Name: ______________

Show an appropriate amount of work. You can (and probably should) check your work with a computer.

1. Compute the magnitude and argument for

1.1. $z = \left(\frac{i}{1+i}\right)^3$

abs(z) =

arg(z) =

1.2. $z = \left(\frac{1+i}{\sqrt{1+3\,i}}\right)^3$

abs(z) =

arg(z) =

1.3. The roots of $z^4 = -64$

abs(z) =

arg(z) =

1.4. The roots of $z^2 + 2\,i\,z + 5 = 0$

abs(z) =

arg(z) =

1.5. $z = i^i$

abs(z) =

arg(z) =

2. Compute the real and imaginary parts for

2.1. $z = \left(\frac{i}{1+i}\right)^3$

$z =$ ☐ $+ i$ ☐

2.2. $z = \left(\frac{1+i}{\sqrt{1+3\,i}}\right)^3$

$z =$ ☐ $+ i$ ☐

2.3. The roots of $z^3 = -64$

$z_1 =$ ☐ $+ i$ ☐

$z_2 =$ ☐ $+ i$ ☐

$z_3 =$ ☐ $+ i$ ☐

2.4. The roots of $z^2 + 2\,i\,z + 5 = 0$

$z_1 =$ ☐ $+ i$ ☐

$z_2 =$ ☐ $+ i$ ☐

2.5. $z = i^i$

$z =$ ☐ $+ i$ ☐

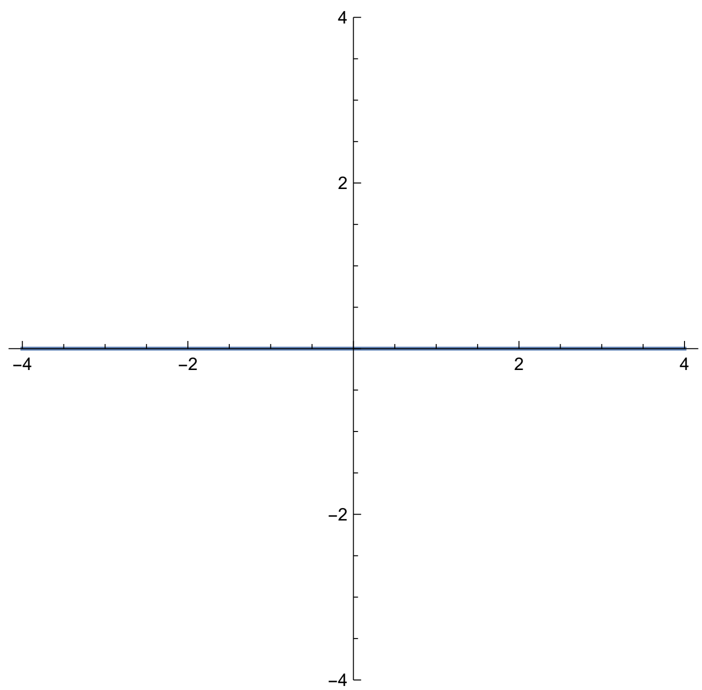
3. Sketch Points

3.1. Sketch the set of points C_1 that satisfy $|z-2|<1$. Make sure you label the set.

3.2. Sketch the set of points C_2 that satisfy $\operatorname{im}(z-2)=1$. Make sure you label the set.

3.3. Sketch the set of points C_3 that satisfy $\arg(z-2)=\pi/4$. Make sure you label the set.

Out[ ]=



4. For $f(z) = u(x, y) + i\, v(x, y)$ and write down the CR equations [ ] and [ ]

4.1. For cos(z) complete the following

$u(x, y) =$ [ ] and $v(x, y) =$ [ ]

$\partial_x u =$ [ ] and $\partial_x v =$ [ ]

$\partial_y u =$ [ ] and $\partial_y v =$ [ ]

Analytic Yes or No [ ]

4.2. For $i\, x^2 - 2\, x\, y - i\, y^2$ complete the following

$u(x, y) =$ [ ] and $v(x, y) =$ [ ]

$\partial_x u =$ [ ] and $\partial_x v =$ [ ]

$\partial_y u =$ [ ] and $\partial_y v =$ [ ]

Analytic Yes or No [ ].

