

Name: scLn

ID #: _____

As always you need to show your work. Fill in the appropriate blanks

1. A pair (λ, v) is an eigen pair if

$$Av = \lambda v$$

and

$$v \neq 0$$

2. For $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

$$(1-\lambda)^2 + 4 = 0$$

$$(\lambda-1)^2 = -4$$

$$\lambda-1 = \pm 2i$$

2.1. Compute the eigenvalues of A

$$1 \pm 2i, 1-2i$$

2.2. compute the eigenvectors of A

$$\begin{pmatrix} 1 \\ -i \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\lambda_1 = 1+2i$$

$$A - \lambda_1 I = \begin{bmatrix} -2i & 2 \\ -2 & -2i \end{bmatrix} \sim \begin{bmatrix} i & 1 \\ 0 & 0 \end{bmatrix}$$

$$x_1 i + x_2 = 0$$

↑
free

$$v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

3. The matrix $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is a scaling by

$$r = \sqrt{a^2 + b^2}$$

and a rotation by

$$\arctan(b/a)$$

4. A scaling by r and a rotation by angle ϕ has matrix $A =$

$$\begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$$

5. The matrix $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ is a scaling by

$$\sqrt{1+4}$$

and a rotation by

$$\arctan(2)$$