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New Discrete Time 2D Chaotic Maps

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ABSTRACT

Chaotic behavior is a term that is attributed to dynamical systems whose solutions are highly sensitive to initial conditions. This means that small perturbations in the initial conditions can lead to completely different trajectories in the solution space. These types of chaotic dynamical systems arise in various natural or artificial systems in biology, meteorology, economics, electrical circuits, engineering, computer science and more. Of these innumerable chaotic systems, perhaps the most interesting are those that exhibit attracting behavior. By that, the authors refer to systems whose trajectories converge with time to a set of values, called an attractor. This can be a single point, a curve or a manifold. The attractor is called strange if it is a set with fractal structure. Such systems can be both continuous and discrete. This paper reports on some new chaotic discrete time two dimensional maps that are derived from simple modifications to the well-known Hénon, Lozi, Sine-sine and Tinkerbell maps. Numerical simulations are carried out for different parameter values and initial conditions and it is shown that the mappings either diverge to infinity or converge to attractors of many different shapes.

KEYWORDS

Attractor, Chaotic Map, Hénon Map, Lozi Map, Sine-sine Map, Tinkerbell Map

1. INTRODUCTION

By chaos and chaotic systems, we generally refer to dynamical systems whose behavior is highly sensitive to initial conditions. That is, small changes in the initial conditions of such a system will lead to completely different trajectories in the solution space, a phenomenon that is widely referred to as the butterfly effect, see for example Alligood et al. (1996). Thus, despite such systems being deterministic in nature with no stochastic parameters, their complex nature constitutes their behavior hard to predict and control. This happens due to the presence of nonlinear terms in the differential equations that describe the system,

Chaotic systems were introduced in the seminal work of Lorenz (1963), that introduced a three dimensional weather model that described forced dissipative hydrodynamic flow. Since then, chaos theory as a discipline has vastly expanded and chaotic systems have been extensively studied over the last 40 years. The advancements in this area have been greatly influenced by the advancement of technology and personal computers, which makes it easier to follow the trajectories of chaotic systems for a longer time period and with a higher precision. Thus, the theoretical analysis of chaos has been assisted by the numerical simulations of such chaotic systems, made possible by the use of computers.

Out of the many chaotic systems, the ones that seem to be the most interesting are those that exhibit an attracting behavior. By that, we refer to systems whose trajectories converge with time to a set of values, called an attractor. This can be a single point, a curve, a surface or a manifold. For example, in a system with a spherical bowl and a rolling marble, the bottom of the bowl is called a fixed point attractor. The attractor is called strange if it is a set with fractal structure, see Grassberger

and Procaccia (2004); Ott (1981). Examples include the aforementioned Lorenz system, as well as the Hénon, Lozi and Tinkerbell maps that will be introduced in the next sections. It should also be noted that a system may have different attractors depending on the choice of initial conditions.

Chaotic systems have found their place in applications spanning a wide variety of natural and artificial systems. These include oscillators (Heagy, 1992; Kengne et al. 2012; Sharma et al. 2012), secure communications (Murali & Lakshmanan, 1998; Zaher, & Abu-Rezq, 2011), controller design (dos Santos Coelho & Mariani, 2012), lasers (Li et al. 2014; Yuan et al. 2014), chemical reactions (Gaspard, 1999), pseudo-random number generation (Stoyanov & Kordov, 2015), optical ring phase resonators (Aboites et al. 2009), biology (Das et al. 2014; Kyriazis, 1991) medicine (Qu, 2011, Witte & Witte, 1991), finance (Guegan, 2009) and more, see also Azar & Vaidyanathan (2014, 2015a, 2015b, 2016); Azar & Zhu (2015); Elhadj (2013); Zhu & Azar (2015) and the references therein.

A major task in the control of chaotic systems, is the one of finding a feedback control of the general form $u(t)=k(x(t))$, where u the input, x the state of the system and k the feedback law, in order to stabilize the system around its unstable equilibrium. Techniques include active control, adaptive control, sliding mode control, fuzzy logic control and more, see Azar & Vaidyanathan, (2014, 2015a, 2015b, 2016); Azar & Zhu, (2015); Vaidyanathan & Azar (2016a), Zhu & Azar, (2015) and the references therein.

Chaos synchronization is another significant problem, which rises when two or more chaotic systems are coupled, or when a chaotic system, called the master system, drives another chaotic system, called the slave system. Due to the aforementioned butterfly effect in chaotic systems which causes the exponential divergence of the trajectories of two identical systems with nearly the same initial conditions, their synchronization is a difficult task and is in itself a subject of extensive research. As with the feedback control, various techniques have been established for chaos synchronization. The choice of the technique used depends on the type of the system and the available resources. For example, the active control method (Rafikov & Balthazar, 2008; Sundarapandian, 2010) is used when all the system parameters are available for measurement and other more robust methods like adaptive control (Boulkroune, Bouzeriba et al. 2016; Vaidyanathan & Azar, 2015d, 2016b, 2016d, 2016e, 2016f), sliding mode control (Vaidyanathan, Sampath & Azar, 2015; Vaidyanathan & Azar, 2015a, 2015b;) and others (Boulkroune, Hamel et al. 2016; Ouannas, Azar & Abu-Saris, 2016; Ouannas, Azar AT & Vaidyanathan, 2016; Vaidyanathan, Idowu & Azar, 2015) are preferred when some or all the system parameters are not available for measurement and estimates of them are used.

A special case of chaotic systems is hyperchaotic systems. They are defined as chaotic systems having more than one positive Lyapunov exponents and were first introduced by Rössler (1979). The minimum dimension for a hyperchaotic continuous time system is four, while for discrete time chaotic maps this is not necessary. As with chaotic systems, hyperchaotic systems have applications in oscillators (Zou et al. 2005), secure communications (Filali et al. 2014; Li et al. 2005), encryption (Zhang et al. 2013) and more. Similar to chaotic systems, various methods have been developed for their synchronization, see for example Vaidyanathan, Azar et al. (2015); Vaidyanathan & Azar (2015c, 2016c, 2016g); Ouannas, Azar & Abu-Saris, (2016) and the references therein.

The main aim of this work is to introduce new chaotic two dimensional discrete time maps with chaotic behavior. These are derived from previously introduced and well-studied maps like the Hénon map, the Lozi map, the Tinkerbell map and the recently introduced Sine-sine map. The new maps are derived from adding nonlinear terms on these systems, or by multiplying any nonlinear term in the equations by a scaling factor. Thus, new chaotic maps are introduced, taking different shapes for different parameter values and different initial conditions. The next step of this work would be to study various qualitative properties of these maps, like their basin of attraction, dissipativity, their Lyapunov exponents and dimension, as well as the problem of adaptive control and synchronization. In addition, the construction of novel discrete hyperchaotic maps from the ones introduced here is a subject of interest.

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