If yes $f(z) =$ [ ]

4.3. For $i\, x^2 - 2\, x\, y - \; y^2$ complete the following

$u(x, y) =$ [ ] and $v(x, y) =$ [ ]

$\partial_x u =$ [ ] and $\partial_x v =$ [ ]

$\partial_y u =$ [ ] and $\partial_y v =$ [ ]

Analytic Yes or No [ ].

If yes $f(z) =$ [ ]

5. Is $e^{x - i y}$ analytic? Yes or No $\boxed{}$. Show your work below.

6. Label poles with $P_1, P_2, \ldots$ Label zeros with $Z_1, Z_2, \ldots$

$\text{Res}(f, P_1) = \boxed{}$

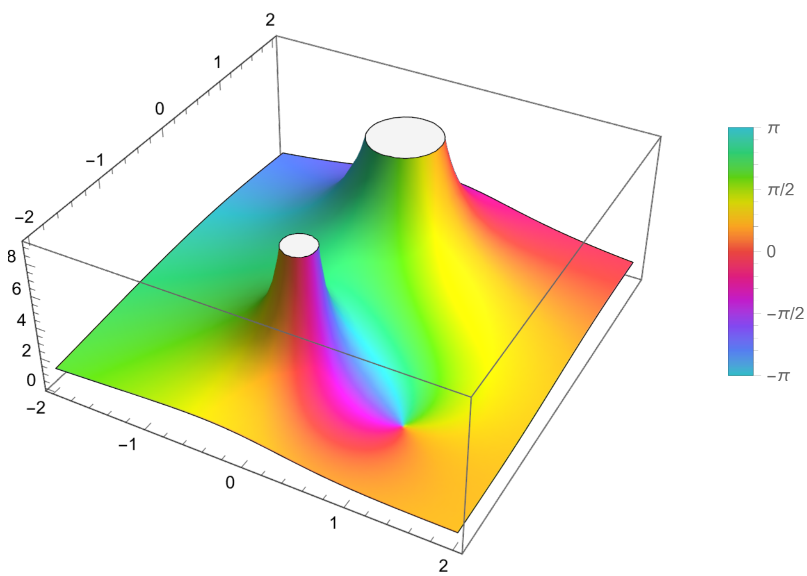
$\text{Res}(f, P_2) = \boxed{}$

In[ ]:=

```
f[z_] := 3 (z - 1 + i)/(1 + z^2)

ComplexPlot3D[ f[z],
  {z, 2}, PlotLegends → Automatic]
```

Out[ ]=



7. Complete the following. The TS

arctan(z) = $\sum_{k=0}^{\infty} a_k z^k$ where a_k = $\boxed{}$

converges for $\boxed{}$. Justify your

convergence statement using a Calculus II theorem. Give a simpler complex analytic justification below.

8. Complete the following. The TS

$\log(2+z) = \sum_{k=0}^{\infty} a_k z^k$ where $a_k =$ $\boxed{}$

converges for $\boxed{}$. Justify your

convergence statement using a Calculus II theorem. Give a simpler complex analytic justification below.

9. Label and explain the singularity and color discontinuity visible in the plot below using appropriate language. The TS $f(z) = \sum_{k=0}^{\infty} a_k (z - z_0)^k$ with $z_0 = -2 + 2\,i$ has

$a_k = \boxed{}$ and converges for

$\left| z - z_0 \right| < \boxed{}$. Explain what the TS converges to around the color discontinuity.

