Mechanics

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Linearity and non-linearity

- Suppose there is a function or mapping that does this $f : x \longrightarrow y$. Function f takes x and gives y
- The function is linear if $f(x) = f(x_1) + f(x_2)$. where $x = x_1 + x_2$
- Suppose we know $f()$ and y but not x. Because the function is linear, we can construct solutions x that may be a combination of different solutions x_i. The idea is that f should not depend on x. It can have other parameters, but not the values of the solution itself. In that case, we would have to find the solution x and also the mapping f

Example

Take function or map ()

Structural non-linearity introduction

- What does non-linearity mean ?
- What makes a structure non-linear ?
- How do you model non-linearity and solve it?

- There are different types of non-linearity in a structure
  - Geometrical - $\mathbf{K}(\mathbf{x})\mathbf{x} = \mathbf{F}$ where here $\mathbf{K}$ depends on $\mathbf{x}$ because $\mathbf{K}$ changes as you chagne x due to the change in geometry and the large deformation.
  - Material - $\mathbf{K}(\mathbf{x})\mathbf{x} = \mathbf{F}$ because $\mathbf{K}$ changes due to the changes in the material property. For eg Youngs modulus may be a function of x.
- **K** is now a function of (x), it means that it can't be solved by using a linear method (Inverting a matrix).
- But the system of equations now has to be linearised about a point. We keep the system of equations as an equality equation $\mathbf{K}(\mathbf{x})\mathbf{x} - \mathbf{F} = 0$.
- We linearise and try to solve the root. At every point we construct the tangent and find a solution, but when we construct the solution at that point, we find the residual and we then use to iterate until convergence.

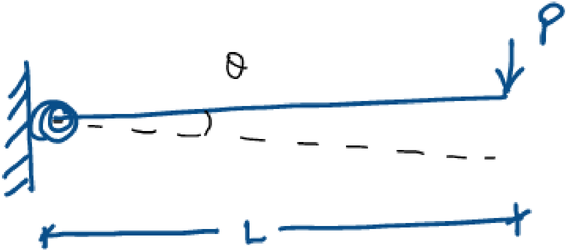


Figure:

Taking a weightless rigid bar with a torsional spring that resists any moment and a load P at the end. Since the bar is rigid so vertical equilibrium is satisfied at the support without any deformation. However, it can rigidly rotate through the spring.

The moment generated at the end support is

$$PLcos\theta = M \tag{1}$$

Taking the equilibrium equation with the spring as $M = K\theta$ we get

$$PLcos\theta = K\theta$$
$$\frac{PL}{K} = \frac{\theta}{cos\theta}$$

- If we take $\theta \rightarrow 0$ then $cos\theta \rightarrow 1$
- So $P = \frac{K}{L}\theta$ which is linear wrt θ

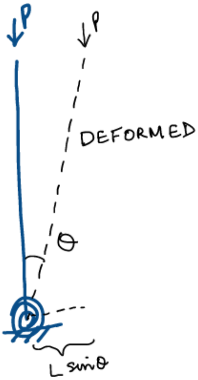


Figure:

Same problem but now the rigid bar is vertical. This is a very common problem for nonlinearity (I think the previos one is better tho!). Nicely represents buckling of columns

Same equilibrium equation but now the lever arm is different (Because the load along the bar)

$$PLsin\theta = M \tag{2}$$

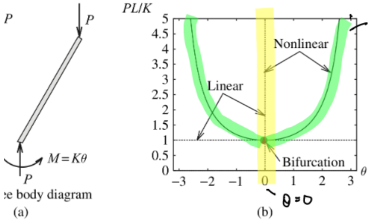
Taking the equilibrium equation with the spring as $M = K\theta$ we get

$$PLsin\theta = K\theta$$

$$\frac{PL}{K} = \frac{\theta}{sin\theta}$$

- If we take $\theta \rightarrow 0$ then $sin\theta \rightarrow 0$ so $M = 0$ (This is one possible equilibrium)
- $\frac{PL}{K} = \frac{\theta}{sin\theta}$: This is the other
- The load $\frac{PL}{K}$ where these two equilibrium equations are possible for the same structure is called the bifurcation point.
- You can imagine that when the load is smaller, it will not buckle and only deform axially so only one equilibrium position ($\theta = 0$) is possible. When $\frac{PL}{K} > 1$ then the other equilibrium comes to play.

Therefore two solutions are there :



ɜimple column.

$$\frac{PL}{K} = \frac{\theta}{\sin\theta}$$

- Below $\frac{PL}{K} < 1$, Only Equilibrium is $\theta = 0$

Because $\frac{PL}{K} < 1$, $\frac{\theta}{\sin\theta} \not< 1$ so $\theta = 0$

- Above $\frac{PL}{K} > 1$, we get two possible solutions
  - $\theta = 0$: $M = 0$
  - $\theta \neq 0$: $\frac{PL}{K} = \frac{\theta}{\sin\theta}$.

- If we linearise our equilibrium equation for small $\theta \rightarrow sin\theta$
- We get $(Pl - K)\theta = 0$ and we get our linear eigen value problem with a trivial solution of $\theta = 0$ and a nontrivial solution of $PL = K$. Again we get two equilibrium solutions. $PL = K$ being the buckling load.
- So when we reach that load, it means we have reached the bifurcation point, where multiple equilibrium solutions exist and the rod may buckle dependant on imperfection, lateral load etc.

Non-Linear strain measures

- Now suppose of being rigid, the body is deformable that is the relative deformation is introducing strain and stresses
- LARGE DISPLACEMENTS + LARGE STRAINS
- The first deals with displacements that are large, and therefore while finding the strains, we have to use higher orders of the displacement derivatives
- The second deals with

- Emphasize its only for One D! But same theory for other dimensions
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