Name: _______________

Show an appropriate amount of work.

1. Compute magnitude and argument for all possible values of $z = \left(\frac{i}{\sqrt{1+i}}\right)^3$

$\operatorname{abs}(z_1) =$ ☐ $\operatorname{abs}(z_2) =$ ☐

$\arg(z_1) =$ ☐ $\arg(z_2) =$ ☐

2. The roots of $z^2 + 2\,i\,z + 5 = 0$ are

$z_1 =$ ☐ $+\,i$ ☐

$z_2 =$ ☐ $+\,i$ ☐

3. $i^{-i} =$ ☐ $+\,i$ ☐

$=$ ☐ $e^{☐}$

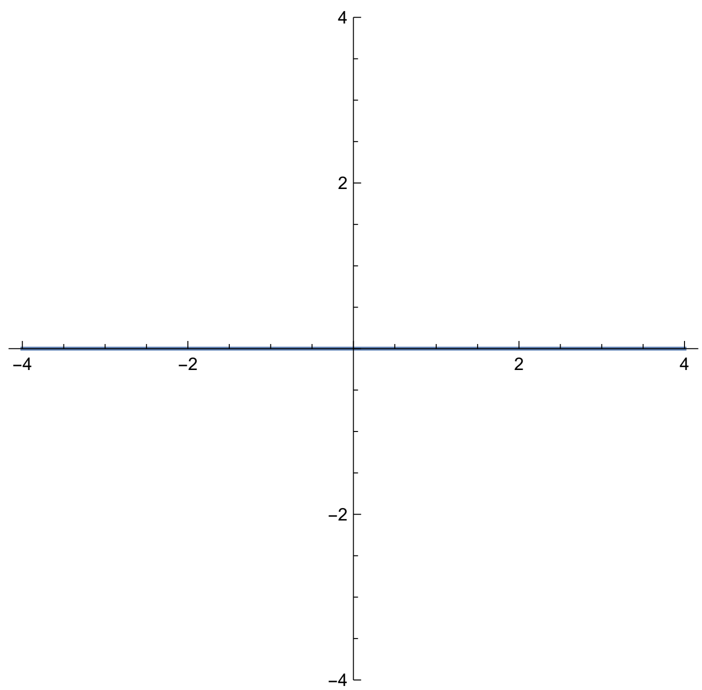
4. Sketch Points

4.1. Sketch the set of points C_1 that satisfy $|z - i| < 2$. Label the set C_1

4.2. Sketch the set of points C_2 that satisfy $\mathrm{re}(z - 2 + i) = 4$. Label the set C_2

4.3. Sketch the set of points C_3 that satisfy $\arg(z - i - 1) = -\pi/4$. Label the set C_3

Out[ ]=



5. For $f(z) = u(x, y) + i\, v(x, y)$ and write down the CR equations

$\boxed{}$ and $\boxed{}$

6. For $2x + i x^3 + 2 i y - 3x^2 y - 3 i x y^2 + y^3$ complete the following

$u(x, y) = \boxed{}$ and $v(x, y) = \boxed{}$

$\partial_x u = \boxed{}$ and $\partial_x v = \boxed{}$

$\partial_y u = \boxed{}$ and $\partial_y v = \boxed{}$

Analytic Yes or No $\boxed{}$.

If yes $f(z) = \boxed{}$

7. Is $\sin(x - i\, y)$ analytic? Yes or No $\boxed{}$. Show any needed work.

8. Label poles with $P_1, P_2, \ldots$ Label zeros with $Z_1, Z_2, \ldots$

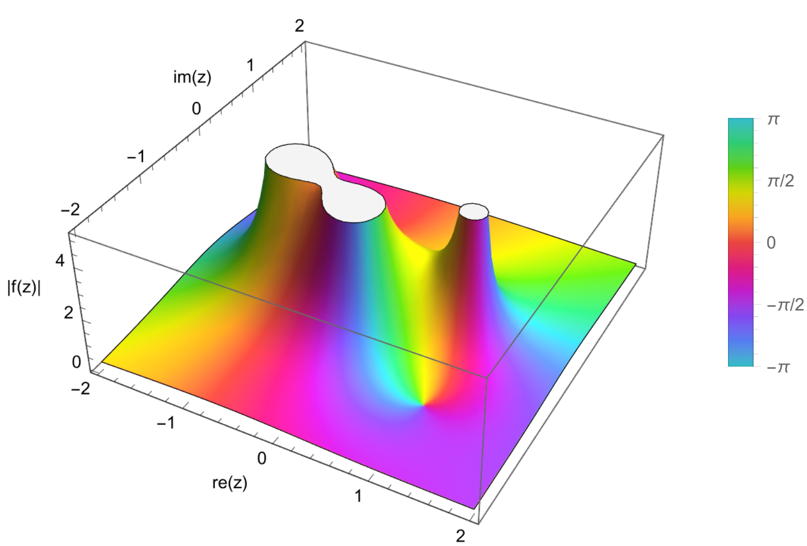
Res(f, P_1) = $\boxed{}$ Res(f, P_2) = $\boxed{}$

