

COMP 360: Algorithm Design

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1 Duality

Standard form:

$$\begin{aligned}
 \max \quad & cx_1 + cx_2 + \dots \\
 \text{s.t.} \quad & a_1x_1 + \dots + a_{1n}x_n \leq b_1 \\
 & x_1, \dots, x_n \geq 0
 \end{aligned} \tag{1}$$

(Or \min, \geq, \geq)

- If $x_1 + x_2 = 0$, replace with $x_1 + x_2 \leq 2, x_1 + x_2 \geq 2$
- If $x_1 \leq 0$, replace with $x'_1 = -x_1$
- If no $x_2 \geq 0$, create $x'_2 \geq 0, x''_2 \geq 0, x_2 = x'_2 - x''_2$

If either primal/dual are unbounded, counterpart is infeasible.

- Weak: $\underbrace{\text{opt}(\text{primal})}_{\max} \geq \underbrace{\text{opt}(\text{dual})}_{\min}$
 - Strong: $\text{opt}(\text{primal}) = \text{opt}(\text{dual})$
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$$\begin{aligned}
 \max \quad & f_{ts} \\
 \text{s.t.} \quad & f_{uv} \leq c_{uv} \quad \forall u \in V \\
 & \sum_{vu \in E'} f_{vu} - \sum_{uw \in E'} f_{uw} = 0 \quad \forall u \in V \\
 & f_{uv} \geq 0 \quad \forall uv \in E'
 \end{aligned}$$

$$\begin{aligned}
 \min \quad & \sum_{uv \in E} c_{uv} x_{uv} \\
 & y_s - y_t \geq 1 \\
 & x_{uv} + y_v - y_u \geq 0 \quad \forall uv \in E \\
 & x_{uv} \geq 0 \quad \forall uv \in E \\
 & y_u \text{ free} \quad \forall u
 \end{aligned} \tag{2}$$

2 P NP