Name: ______________

Show an appropriate amount of work. You can (and probably should) check your work with a computer.

1. Compute and simplify a bit all possible values of

1.1. $z = \left(\frac{i}{1+i}\right)^3 =$ $\boxed{}$

1.2. Roots of $z^5 = -32\,i$

$z =$ $\boxed{}$

1.3. Roots of $(z - i)\left(z^2 - 4\,i\,z + 5\right) = 0$

$z =$ $\boxed{}$

2. Sketch and label the sets

2.1. Points C_1 satisfying $|z + i| < 2$.

2.2. Points C_2 satisfying $\mathrm{re}(z - i) = 1$.

2.3. Points C_3 satisfying $\arg(z + i) = 3\pi/4$.

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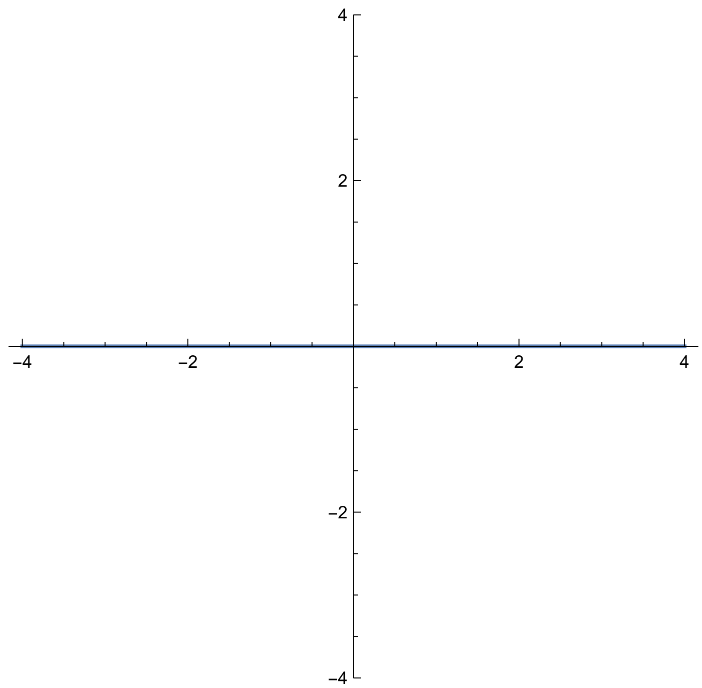

3. For $f(z) = u(x, y) + i\, v(x, y)$ and write down the CR equations

$\boxed{}$ and $\boxed{}$

3.1. Explain why u satisfies $u_{xx} + u_{yy} = 0$.

3.2. Is $i\, x^3 - 3\, x^2\, y - 3\, i\, x\, y^2 + y^3$ analytic? If it is write down $f(z)$.

3.3. Is $i\, x^3 + 3\, x^2\, y - 3\, i\, x\, y^2 + y^3$ analytic? If it is write down $f(z)$.

3.4. Is $e^{x - i\, y}$ analytic? $\boxed{}$.

4. Explain steps and show appropriate work.

4.1. Compute $\int_O e^{1/z}\, dz$ for the ccw unit circle. #5.6.1 p166

4.2. Compute $\int_0^\infty \frac{dx}{x^4+2\,x^2\cos(2\,\alpha)+1}$ #5.6.4 p166

As extra credit if you want to practice trig identities show it equals $\left|\ \frac{\pi}{4\cos(\alpha)}\ \right|$

4.3. Extra credit. Show $\int_0^\infty \frac{e^{-x}-\cos(x)\,dx}{x} = 0$

#5.6.5 p166

4.4. Compute $\int_{-\infty}^\infty \frac{dx}{1+x^{2\,p}}$ #5.6.7 p166

As extra credit if you want to practice trig identities show it equals $\left|\ \frac{\pi}{p\sin\left(\frac{\pi}{2\,p}\right)}\ \right|$

5. Reminder $\frac{d}{dz}(\arcsin(z)) = \frac{1}{\sqrt{1-z^2}}$

5.1. Discuss the singularities of $f(z) = \arcsin(z)$ for $z \in \mathbb{C}$

5.2. Compute two non-zero terms of the Taylor Series for f(z) about $z = 0$.

5.3. Compute the general term of the Taylor Series for f(z) about $z = 0$.

5.4. What is the radius of convergence for the TS about $z = 0$?

5.5. What would be the radius of convergence for the TS about $z = i$?

6. $f(z) = \sum_{n=-\infty}^{n=\infty} a_n z^n$ is the Laurent Series for f about $z = 0$.

