

Rössler Attractor Circuit

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The Sales Pitch

- Want to solve ODEs without pesky rounding errors?
 - Want to solve them...*fast*?
 - ...with infinite precision*?
- Say goodbye to digital computation and hello to analog mathematics!

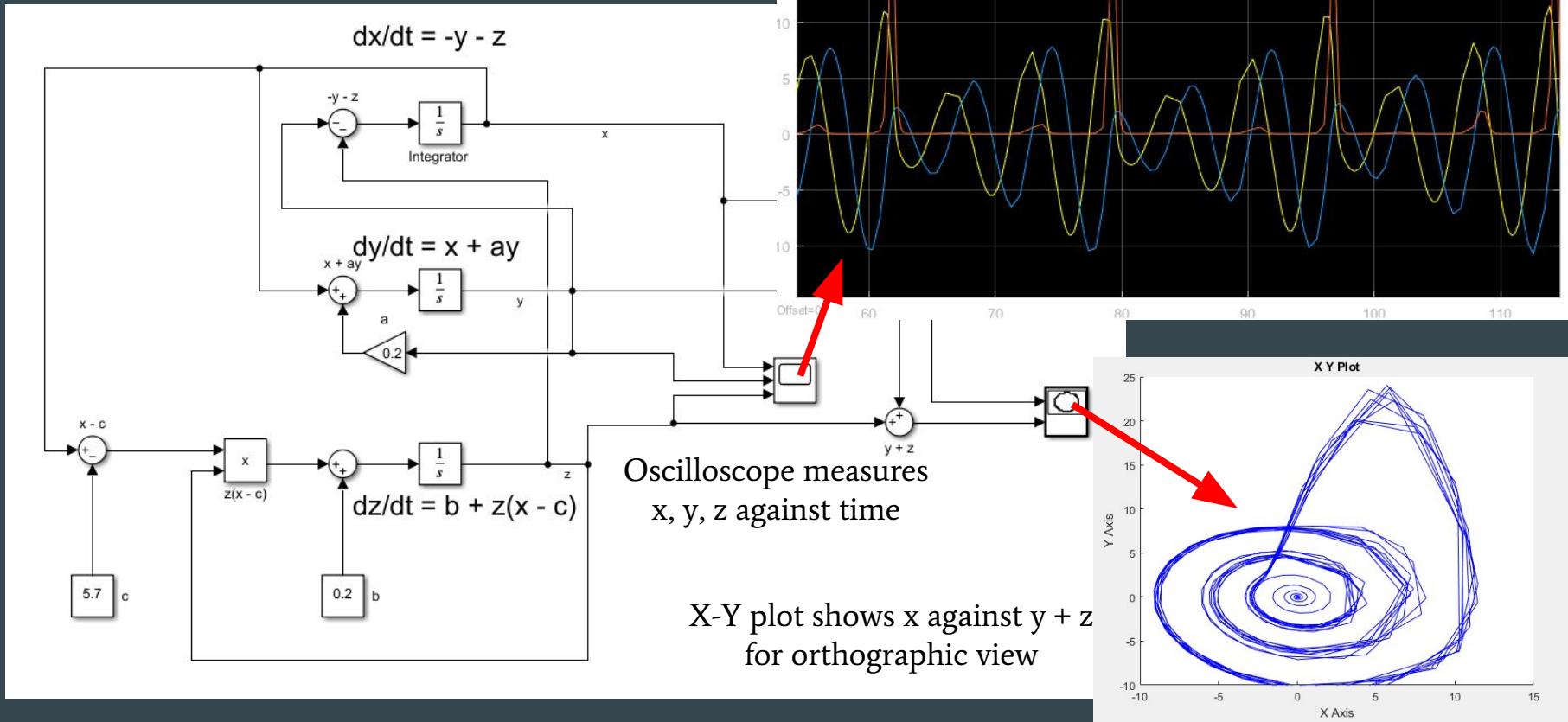
*Technically you trade digital reliability for analog computation speed and accuracy limited only by hardware

