

DIFFERENTIAL GEOMETRY : CURVED COMPUTING :

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- A shell is comprised of its reference surface, thickness and edges. The thickness of the shell at a point is found as the distance between the bounding surface (Top and bottom surfaces) as measured along a normal to the reference surface that passes through the point.
- FIX ME???????????? New things also have to be added

- A 3d curve in a rectangular coordinate system (x_1, x_2, x_3) can be represented by the locus of the end point of the position vector

$$\mathbf{x} = x_1(t)\mathbf{e}_1 + x_2(t)\mathbf{e}_2 + x_3(t)\mathbf{e}_3 \quad (1)$$

for all the values of the parameter t that are in the interval $t_1 < t < t_2$. Suppose that \mathbf{x} takes only a single value of the parameter t , then we insure it's uniqueness.

- We take s as the variable of the arc length along the space curve. The derivative of the position vector \mathbf{x} with respect to s is given as

$$\frac{d\mathbf{x}}{ds} = \frac{dx_1}{ds}\mathbf{e}_1 + \frac{dx_2}{ds}\mathbf{e}_2 + \frac{dx_3}{ds}\mathbf{e}_3 \quad (2)$$

- If we form a scalar product with itself we get

$$\frac{d\mathbf{x}}{ds} \cdot \frac{d\mathbf{x}}{ds} = \left(\frac{dx_1}{ds}\right)^2 + \left(\frac{dx_2}{ds}\right)^2 + \left(\frac{dx_3}{ds}\right)^2 \quad (3)$$

- From differential calculus we know that

$$(ds)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 \quad (4)$$

Hence

$$\frac{d\mathbf{x}}{ds} \cdot \frac{d\mathbf{x}}{ds} = 1 \quad (5)$$

This confuses me how the magnitude of the tangent becomes 1, because sometimes in calculus some manipulations seem very devious

- We can think of a vector $\Delta \mathbf{x}$ joining two points Q and Q' on a curve. The vector $\Delta \mathbf{x} / \Delta s$ has the same direction as $\Delta \mathbf{x}$ and as Δs approaches 0, $\Delta \mathbf{x} / \Delta s$ becomes the tangent vector to the curve at the point Q.
- The vector \mathbf{T} is as the limit of $\Delta s \rightarrow 0$ we get

$$\mathbf{T} = \frac{d\mathbf{x}}{ds} \quad (6)$$

- We also note that

$$\mathbf{x}' = \frac{d\mathbf{x}}{dt} = \frac{d\mathbf{x}}{ds} \frac{ds}{dt} \quad (7)$$

This comes from chain rule by the way the parametrisation is done originally. This is also a tangent vector but it is not of unit length

- The tangent to the Curve at Q is found to be the limiting position of the line connecting the points Q and Q'. Now if we consider the limiting position of a plane passing through three consecutive places. We will get the osculating plane joining the normal and the tangent of a curve.
- The tripple scalar produc of three coplanar vectors is zero and the expression of the osculating plane is as

$$(\mathbf{X} - \mathbf{x}) \cdot (\mathbf{x}' \times \mathbf{x}'') \quad (8)$$

- By taking the dot product with itself $\mathbf{T} \cdot \mathbf{T} = 1$. If we differentiate this scalar product with respect to the arc length we get

$$\frac{d}{ds}(\mathbf{T} \cdot \mathbf{T}) = 2\mathbf{T} \cdot \mathbf{T}^o = 0 \quad (9)$$

where \mathbf{T}^o is the derivative with respect to s . Now this tells us that the tangent is perpendicular to the curvature. We can also then write

$$\mathbf{T} = \frac{d\mathbf{x}}{ds} = \frac{d\mathbf{x}}{dt} \frac{dt}{ds} = \mathbf{x}' t^o \quad (10)$$

where this comes from chain rule but the parameterisation is messed up, originally it is $\frac{d\mathbf{x}}{dt} = \frac{d\mathbf{x}}{ds} \frac{ds}{dt}$. I don't know when things can be used like fractions!! But it checks out here. Also note that s and t will be different functions for different components of \mathbf{x}

- Therefore we can say

$$\mathbf{T}^o = \mathbf{x}' t^{oo} + \mathbf{x}'' (t^o)^2 \quad (11)$$

Which shows that the curvature also lies in the plane of \mathbf{x}' and \mathbf{x}'' , or the osculating plane.

- Since the curvature has been shown to be perpendicular to the tangent \mathbf{T} we can say that the curvature is parallel to the principal normal given by

$$\mathbf{T}^o = k\mathbf{N} = \mathbf{k} \quad (12)$$

where \mathbf{N} is a unit normal vector in the direction of the principal normal to the curve at a point $(1/R)$. We can see that, k is the curvature and $k\mathbf{N}$ is the curvature vector.

- Although the sense of \mathbf{T}^o is determined solely by the curve, the sense of the principal normal \mathbf{N} is arbitrary. k therefore depends on the sense of \mathbf{N} .