Name: Key ID #:

Fill in all the gaps. Show your work.

1. (10pts)

1.1. The Normal Equations for the least squares solution of $A x = b$ are $\boxed{A^T A x = A^T b}$

1.2. $\boxed{\text{Nul}(A)}$ the set of all solutions of $\boxed{A x = 0}$.

1.3. A eigenpair λ v for A satisfies $\boxed{A v = \lambda v}$ with $\boxed{v \neq 0}$.

1.4. An AM for Ax = b row reduces to $\begin{pmatrix} 1 & 3 & -3 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Explain why the system is inconsistent.

Last eq says $0x_1 + 0x_2 + 0x_3 = 1$ which is impossible. There are no solns.

1.5. Finish row reducing the AM for Ax = b

$$\begin{pmatrix} 1 & 3 & -3 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 0 & 14 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

. Give sols of Ax = b

$x_1 = 14 - 3x_2$
x_2 free
$x_3 = 3$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} + \boxed{x_2} \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + \boxed{} \begin{pmatrix} \\ \\ \end{pmatrix}$$

2. (10pts) $(1-\lambda)(7-\lambda)+10=0 \quad \lambda^2-8\lambda+17=(\lambda-4)^2+1=0$

$(\lambda-4)^2=-1$

2.1. The eigenvalues of $\begin{pmatrix} 1 & -5 \\ 2 & 7 \end{pmatrix}$ are $\boxed{\lambda = 4 \pm i}$

2.2. For $A = \begin{pmatrix} 8 & -5 \\ 10 & -7 \end{pmatrix}$ the evals are $\boxed{3,\ -2}$ and $A = P.D.P^{-1}$

with $P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$

$(8-\lambda)(-7-\lambda)+50 = \lambda^2-\lambda-6 = (\lambda-3)(\lambda+2)=0 \quad \lambda = 3 \text{ or } -2$

$A-3I = \begin{bmatrix} 5 & -5 \\ 10 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad x_1 - x_2 = 0,\ x_1 = x_2$

$A+2I = \begin{bmatrix} 10 & -5 \\ 10 & -5 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad 2x_1 - x_2 = 0,\ x_2 = 2x_1$

3. (15pts) For A = $\begin{pmatrix} -2 & 0 & 0 & 0 \\ 4 & -1 & 0 & 2 \\ -14 & 5 & -2 & -7 \\ 14 & -5 & 0 & 5 \end{pmatrix}$

$v = \begin{pmatrix} 0 \\ 3 \\ -5 \\ 5 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

3.1. A. $\begin{pmatrix} 0 \\ 3+i \\ -5 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 7-i \\ -10+5i \\ 10-5i \end{pmatrix}$. Is it an evec? Yes or No $\boxed{\text{yes}}$

If it is $\lambda = \boxed{2-i}$

e.g. $(3+i)(2-i) = 6+2i-3i+1$ etc.

3.2. A. $\begin{pmatrix} 1 \\ 1-i \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 7+i \\ -25-5i \\ 19+5i \end{pmatrix}$. Is is an evec? Yes or No $\boxed{\text{NO}}$

If it is $\lambda = \boxed{}$

3.3. $\lambda = -2$ is an eval of A. If $a = \boxed{2}$, $b = \boxed{-1}$ and

P = $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 1 & 0 & -5 & 0 \\ 0 & -2 & 5 & 0 \end{pmatrix}$ then $A = P.\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & a & -b \\ 0 & 0 & b & a \end{pmatrix}.P^{-1}$

$$A+2I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 2 \\ -14 & 5 & 0 & -7 \\ 14 & -5 & 0 & 7 \end{bmatrix} \sim \begin{bmatrix} 4 & 1 & 0 & 2 \\ 14 & -5 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 4 & 1 & 0 & 2 \\ 2 & -8 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & -8 & 0 & 1 \\ 0 & 17 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 and x_4 free $\quad 2x_1 = -x_4$, $x_2 = 0$

$v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

(15pts) For A = $\begin{pmatrix} -4 & 4 & 8 & 0 \\ -4 & 6 & 4 & 0 \\ -4 & 2 & 8 & 0 \\ -8 & 4 & 8 & 4 \end{pmatrix}$

4.1. A . $\begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2 \\ 4 \end{pmatrix}$. Is it an evec? Yes or No $\boxed{\text{yes}}$

If it is λ = $\boxed{2}$

$V_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

4.2. A . $\begin{pmatrix} 1 \\ i \\ 1+i \\ i \end{pmatrix} = \begin{pmatrix} 4+12\,i \\ 10\,i \\ 4+10\,i \\ 16\,i \end{pmatrix}$. Is it an evec? Yes or No $\boxed{\text{No}}$

If it is λ = $\boxed{///}$

4.3. $\lambda = 4$ is an eval of A. If d = $\boxed{2}$ and $P = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{pmatrix}$

then $A = P.\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & d \end{pmatrix}.P^{-1}$

$A - 4I = \begin{bmatrix} -8 & 4 & 8 & 0 \\ -4 & 2 & 4 & 0 \\ -4 & 2 & 4 & 0 \\ -8 & 4 & 8 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$2x_1 = x_2 + 2x_3$ $\quad x_2, x_3, x_4$ free

$v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ $\quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$