The Rossler Attractor

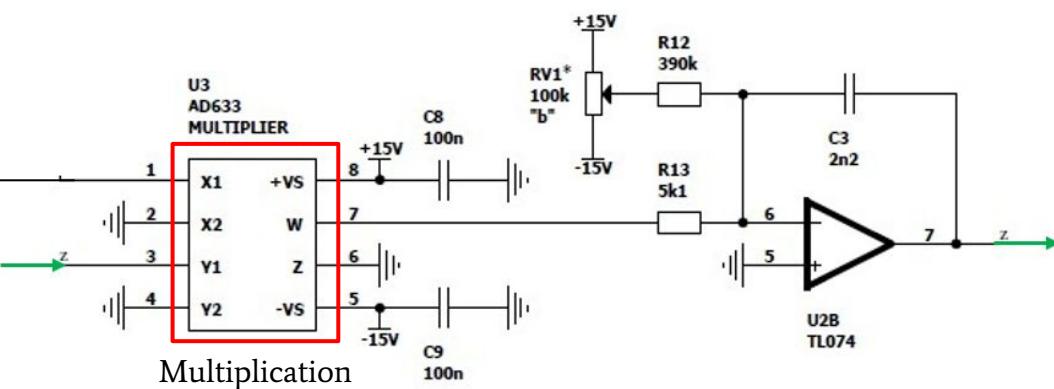
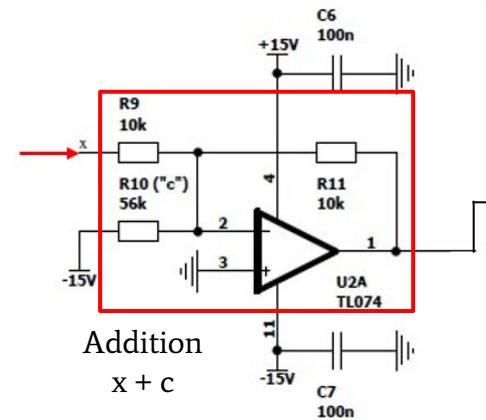
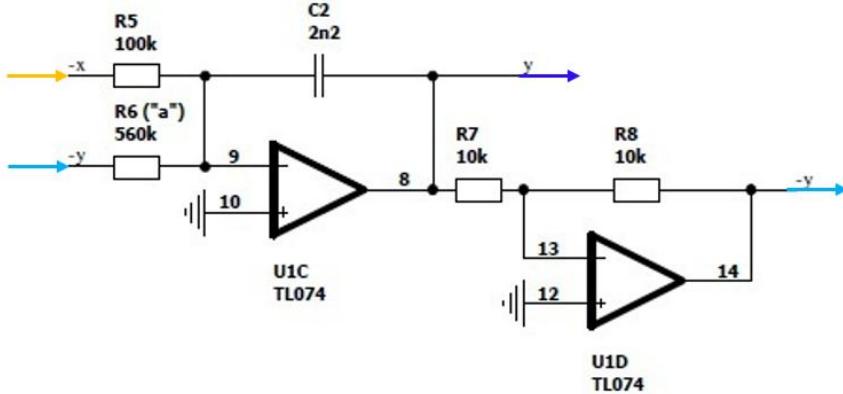
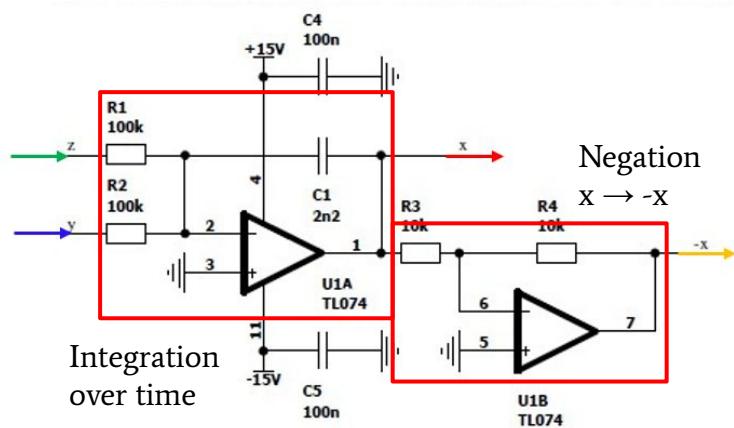
- Three coupled first-order ODEs
 - Nonlinear system
 - Exhibits chaotic behavior
- Integrating each variable over time yields trajectory
- Feeding variables back into integrator is the basis for a physical ODE solver

$$\left\{ \begin{array}{l} \frac{dx}{dt} = -y - z \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z(x - c) \end{array} \right.$$