In[46]:=
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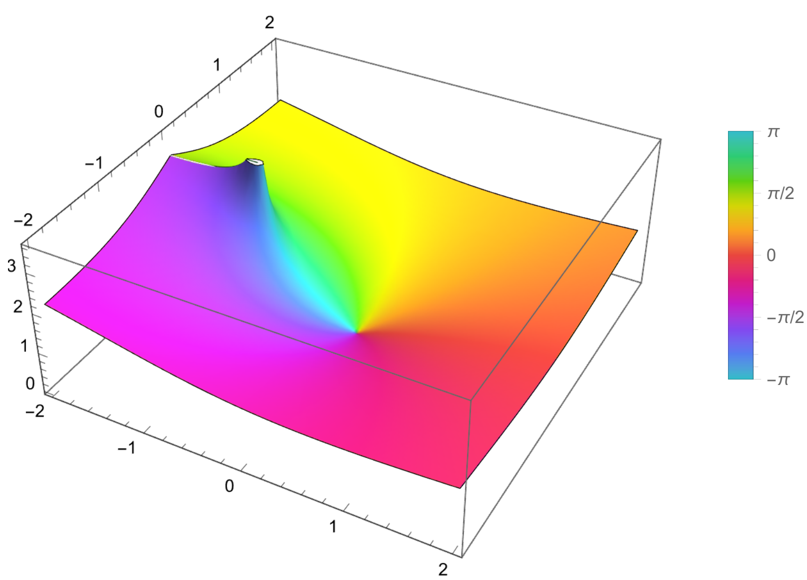
f[z_] := Log[1 + z]
ComplexPlot3D[ f[z],
 {z, 2}, PlotLegends → Automatic]
```

Out[47]=



10. Label and explain the singularity and color discontinuity visible in the plots below using appropriate language. Explain why you might expect f_1 to match f_2. Explain why they do not match using appropriate language.

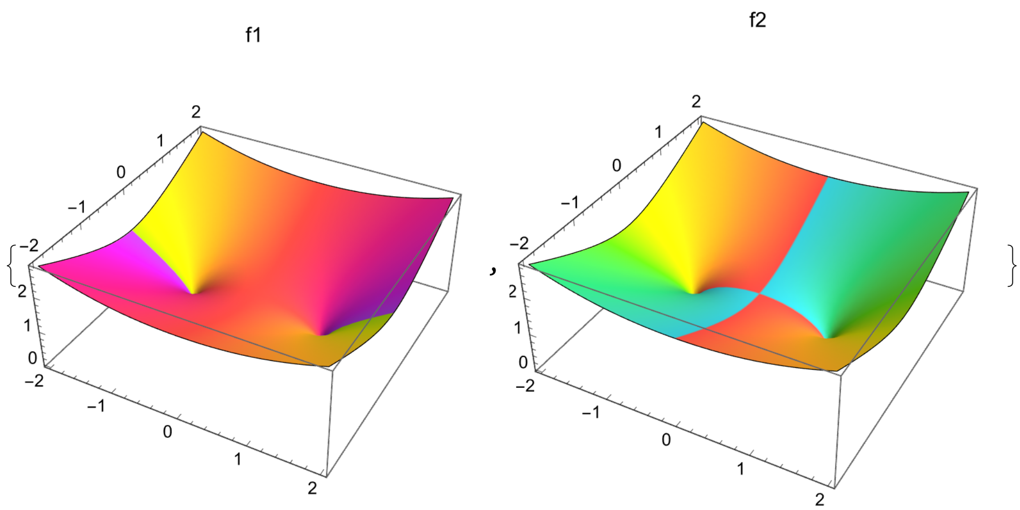
In[138]:=

$$\texttt{f1[z\_] := } \sqrt{1 - z^2}$$

$$\texttt{f2[z\_] := } i \sqrt{z^2 - 1}$$

