Scaling : *March* 3, 2021

Scaling (Fox Stanton 1968)

- Sometimes we want to keep all the variables in the same order of magnitude (suppose forces, moments. Displacements, rotations)
- We can simply scale bya simple multiplication of the individual dof with a non singular diagonal transofmratio matrix
- Eg we can have zero eigen values for rigid body displacements. Sometimes we can have the non zero eigen values range over six orders and some dof have larger strain energies than others.

- This theorem states that every eigenvalue of a matrix **A** with enteties a_{ij} lies in at leasat one of the disks centered at a_{ii} and of radii R_i

$$R_i = \sum_{j \neq i} |a_{ij}| \tag{1}$$

where R_i is the sum of the absolute values of the non diagonal enteries in the ith row

- Let $\mathrm{D}(a_{ii}, R_i)$ be a closed disc centered at a with radius R. Such a disc is a Gershgorin disc.
- Every eigen value of A lies within at least one of the Gersgorin discs.
- Shift in orders can be because of the shift in the center without the raius isze also. This happens when the condition number is affected by change in phyical properties (Like size of the problem)

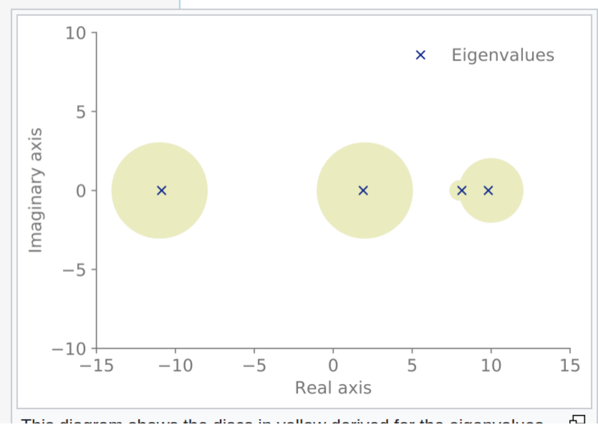
Eigenvalues
Imaginary axis
Real axis
10
5
0
−5
−10
−15
−10
−5
0
5
10
15

Figure:

- Suppose we have the potential energy in the form

$$\Pi(X) = \frac{1}{2}X^T KX - X^T F \tag{2}$$

- Operated in scale coordiantes as

$$\Pi(Y) = \frac{1}{2}Y^T K' Y - Y^T F' \tag{3}$$

where $K' = D^T KD \qquad F' = D^T F$
$d_{ii} = \frac{1}{c(k_i i)^{1/2}} \qquad d_{ij} = 0 \qquad i \neq j$

- This centers all the Gerschogorin disks at the same point namely $1/c^2$
- c $=1/n^{1/2}$ where n is the max no of nonzero elements is nonzero is a good choice.
- Can also be shown that they are bounded. BUT WHAT ABOUT NONLINEAR
- The existence of the scaling transformation requires that the diagonals > 0. Which is satisfied in positive definite matrixes.
- Can read Developments in structural analysis by direct energy minimization for more details.