Simulink Model Proof-of-Concept



Circuit Components

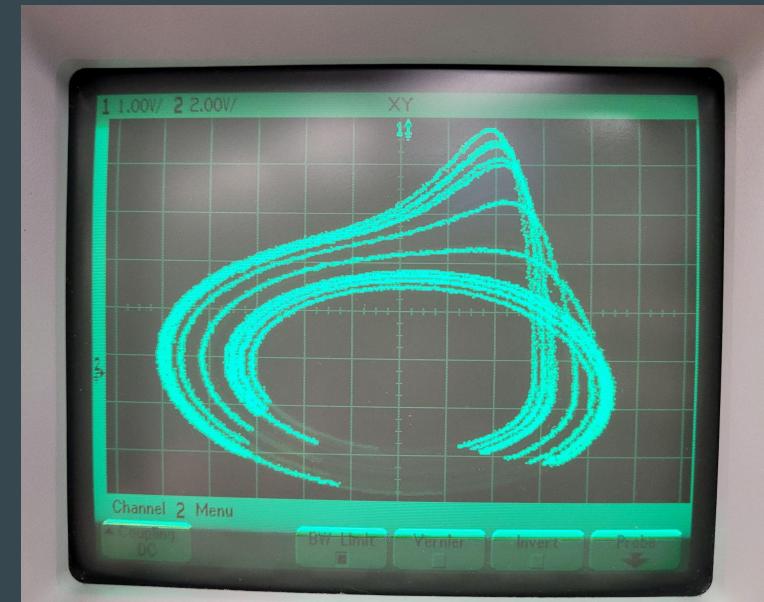
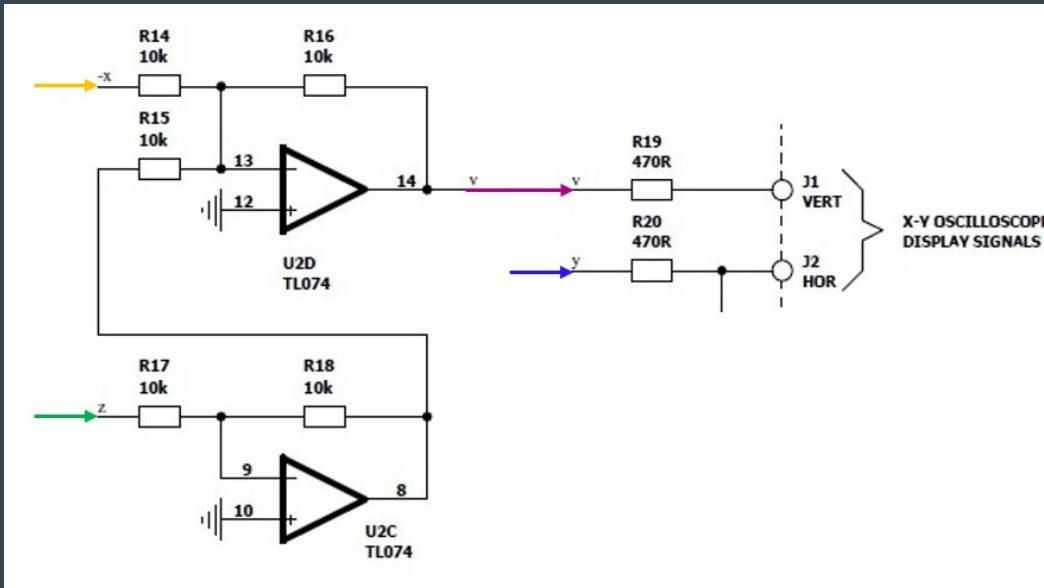


Circuit Components (cont.)

