1 Tangent matrix : February 18, 2021

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- Linearity allows us to know the solution at any value. What I mean is if we have a function f(x), we can find the slope at any point of f and then get the solution
- In nonlinear cases, the slope at one point may not represent the actual solution at another so we have an issue. So we linearise the solution at some point and then find how far off is the linear solution from the nonlinear one.
- We will focus here on different derivations of the tangent stiffness matrix for different domains

- For the euler beam element we get the stiffness matrix as follows $\begin{bmatrix} K^{11} & K^{12} \\ K^{21} & K^{22} \end{bmatrix} \begin{bmatrix} \Delta^1 \\ \Delta^2 \end{bmatrix} = \begin{bmatrix} F^1 \\ F^2 \end{bmatrix}$ where we see a coupling between the two dof here (Note there are more dof inside each one)
- Sometimes we can derive the stiffness matrix terms involving the von karman nonlinearity $\frac{\partial dw}{\partial dx}$ insuch a way that $K^{12} = 0$ and so the nonlinear equations become uncoupled and we can solve them iteratively. In this case u and w will become nonlinear terms. However the tangent matrix will remain the same.

- In direct method, we just keep looping through the process till the nonlinear terms or the residual stabilise.
- Here we talk about the NR method:
- There are two things which are a bit complicated. One is the position of a body and the other is the displacement whereby it moves towards the equilibrium state.
- We see from the directional derivative notes, that the directional derivative is used to linearise two things, one is the functional of the potential energy which gives us our equilibrium equations
- The other linearisation is of these equilibrium equations giving us a linearised version of the equations at a location.

- The main problem lies I guess in the assumption of small deformations. FIX ME!!!!!!!!!!!!!!!!!
- We have a functional which is has both internal and external work . $\Pi = d^T Kd - Fd$
- This energy functional is therefore dependant on the intial undeformed geometry. Based on it if we move by some displacement d, we change the internal energy and potential of the external work.
- In the Updated weight method, we however keep the energy in terms of the location in space. Once we found the first directional derivative, invariably we became intersted in displacements. But these displacements were of an energy functional that depended only on the position.
- What is then the difference between du and dx??? I think that u is the way to think about it.
- The first directional derivative gives you the direction that you need to move your displacement such that your potential energy gets minimized. Again note that the potential is ususally referenced to an undeformed shape.

- We have the residual equation given as $\mathbf{R} = \mathbf{F} - \mathbf{K}\Delta$
- Linearising this we get $\mathbf{R^{i+1}} = \mathbf{R^i} + \frac{dR}{d\Delta}|_i \delta\Delta$
  Interesting thing to note is that the derivative of the residual is with respect to the displacements, and the small amount we move is in a small direction in that displacement we're actually on. That is the displacement increment !!! ΔU

