sept 23rd

Name: ____________________ ID #: ________________________

As always you need to show your work. Fill in the appropriate blanks

1. A pair (λ, v) is an eigen pair if $\boxed{Av = \lambda v}$ and $\boxed{v \neq 0}$

2. For $A = \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

2.1. Is $v = \{1, 4, 2\}$ an evec of A? $\boxed{\text{yes}}$ if it is compute the eval $\lambda = \boxed{2}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

2.2. Is $v = \{1, 1, -1\}$ an evec of A? $\boxed{\text{NO}}$ if it is compute the eval $\lambda = \boxed{}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

3. For $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$.

3.1. Is $\lambda = 2$ an eval of A? $\boxed{\text{yes}}$ if it is compute the evec $v = \boxed{\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}}$

$$A - 2I = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(A - \lambda I)X = 0$$

$$2x_1 + 0x_2 + x_3 = 0$$

$$x_2 = 0$$

x_3 free

e.g. $x_3 = 1$, $x_1 = -1/2$, $x_2 = 0$

e.g. $x_3 = -2$, $x_2 = 0$, $x_1 = 1$

Sept 23rd

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1. The characteristic equation of a matrix A is $\boxed{\det(A - \lambda I) = 0}$.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Eigenvalues are roots of $\boxed{\det(A - \lambda I) = 0}$.

3. The char eqn of $A = \begin{pmatrix} 2 & 1 \\ -7 & 3 \end{pmatrix}$ is $\boxed{(2-\lambda)(3-\lambda) + 7 = 0}$ with evals $\boxed{}$.

$$\det\begin{pmatrix} 2-\lambda & 1 \\ -7 & 3-\lambda \end{pmatrix} = (2-\lambda)(3-\lambda) - (-7)$$

$$= 6 - 5\lambda + \lambda^2 + 7$$

$$0 = 13 - 5\lambda + \lambda^2$$

$$\lambda = \frac{5 \pm \sqrt{25 - 4(13)}}{2(1)}$$

4. The char eqn of $A = \begin{pmatrix} 6 & 0 & 1 \\ 1 & 6 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ is $\boxed{(3-\lambda)(6-\lambda)^2 = 0}$ with evals $\boxed{3, 6, 6}$

$$\det\begin{pmatrix} 6-\lambda & 0 & 1 \\ 1 & 6-\lambda & 2 \\ 0 & 0 & 3-\lambda \end{pmatrix} = (3-\lambda)\begin{vmatrix} 6-\lambda & 0 \\ 1 & 6-\lambda \end{vmatrix} = (3-\lambda)(6-\lambda)^2 = 0$$