- 3 Main Components
 - Summing Amplifiers
 - Integrators
 - Analog Multiplier
- Components powered by +15 VDC and -15 VDC
- Variables a and c are constant resistors, b is variable resistor

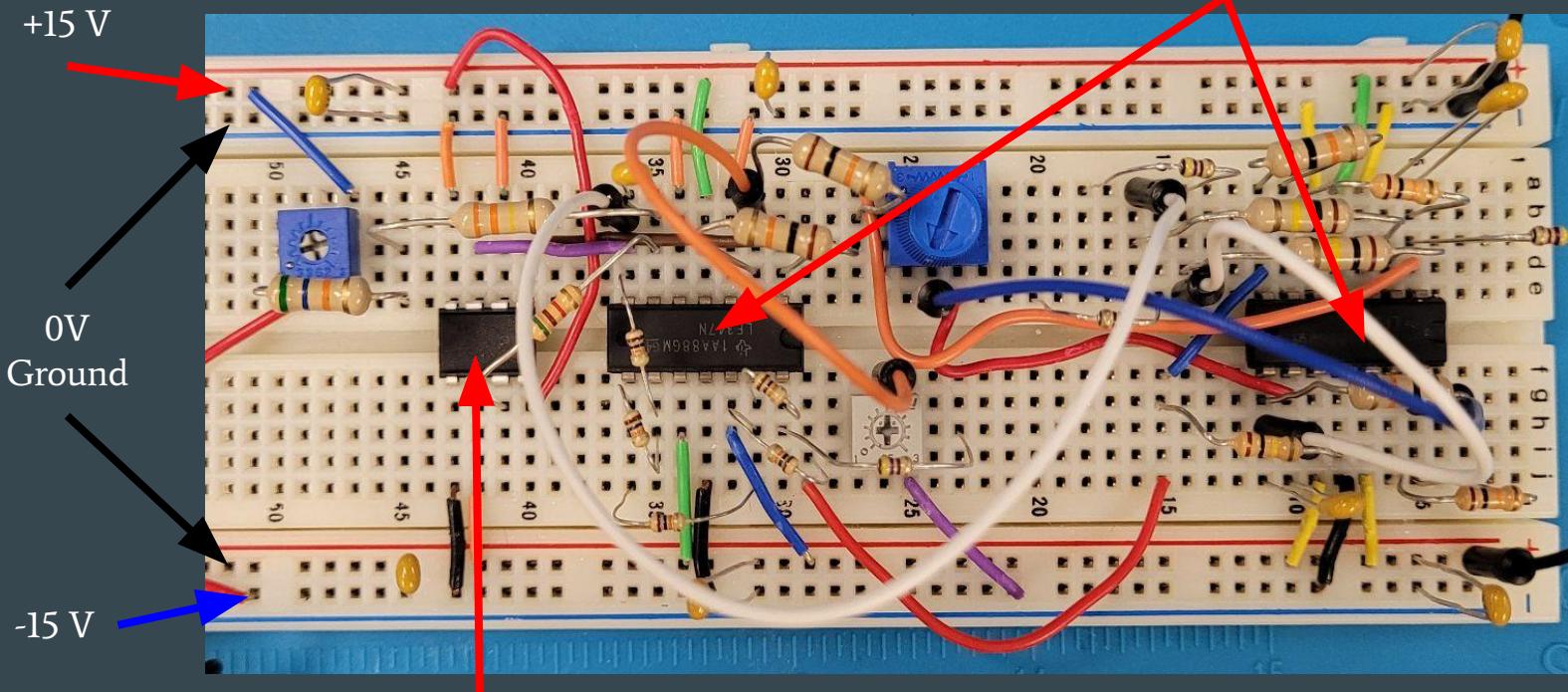
3D Display on 2D Oscilloscope

- Create a new basis vector in the x + z direction
- Pseudo-3D view of the system