6.1. What is the name for a_{-1}? Why is it particularly important?

6.2. Write down a simple formula for a_n with $n \geq 0$ valid when f is analytic near $z = 0$.

6.3. Write down a formula for a_n valid when f is not analytic near $z = 0$. Explain when this formula is valid.

7. Compute $\lim_{n \to \infty} \sum_{k=-n}^{k=n} \frac{1}{n^4+1}$ using a contour integral. Explain your steps and show appropriate work. In this problem you do not need to compute residues. If you have defined f and z_i you can just write res(f, z_i) for the residue.

8. For the closed counter clockwise contour Γ shown below

$p_1(t) = \boxed{}$ for $0 \le t < 1$

$p_2(t) = \boxed{}$ for $0 \le t < 1$

$\int_\Gamma f(z)\,\mathrm{dz} = \int_0^1 \boxed{}\,\mathrm{dt} + \int_0^1 \boxed{}\,\mathrm{dt}.$

For $f(z) = \frac{z^2}{z - i}$ evaluate $\int_\Gamma f(z)\,\mathrm{dz} = \boxed{}$. Show appropriate work below.

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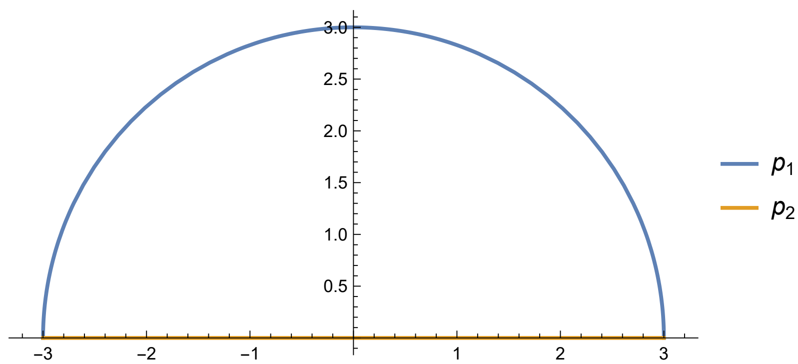

9. For the closed counter clockwise contour Γ shown below

$p_1(t) = \boxed{}$ for $0 \leq t < 1$

$p_2(t) = \boxed{}$ for $0 \leq t < 1$

$p_3(t) = \boxed{}$ for $0 \leq t < 1$

$\int_\Gamma f(z)\,dz = \int_0^1 \boxed{}\,dt +$

$\int_0^1 \boxed{}\,dt + \int_0^1 \boxed{}\,dt$

For $f(z) = \frac{i}{z-(1+2i)}$ evaluate $\int_\Gamma f(z)\,dz = \boxed{}$.

Explain the need for care when evaluating these individual line integrals using FTC and logs.

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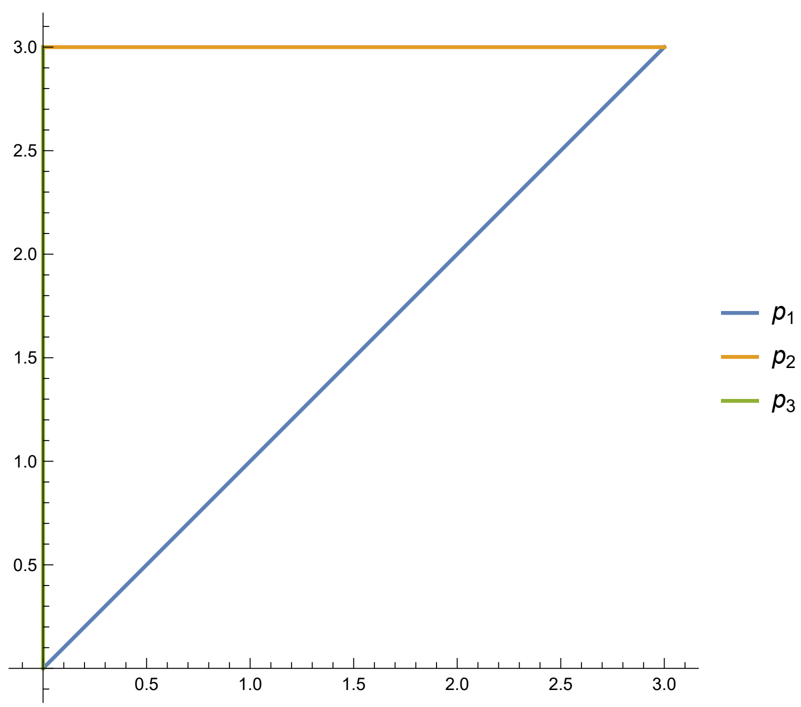

10. Compute $\int_\Gamma \frac{e^z}{(z^2+9)(z-3)}\,dz =$ $\boxed{}$ for the counter clockwise contour Γ. Show appropriate work.

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