Res(f, P_3) = $\boxed{}$

In[ ]:=
```
f[z_] := 3 (z - 1 + I)/((1 - z^2) (I + 2 z))

ComplexPlot3D[ f[z],
 {z, 2}, PlotLegends → Automatic,
 AxesLabel → {"re(z)", "im(z)", "|f(z)|"}]
```

Out[ ]=



9. Complete the following. The TS

$\log(i + z) = \sum_{k=0}^{\infty} a_k z^k$ where $a_k =$ $\boxed{}$

converges for $\boxed{}$. Justify your

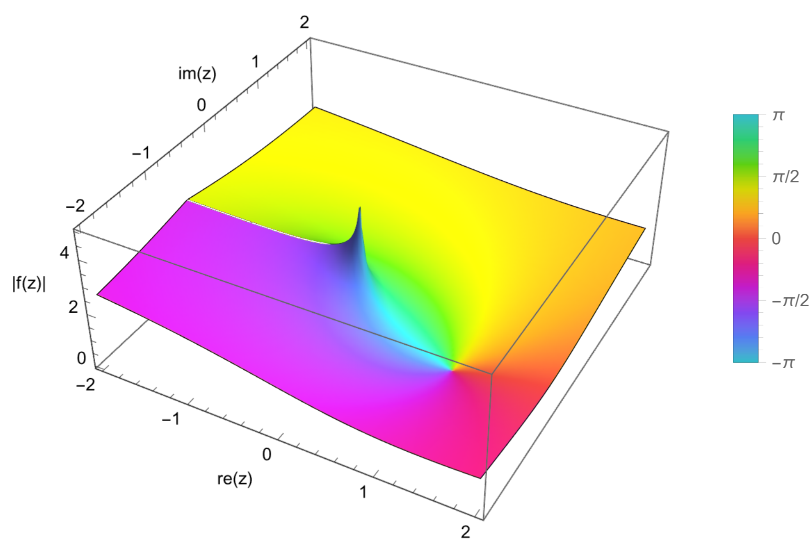
convergence statement using a Calculus II theorem. Give a simpler complex analytic justification below.

10. Label and explain the singularity and color discontinuity visible for $\log(i/2+z)$ in the plot below using appropriate language. The radius of convergence for the TS $f(z)=\sum_{k=0}^{\infty} a_k(z-z_0)^k$ with $z_0 == 1$ is $R = \boxed{}$.

Explain how you know and explain what the TS converges to around the color discontinuity.

In[ ]:=
```
f[z_] := Log[i / 2 + z]
ComplexPlot3D[f[z], {z, 2}, PlotLegends → Automatic,
 AxesLabel → {"re(z)", "im(z)", "|f(z)|"}]
```

Out[ ]=



11. For the closed CCW contour Γ shown below

$p_1(t) = \boxed{}$ for $0 \le t < 1$

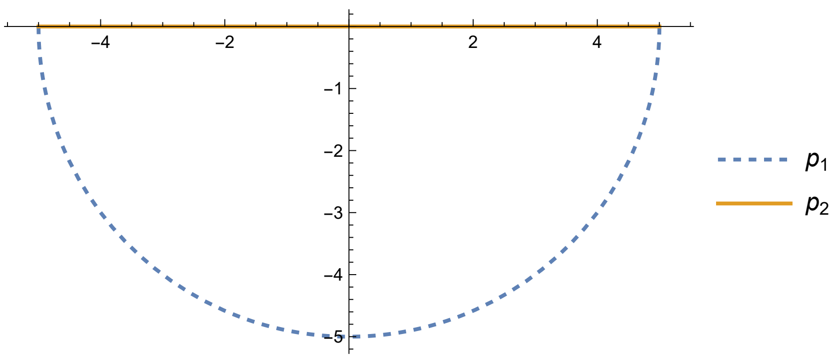
$p_2(t) = \boxed{}$ for $0 \le t < 1$

$\int_\Gamma f(z)\, \mathrm{dz} = \int_0^1 \boxed{}\, \mathrm{dt} + \int_0^1 \boxed{}\, \mathrm{dt}.$

For $f(z) = z^2$ evaluate $\int_\Gamma f(z)\, \mathrm{dz} = \boxed{}$ and

$\int_{\Gamma_2} f(z)\, \mathrm{dz} = \boxed{}$. Show appropriate work below.

Out[10]=



12. Compute $\int_\Gamma \frac{e^z}{(z^2+2z+1)(z+i)}\, dz = \boxed{}$ for the CCW contour Γ. Show appropriate work.

In[∘]:=
```
p1[t_] := 2 + (E^(2 π i t) + 1.2)^2
ParametricPlot[ReIm[p1[t]], {t, 0, 1}, PlotLegends → {"Γ"},
 AxesOrigin → {0, 0}]
```

Out[∘]=