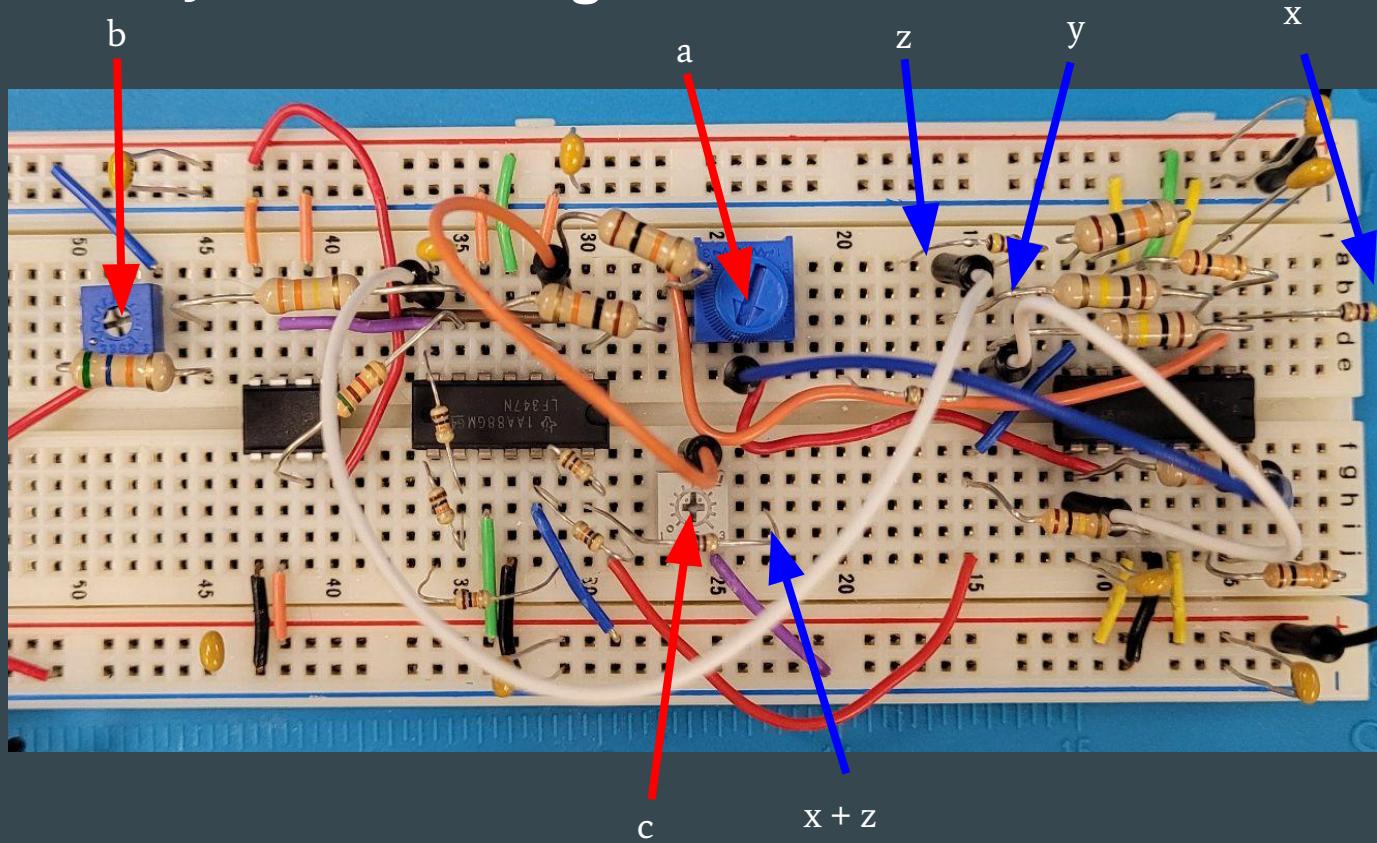
Power Supply & Integrated Circuits

Operational
Amplifiers



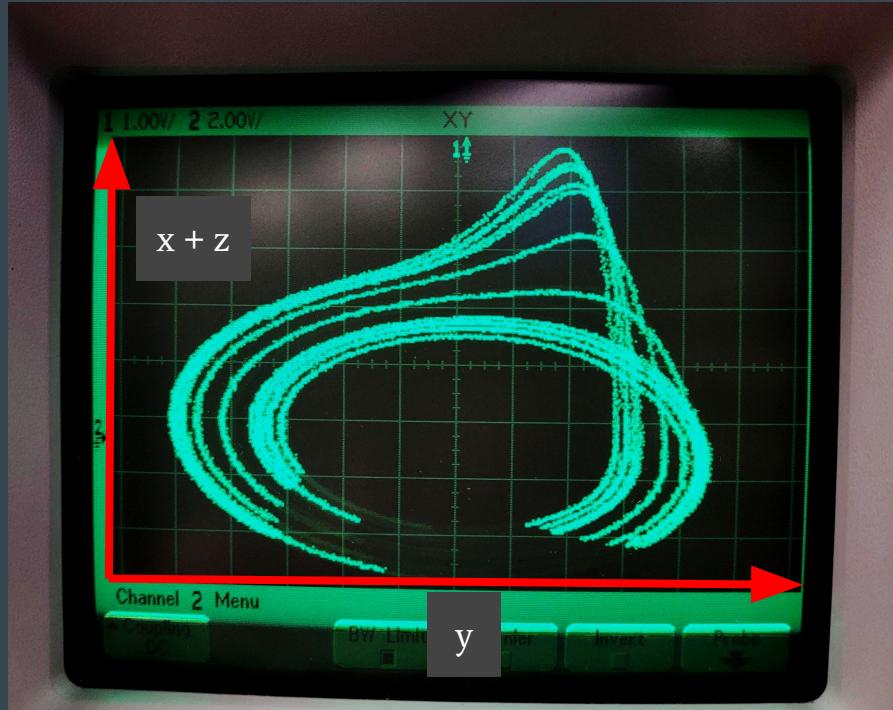
Multiplier

Parameter Adjustments & Signals

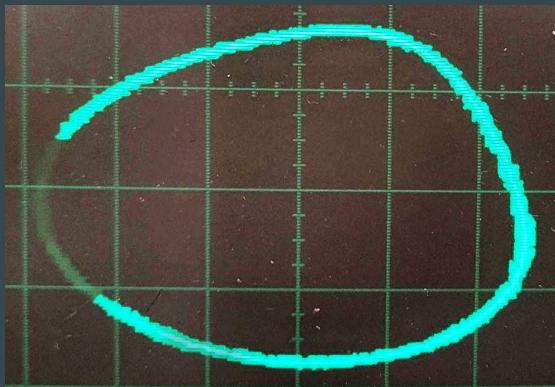


Plotting the Attractor

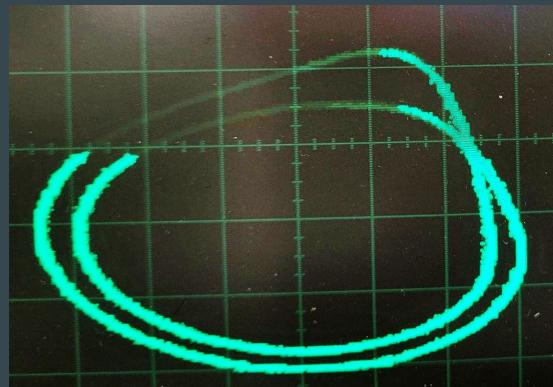
- Used oscilloscope in the new EERC Plexus Lab
- Plots one variable on each axis
 - Effective projection from $\mathbb{R}^3 \rightarrow \mathbb{R}^2$
 - y is horizontal, $x + z$ is vertical
- Potentiometers for a , b , and c were adjusted to change equation
- Some quirks due to taking pix of CRT display