```
{ComplexPlot3D[ f1[z], {z, 2}, PlotLabel → "f1"],
 ComplexPlot3D[ f2[z], {z, 2}, PlotLabel → "f2"]}
```

Out[140]=



11. For the closed counter clockwise contour Γ shown below

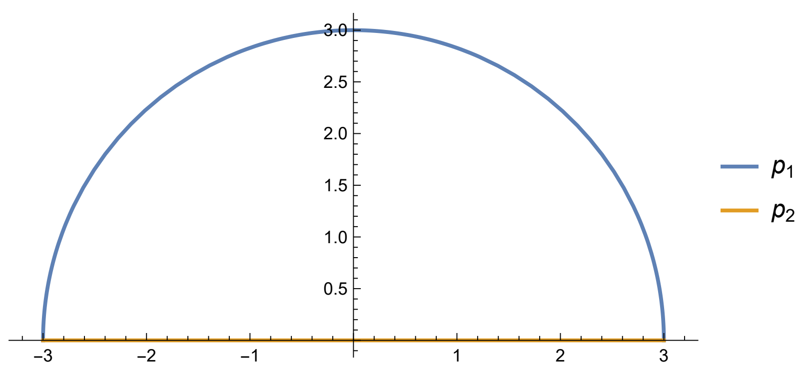
$p_1(t) = \boxed{}$ for $0 \leq t < 1$

$p_2(t) = \boxed{}$ for $0 \leq t < 1$

$\int_\Gamma f(z)\, dz = \int_0^1 \boxed{}\, dt + \int_0^1 \boxed{}\, dt.$

For $f(z) = z^2$ evaluate $\int_\Gamma f(z)\, dz = \boxed{}$ and $\int_{\Gamma_1} f(z)\, dz = \boxed{}$. Show appropriate work below.

Out[93]=



12. For the closed counter clockwise contour Γ shown below

$p_1(t) = \boxed{}$ for $0 \leq t < 1$

$p_2(t) = \boxed{}$ for $0 \leq t < 1$

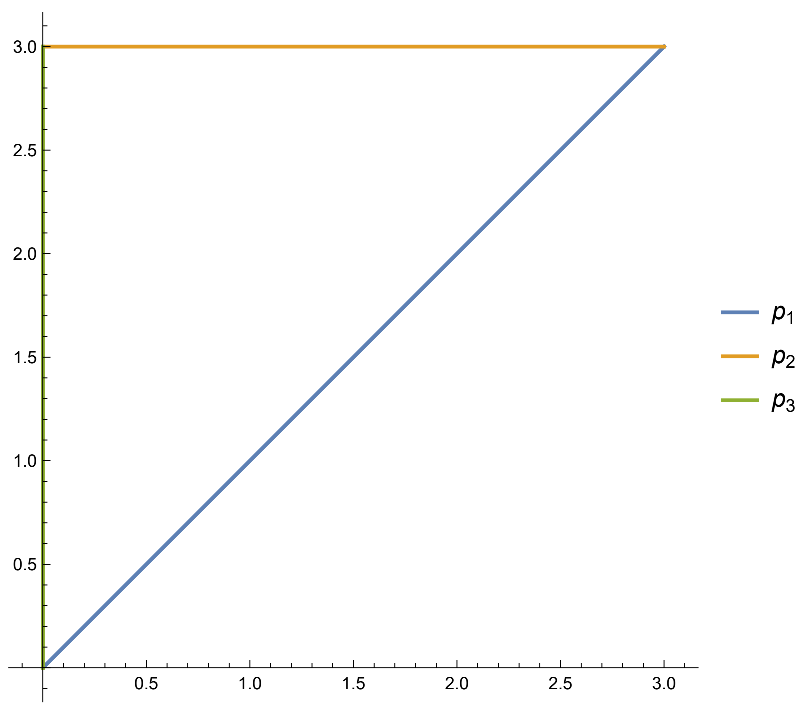
$p_3(t) = \boxed{}$ for $0 \leq t < 1$ $0 \leq t < 1$

$\int_\Gamma f(z)\,dz = \int_0^1 \boxed{}\,dt +$

$\int_0^1 \boxed{}\,dt + \int_0^1 \boxed{}\,dt$

For $f(z) = \frac{i}{z-(1+2\,i)}$ evaluate $\int_\Gamma f(z)\,dz = \boxed{}$ and $\int_{\Gamma_3} f(z)\,dz = \boxed{}$. Show appropriate work below.

Out[117]=



13. Compute $\int_\Gamma \frac{e^z}{(z^2+4)(z-1)}\, dz =$ $\boxed{}$ for the counter clockwise contour Γ. Show appropriate work.

```
p1[t_] := (E^(2 π i t) + 1.2)^2
ParametricPlot[ReIm[p1[t]], {t, 0, 1}, PlotLegends → {"Γ"}]
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Out[124]=