

Runge-Kutta Method Presentation

Using Beamer

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Introduction

Runge-Kutta Method is a series of methods used to solve differential equations with more accuracy without performing many calculations. It assumes the following solutions:

- 1st Order (Eulers Method)
- 2nd Order Runge-Kutta Method
- 4th Order Runge-Kutta Method (RK4)

1st Order Runge-Kutta Method

It uses the following formular:

$$y(x + h) = y(x) + hf(x, y)$$

To construct the tangent at the point x and obtain the value of $y(x + h)$ whose slope is:

$$f(x, y) \text{ or simply, } \frac{dy}{dx}$$

General formula: $y_{i+1} = y_i + hf(x_i, y_i)$

NB

This is the easiest of all the methods but has an error margin

2nd Order Runge-Kutta Method

The Runge-Kutta method finds an approximate value of y for a given x . Only first-order ordinary differential equations can be solved by using the Runge Kutta 2nd order method.

Below is the formula used to compute next value: y_{n+1} from previous value

y_n : y_{n+1} = value of y at $(x = n + 1)$

y_n = value of y at $(x = n)$ where:

$0 \leq n \leq (x - x_0)/h$; h is step height

$x_{n+1} = x_0 + h$

cont...

The essential formula to compute the value of $y(n+1)$:

$$K_1 = h * f(x_n, y_n)$$

$$K_2 = h * f\left(x_n + \frac{h}{2}, y_n + \frac{K_1 * h}{2}\right)$$

$$y_{n+1} = y_n + K_2 + (h^3)$$

2nd order Continuation

- The formula basically computes the next value y_{n+1} using current y_n plus the weighted average of two increments:
- K_1 is the increment based on the slope at the beginning of the interval, using y .
- K_2 is the increment based on the slope at the midpoint of the interval, using $(y + h * K_1/2)$.
- The method is a second-order method, meaning that the local truncation error is on the order of $O(h^3)$, while the total accumulated error is order $O(h^4)$.

4th Order Runge-Kutta Method

To be presented by Nelson

TIP

4th Order Runge-Kutta method also called RK4 is the most important in the series