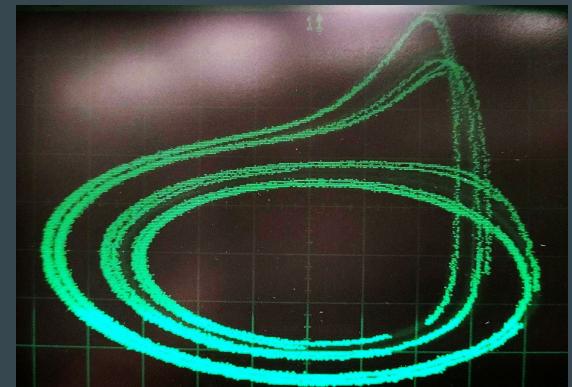
Stable Solutions



Period 1

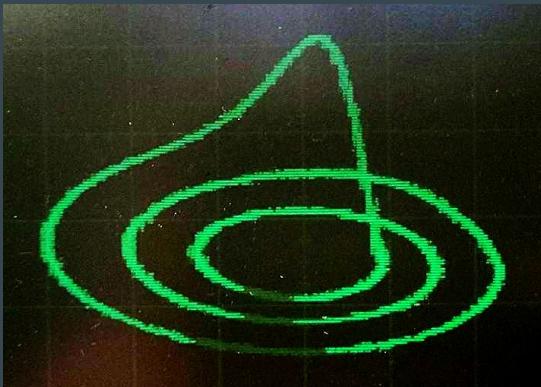


Period 2



Period 4→8
transition!

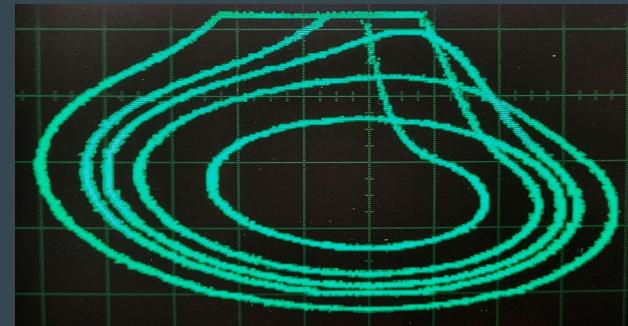
Stable Solutions (cont.)



Period 3



Period 6

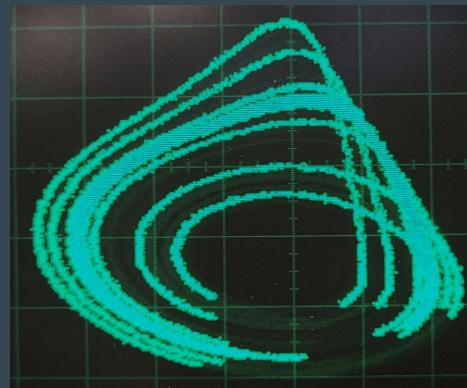


Period 5

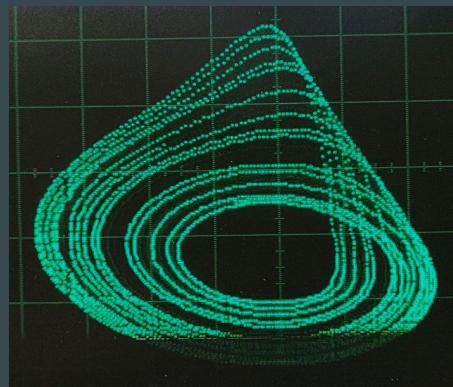
Clipping causes
distortion of solution

Chaotic Solutions!

