{\text{orthonormal}}$ Q

satisfying $\boxed{\text{col}(A) = Col(Q)}$.

1.4. The $\boxed{\text{null space}}$ of A is the set of all solutions of $A\,v = 0$.

1.5. The $\boxed{\text{column space}}$ of A is the span of the columns of A.

1.6. ...

2. (XX) Compute the eigenvalues and eigenvectors of

$A = \begin{pmatrix} 7 & -5 \\ 10 & -8 \end{pmatrix}$ $\det \begin{pmatrix} 7-\lambda & -5 \\ 10 & -8-\lambda \end{pmatrix} = (7-\lambda)(-8-\lambda) + 50$

$= -56 - 7\lambda + 8\lambda + \lambda^2 + 50$

$= \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2)$

$\lambda = -3$ or 2

2.1. Eigenvalues of A are $\boxed{-3, 2}$

$A - 2I = \begin{bmatrix} 5 & -5 \\ 10 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $x_1 - x_2 = 0$, $x_1 = x_2$

$A + 3I = \begin{bmatrix} 10 & -5 \\ 10 & -5 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$ $2x_1 = x_2$

2.2. Eigenvectors of A are

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \ \end{pmatrix}, \begin{pmatrix} \ \end{pmatrix}$

2.3. $A = P.D.P^{-1}$ with P = $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$

$(7-\lambda)(1-\lambda)+18 = \lambda^2+7-\lambda-7\lambda+18 = \lambda^2-8\lambda+25$
$= (\lambda-4)^2+9=0$
$\lambda = 4\pm 3i$

3. (XX) Compute the eigenvalues and eigenvectors of $A = \begin{pmatrix} 7 & 3 \\ -6 & 1 \end{pmatrix}$

3.1. Eigenvalues of A are $\boxed{4 \pm 3i}$

$A-(4-3i)I = \begin{bmatrix} 3+3i & 3 \\ -6 & -3+3i \end{bmatrix}$

$\sim \begin{bmatrix} 1+i & 1 \\ 0 & 0 \end{bmatrix}$ $(1+i)x_1+x_2=0$
$x_2 = -(1+i)x_1$

3.2. Eigenvectors of A are

$\begin{pmatrix} 1 \\ -(1+i) \end{pmatrix}, \begin{pmatrix} 1 \\ -(1-i) \end{pmatrix}, \begin{pmatrix} \cancel{} \end{pmatrix}, \begin{pmatrix} \cancel{} \end{pmatrix}$

3.3. $A = P.D.P^{-1}$ with P = $\begin{pmatrix} 1 & 1 \\ -(1+i) & -(1-i) \end{pmatrix}$ and $D = \begin{pmatrix} 4-3i & 0 \\ 0 & 4+3i \end{pmatrix}$

or if you want

$P = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$

and you hate complex #'s

(XX) Compute the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} -10 & 0 & -51 \\ 1 & 1 & -9 \\ 3 & 0 & 14 \end{pmatrix}$$

$$0 = (1-\lambda)\begin{vmatrix} -10-\lambda & -51 \\ 3 & 14-\lambda \end{vmatrix} = (1-\lambda)\left[\begin{matrix} -(10+\lambda)(14-\lambda) \\ +153 \end{matrix}\right]$$

4.1. Eigenvalues of A are $\boxed{1,\ 2 \pm 3i}$

$$= (1-\lambda)\left(-\left(140 - 10\lambda - \lambda^2 + 14\lambda\right) + 153\right]$$

$$= (1-\lambda)\left(13 + \lambda^2 - 4\lambda\right)$$

$$= (1-\lambda)\left((\lambda-2)^2 + 9\right)$$

4.2. Eigenvectors of A are

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4-i \\ -1-4i \\ 1 \end{pmatrix}, \begin{pmatrix} -4+i \\ -1+4i \\ 1 \end{pmatrix}, \begin{pmatrix} \\ \\ \end{pmatrix}$$

$\lambda = 2 \pm 3i$

$\lambda = 1$

$x_1 = 0$

$x_3 = 0$

$$A - (1)I = \begin{bmatrix} -11 & 0 & -51 \\ 1 & 0 & -9 \\ 3 & 0 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -9 \\ -11 & 0 & -51 \\ 3 & 0 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -9 \\ 0 & 0 & -51-99 \\ 0 & 0 & 13+27 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A - (2-3i) = \begin{pmatrix} -12+3i & 0 & -51 \\ 1 & -1+3i & -9 \\ 3 & 0 & 12+3i \end{pmatrix} \sim \begin{pmatrix} 1 & -1+3i & -9 \\ 3 & 0 & 12+3i \\ -12+3i & 0 & -51 \end{pmatrix} \sim \begin{pmatrix} 1 & -1+3i & -9 \\ 0 & 3-9i & 39+3i \\ 0 & -3+39i & -159+27i \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1+3i & -9 \\ 0 & 1 & 1+4i \\ 0 & 1 & 1+4i \end{pmatrix} \sim \begin{pmatrix} 1 & -1+3i & -9 \\ 0 & 1 & 1+4i \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4+i \\ 0 & 1 & 1+4i \\ 0 & 0 & 0 \end{pmatrix}$$

$x_1 = -(4+i)x_3$

$x_2 = -(1+4i)x_3$

$x_1 = -4-i$

$x_2 = -1-4i$

$x_3 = 1$