1 Nonlinear bending of shells

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- Shells are much like plates except that they are curved.
- FEM models of shells are developed using (1) shell theory or (2) 2-D equations obtained from a degenerated 3D elasticity model
- Shell theory are developed, are origninally based on Kirchoff-Love kinematic hypothesis
- Some group of shell theories is based on order magnitude on strains and rotations in full nonlinear equations (called finite rotation theories).
    - Strains and rotations about the normal to he surface are assumed to be of order $\epsilon << 1$
    - Roations about tangents to the surface are organised in a consistent classiciation where for each range of magnitude of ratioans specific shell euqations are obtained.
- Shells can be synclastic or anticlastic. A curved surface is developable if it can be developed to a plane without sretching. Nondevelopable requires cuting or deforming. These are stronger than developable because they need additional forces to collapse to planar surfaces.

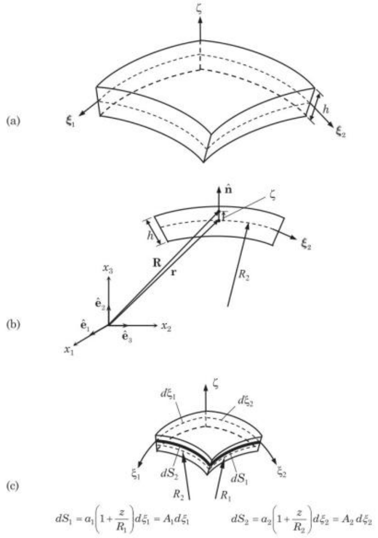

Fig. 8.2.1: (a) Shell geometry. (b) Position vector of a point and coordinates on the middle surface. (c) Position vectors of points on the middle surface and above the middle surface.

- Take a curved shell element of uniform thickness. Here (ξ_1, ξ_2, ζ) which denote the curvilinear coordinates such that ξ_1, ξ_2 curves are the lines of crvature of the middle surface $(\zeta = 0)$.
- The position vector of a point $(\xi_1, \xi_2, 0)$ is denote by $\mathbf{r}$ and any other arbitary point is denoted by $\mathbf{R}$
- A differential line element on the middle suface can be written as

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial \xi_a} d\xi_a = \mathbf{g}_a d\xi_a, \quad \mathbf{g}_a = \frac{\partial \mathbf{r}}{\partial \xi_a} \quad (a = 1, 2) \tag{1}$$

where the vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ are tangent to the ξ_1, ξ_2 coordinates lines as shown.

- The components of the metric tensor g_{ab}, (a,b=1,2) are

$$\mathbf{g}_a . \mathbf{g}_b = g_{ab}, \qquad p_1 = \sqrt{g_{11}}, \qquad p_2 = \sqrt{g_{22}}, \qquad \mathbf{g_1} . \mathbf{g_2} = p_1 p_2 cos\chi \tag{2}$$

where χ denotes the angle between the coordinate curves. [1]

- The normal vector is found as

$$\hat{\mathbf{n}} = \frac{\mathbf{g_1} \times \mathbf{g_2}}{p_1 p_2 sin\chi} \tag{3}$$

[1] Note that $\hat{\mathbf{n}}$, p_1, p_2 are functions of ξ_1, ξ_2

- The square of the distance dS,say between points $(\xi_1, \xi_2, 0)$ and $(\xi_1 + d\xi_1, \xi_2 + d\xi_2, 0)$ on th emiddle surface is given by

$$(ds)^2 = d\mathbf{r}.d\mathbf{r} = g_{ab} d\xi_a d\xi_b = p_1^2 (d\xi_1)^2 + p_2^2 (d\xi_2)^2 + 2a_1 a_2 cos\ \chi\ d\xi_1 d\xi_2 \qquad (4)$$

  The RHS is called the first quadratic form of the surface which allows us to find infinitesimal lengths, angles and area. The terms $p_1^2,\ p_2^2,\ p_1 p_2 cos\chi$ are called the first fundamental quantities

- Let $\mathbf{r} = \mathbf{r}(s)$ be the equation of a curve s on the surface. The unit vector tangent to the curve is

$$\hat{\mathbf{t}} = \frac{d\mathbf{r}}{ds} = \frac{\partial \mathbf{r}}{\partial \xi_a} \frac{\partial \xi_a}{\partial s} = \mathbf{g}_a \frac{\partial \xi_a}{\partial s} \qquad (5)$$

Im not writing this because the conventions are too long. Please check the annotated book