Linearisation : February 18, 2021

- In the directional derivative section, we have seen how we can linearise the potential energy or any function, functional
- We get our equilibrium equation $\mathbf{Kx} = \mathbf{F}$
- This equation can be nonlinear with respect to x. So we have a residual function $\mathbf{R}(\mathbf{x}) = \mathbf{Kx} - \mathbf{F}$ which again can be thought as something that we can find the linear change

- Suppose $R = \begin{bmatrix} R_1(x_1 \; x_2) \\ R_2(x_1 \; x_2) \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\mathbf{R}(\mathbf{x_{i+1}}) \approx \mathbf{R}(\mathbf{x_i}) + \mathbf{DR}(\mathbf{x_k})[\mathbf{u}]$$
$$D\mathbf{R}(x_k)[u] = \frac{d}{d\epsilon} R(\mathbf{x_k} + \epsilon \mathbf{u})$$
$$= \frac{d}{d\epsilon} \begin{bmatrix} R_1(x_1 + \epsilon u_1 \; x_2 + \epsilon u_2) \\ R_2(x_1 + \epsilon u_1 \; x_2 + \epsilon u_2) \end{bmatrix}$$
$$R(x_i) = \mathbf{K_T u} \qquad \textit{Taking} \quad \mathbf{R}(\mathbf{x_{i+1}}) = \mathbf{0}$$

- Where K_T is the tangent stiffness, and then we can find **u** until R is zero [1]
- $K_T = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$ and $u = K_T^{-1} F$

[1] $R_1(x + \epsilon u) = 2k(x_1 + \epsilon u_1) - k(x_2 + \epsilon u_2)$ and $R_2(x + u) = -k(x_1 + \epsilon u_1) + k(x_2 + \epsilon u_2) - F$