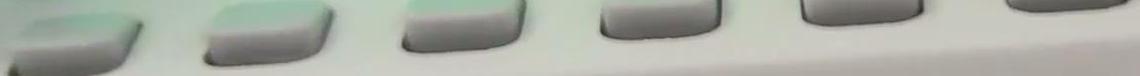
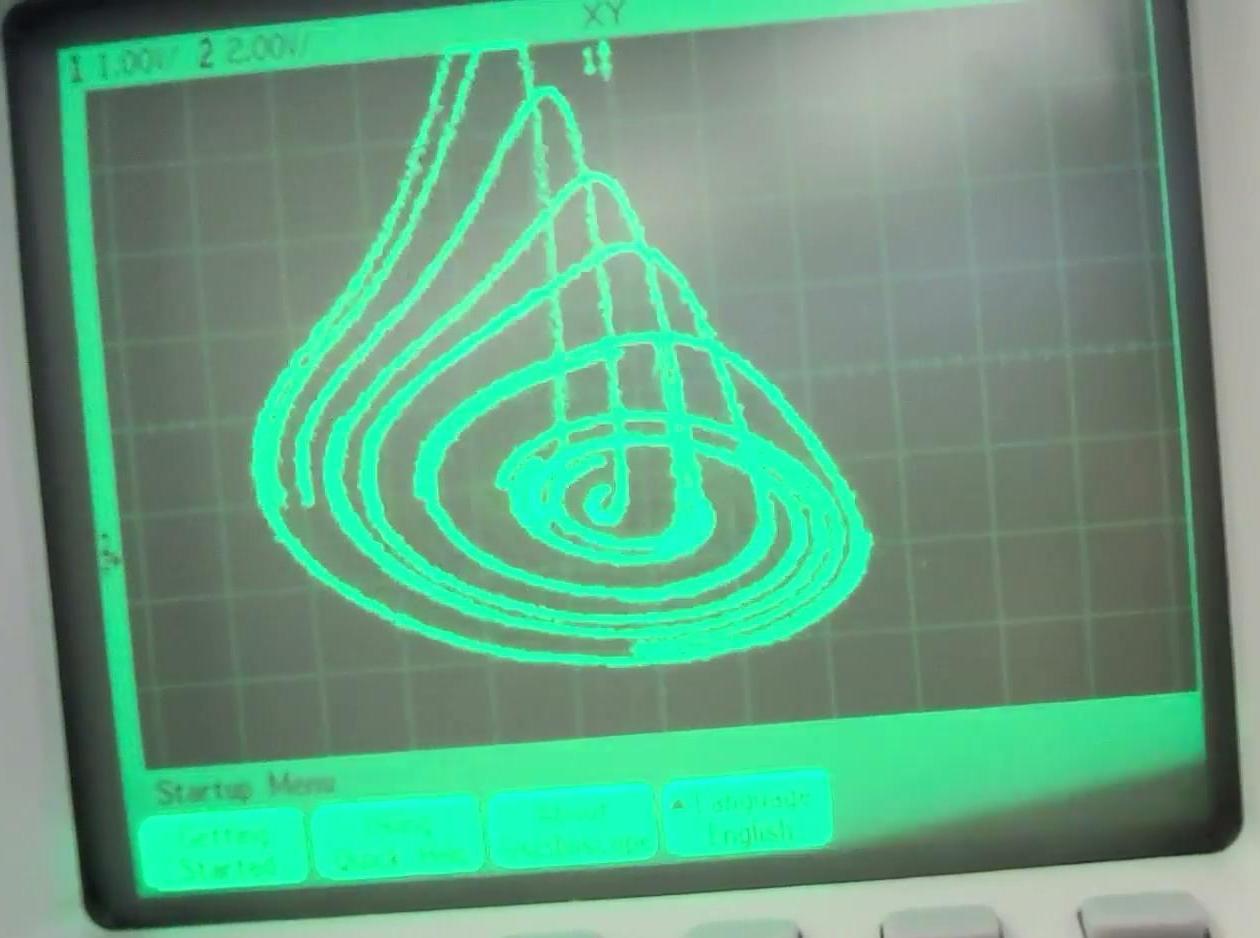
- Observed for a variety of a, b, c
- Trace length limited by memory depth of the ‘scope & display persistence
- Line fuzz from measurement noise
- Trade digital machine precision for analog machine accuracy



Densely-packed curves



Chaotic Regime
with trace averaging



1

X

1

1

Anal

Auto-Scale

5V 1mV

Conclusion

- Analog computation solves ODEs quickly
 - Accuracy limited by thermal noise, component non-idealities
 - Numerical analysis requires accurate measurement devices
- Rossler system can be implemented in electronics!
- Easy to investigate effects of changing parameters
- Analog electronics are cool!

Project Inspiration

Credit to Glen Kleinschmidt for publication of the original [circuit design](#) and more!