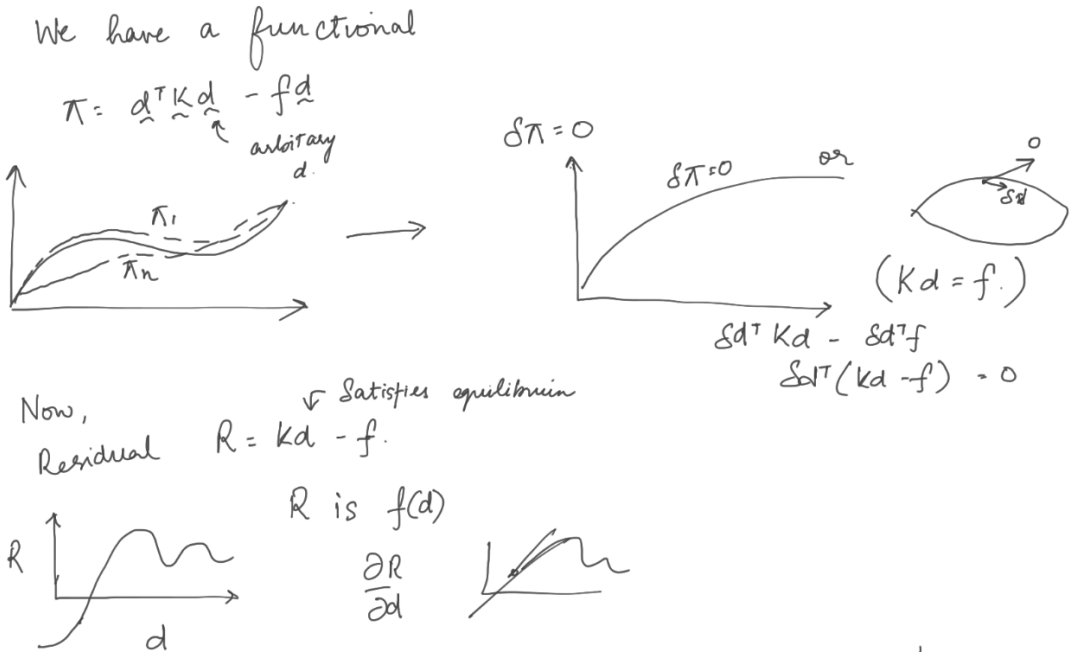

Figure:

- In Reddy 5.2.43 we get our displacement increment, which is $\Delta U = -\left(T(U^{r-1})\right)^{-1} R^{r-1}$
- Where the next solution is then found as $\mathbf{U^r} = \mathbf{U^{r-1}} + \Delta\mathbf{U}$
- The coefficients of the tangent stiffness amtrix has a similar structure to **K** so we can have the form $\begin{bmatrix} T^{11} & T^{12} \\ T^{21} & T^{22} \end{bmatrix} \begin{bmatrix} \delta\Delta^1 \\ \delta\Delta^2 \end{bmatrix} = - \begin{bmatrix} R^1 \\ R^2 \end{bmatrix}$
- Note that each coeffcients in both **T** and Δ are matrices and vectors.
- So lets say that $T^{11}\delta\Delta^1 + T^{12}\delta\Delta^2 = -R^1$ or $T^{11}_{ij}\delta\Delta^1_j + T^{12}_{ij}\delta\Delta^2_j = -R^1_i$
- Here we note that each component of T and Δ are matrices.

where symbol δ is used in place of Δ, for obvious reason, to denote the increment of the displacements. Also, note that $\boldsymbol{\Delta}^1 = \mathbf{u}^e$ and $\boldsymbol{\Delta}^2 = \bar{\boldsymbol{\Delta}}^e$. Then we can compute $\mathbf{T}^{\alpha\beta}$ from the definition

$$T_{ij}^{\alpha\beta} = \frac{\partial R_i^\alpha}{\partial \Delta_j^\beta}, \quad \alpha, \beta = 1, 2 \tag{5.2.46}$$

The components R_i^α of the residual vector $\mathbf{R}$ can be expressed as

$$\begin{aligned} R_i^\alpha &= \sum_{\gamma=1}^{2} \sum_{p=1}^{n(\gamma)} K_{ip}^{\alpha\gamma} \Delta_p^\gamma - F_i^\alpha = \sum_{p=1}^{2} K_{ip}^{\alpha 1} \Delta_p^1 + \sum_{P=1}^{4} K_{iP}^{\alpha 2} \Delta_P^2 - F_i^\alpha \\ &= \sum_{p=1}^{2} K_{ip}^{\alpha 1} u_p + \sum_{P=1}^{4} K_{iP}^{\alpha 2} \bar{\Delta}_P - F_i^\alpha \end{aligned} \tag{5.2.47}$$

where $n(\gamma)$ denotes the number of element degrees of freedom [$n(1) = 2$ and $n(2) = 4$]. We have

$$T_{ij}^{\alpha\beta} = \frac{\partial R_i^\alpha}{\partial \Delta_j^\beta} = \frac{\partial}{\partial \Delta_j^\beta} \left(\sum_{\gamma=1}^{2} \sum_{p=1}^{n(\gamma)} K_{ip}^{\alpha\gamma} \Delta_p^\gamma - F_i^\alpha \right)$$

- Here we see that the top indices are for the smaller condensed matrix while the lower one takes care of every degree of freedom.
- Note that $n(\gamma)$ means the summation differs depending on whether its u(2 dof per node) and $\bar{\Delta}$ having 4 dof per node.
- Pretty interesting