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Example A1. Suppose the RREF of the augmented matrix for a system is

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 3 \end{pmatrix}$$

$x_1 = 2$

$x_2 + x_3 = 3 \Rightarrow x_2 = 3 - x_3$

then $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

Example A2. Suppose the RREF of the augmented matrix for a system is

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 2 & 6 \end{pmatrix}$$

x_4 free

$x_1 = 3 - 2x_4$

$x_2 = 4 - 3x_4$

$x_3 = 6 - 2x_4$

$x_4 = 0 + 1x_4$

then $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -2 \\ -3 \\ -2 \\ 1 \end{pmatrix}$

Example A3. Which are the free and pivot variables for

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 2 & 6 \end{pmatrix}$$

x_4 free

x_1, x_2, x_3 are pivot

Example A4. Is this matrix in REF or RREF or neither.

$$\begin{pmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 2 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 6 \end{pmatrix}$$

[redacted] Echelon Form not Reduced Echelon Form

Example A5. Is this matrix in REF or RREF or neither.

$$\begin{pmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 6 \end{pmatrix}$$

[redacted] Echelon Form not reduced Echelon Form

Example A6. The augmented matrix

$$\begin{pmatrix} 1 & h & 3 \\ 2 & 4 & k \end{pmatrix}$$

is degenerate for $\boxed{h=2}$. The degenerate system with this h value has no solution unless $\boxed{k=6}$

With the two special values of h and k the set of all solutions is

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$x_1 + 2x_2 = 3$$
$$2x_1 + 4x_2 = 6$$

$$x_1 = 3 - 2x_2$$
$$x_2 = 0 + 1x_2$$