1. Object

2. 1st order tensors: Vectors

3. 2nd order tensors : (Many types of objects!)

Tensors

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2. 1st order tensors: Vectors

3. 2nd order tensors : (Many types of objects!)

Object

- Tensors are objects. An object is a some physical quantity that you can say exists in real life. Examples are forces, stresses etc.
- Each tensor has a co-ordinate frame where you can describe it. For eg : You would need to define a vector by how much it goes in unit basis directions like e^1, e^2, e^3. So we can say that a vector may be defined as (3,4,5) describing how we move in e^1, e^2, e^3 to represent the vector in the basis and so on.

1st order tensors: Vectors

- A vector can be representated by some components. These components are related with a basis.

$$\mathbf{v} = v_i e_i \tag{1}$$

- The basis is defined by the right hand rule
- Main operations in vectors are
  - Scalar product or the dot product
  - Cross product
  - Vector basis transformation

- The dot product is defined as such

$$\begin{aligned} \mathbf{v}.\mathbf{u} = v_i e_i . u_j e_j = v_i u_j \; e_i . e_j = v_i u_i \delta_{ij} = v_i u_i \\ \textit{or} \\ v = (v_1 e_1 + v_2 e_2 + v_3 e_3)(u_1 e_1 + u_2 e_2 + u_3 e_3) \end{aligned} \tag{2}$$

- e_1, e_2, e_3 are orthogonal to each other therefore $\delta_{ij} = 1, 0$

- As explained, every vector is a physical object but the way we define it depends on us.
- The way a vector is defined is with respect to the components for any basis. For some basis $\mathbf{e^1}, \mathbf{e^2}, \mathbf{e^3}$, we can write as follows

$$\mathbf{v} = \begin{bmatrix} \mathbf{e_1} & \mathbf{e_2} & \mathbf{e_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- Therefore in some other basis the vector can be defined with different components as :

$$\mathbf{v} = \begin{bmatrix} \mathbf{e'_1} & \mathbf{e'_2} & \mathbf{e'_3} \end{bmatrix} \begin{bmatrix} v'_1 \\ v'_2 \\ v'_3 \end{bmatrix} = \mathbf{Q} \begin{bmatrix} v'_1 \\ v'_2 \\ v'_3 \end{bmatrix}$$

- Therefore $\begin{bmatrix} v'_1 \\ v'_2 \\ v'_3 \end{bmatrix} = \mathbf{Q^{-1}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

- $\mathbf{Q}$ is the position of the new basis with respect to the old one. That's why we can operate $\mathbf{Q}$ on $\mathbf{v}$

$$\mathbf{Q} = \begin{bmatrix} e1.e1' & e1.e2' & e1.e3' \\ e2.e1' & e2.e2' & e2.e3' \\ e3.e1' & e3.e2' & e3.e3' \end{bmatrix} \tag{3}$$

- The first column, gives the location of e1 with respect to the old basis. As we can see it gives the direction cosines. And so on
- If we do choose the new basis an orthogonal basis, we get $\mathbf{Q^{-1}} = \mathbf{Q^T}$
- $[v]' = \mathbf{Q^T}[v]$
- $[v] = \mathbf{Q}[v]'$
- [] denotes that we are working with only the components, in basic matrix form

2nd order tensors : (Many types of objects!)

- A second order tensor can be a linear map from one vector to another vector
- It can be a mapping that takes two vectors and gives a scalar
- But for now we'll focus mainly on the first thing

Linear map $\mathbf{S}$

$\mathbf{v} = \mathbf{S}\mathbf{u}$

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