Name: KEY ID #:________________________

Fill in all the gaps. Show your work.

**1.** (6pts) For $A = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

$(1 0 1) \cdot (1,2,1) = 2$ $(1 0 1) \cdot (2 0 0) = 2$

x.1) Compute BA = $\begin{pmatrix} 2 & 2 \\ -2 & 0 \end{pmatrix}$

$(0 -1 0) \cdot (1,2,1) = -2$ $(0,-1,0) \cdot (2 0 0) = 0$

x.2) Fill in the gaps in AB = $\begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

$(2,0) \cdot (1,0) = 2$

$(1,0) \cdot (1,0) = 1$

**2.** (15pts) The AM of $(a1 \;\; a2 \;\; a3 \;\; a4 \;\; b)$ for Ax = $b$ row reduces to

$$\begin{pmatrix} \boxed{1} & 0 & 3 & 1 & 1 \\ 0 & \boxed{1} & 2 & 5 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 = 1 - 3x_3 - x_4$
$x_2 = 2 - 2x_3 - 5x_4$
$x_3 = 0 + x_3 + 0x_4$
$x_4 = 0 + 0x_3 + x_4$

x.1) Circle pivot columns and put a box around pivot entries.

x.2) Pivot variables are $\boxed{x_1, x_2}$. Free variables are $\boxed{x_3 \;\; x_4}$

x.3) Write down a formula for all solutions to Ax = 0. note $Ax = 0$.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \boxed{x_3} \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \boxed{x_4} \begin{pmatrix} -1 \\ -5 \\ 0 \\ 1 \end{pmatrix}$$

x.4) Explain why there are no solutions of Ax = $b$.

3rd row says

$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$

No solution!

**3.** (14pts) The AM of $(a1\ \ a2\ \ a3\ \ a4\ \ a5\ \ b)$ for $Ax = b$ row reduces to

$$\begin{array}{c}\begin{pmatrix} 1 & 0 & 8 & 0 & -4 & 5 \\ 0 & 1 & 6 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \text{rhs} \end{array}\end{array}$$

$x_1 = 5 - 8x_3 + 4x_5$

$x_2 = 7 - 6x_3 - 2x_5$

$x_3$ ... free $\quad x_3 = 0 + 1x_3$

$x_4 = 2 - x_5$

$x_5 = 0 + 1x_5$ free

x.1) Write down a formula for all solutions to $Ax = b$.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 0 \\ 2 \\ 0 \end{pmatrix} + \boxed{x_3}\begin{pmatrix} -8 \\ -6 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \boxed{x_5}\begin{pmatrix} 4 \\ -2 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \cancel{\boxed{\phantom{x}}\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}} + \cancel{\boxed{\phantom{x}}\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}}$$

x.2) Write down two different non-trivial solutions to $Ax = 0$. $\begin{pmatrix} -8 \\ -6 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

x.3) Are the vectors a1 and a2 LI or LD? Explain how you know.

$$[a_1 \mid a_2] \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LI because there are no free variables.

**4.** (15pts) Is the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ invertible? If it is compute

$$A^{-1} = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}.$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$