

# Towards the Notion of Average Trajectory of the Repeating Motion of Human Limbs

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**Abstract.** Average trajectory of the repeating motion of the limb joint defined in this paper. Majority of the existing results communicate analysis of the motions by means of different numeric parameters which does not necessary provide desirable feedback for those without deep knowledge of the subject. The notion of the average trajectory defined as the pair, consisting of trajectory, describing the shape of the joint motion, and pipe-shaped neighbourhood, describing variability of the observed motions. Proposed notion allows visualisation in three-dimensional space, which is easily interpretable and in turn may be used as feedback communicating results of the training or therapy session. Numeric parameters are associated with the average trajectory to validate proposed definition.

**Keywords:** Average trajectory · Limb motion · Motor performance

## 1 Introduction

Medicine, psychology and sport are the main areas where analysis of human motor functions used to determine condition of the individual (trainee or patient). Trainees or patients (undergoing rehabilitation of their motor functions) may benefit from the visualization of a training or therapy session. Another direction demonstrating growing interest towards evaluation of motor performance is the learning of specific movements, for example in laparoscope surgery [3, 5] or military training [4]. It is assumed that in these areas condition (either level of training or stage of the disease) of the individual related to the motor performance. Motion capture system records the motions of the joints of interests, observed during one session. Such sessions usually consist of number repeated exercises, which constitute object of the research reported in this paper.

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The notion of *average trajectory of the motion* introduced in the present paper. Its main purpose is to describe overall shape and deviation of the repeated motion captured during training or therapy session in an easily interpretable way. Nowadays majority of techniques describing motions of the human limbs rely on different numeric parameters. Main drawback of this approach is that it may be difficult to interpret and explain to those without scientific background of the field. Majority of the existing results, describing and comparing motor performance, are based on feature extraction techniques [2], where some parameters are assigned to the certain time instances. Another approach [6] proposed a set of so called motion mass parameters to describe amount and smoothness of the motion observed over a certain time interval and not at a particular time instance. Projections of the trajectories onto two dimensional planes and even coordinate wise time parameterizations are analyzed [7]. The main drawback of such approach is that it is difficult to interpret which in turn motivates the idea of more understandable geometric representation of the results. While the idea of the geometric approach is not new, majority of the available results achieved in the domain of video processing and not suited for evaluation of motor performances. In [1] along with parametric approach, trajectories of a certain joints presented in the form of 3D diagram. In [10, 11] the idea of motion region is introduced. The latest, by its nature, is the closest to the results reported in the present paper.

The paper organized as follows. Formal problem statement, background behind this research, conditions and limitations are explained in Sect. 2. Average trajectory formally defined in Sect. 3. Illustrative examples are presented in Sect. 4. Section 5 discusses different aspects of proposed notion. Concluding remarks presented in the last section.

## 2 Background and Problem Statement

Formally, the problem may be stated in the following way.

- Let  $T$  is the set of  $n$  trajectories corresponding to the  $n$  captures of the motion of interest observed during one session. Define a trajectory  $\bar{T}$ , such that its each point would be the closest to the corresponding points of  $n$  captured trajectories, with respect to the certain parameter.
- For each point of the trajectory  $\bar{T}$  define a convex shape describing the variance of the corresponding points of the captured trajectories.
- Associate the set of numeric parameters enabling to evaluate proposed technique.

Last condition refers the necessity to validate achieved results. And will be discussed in Sect. 3.

One of the motivations behind present research was the desire to visualise changes in motor functions caused by training or therapy. Within the frameworks of the present research, learning of a new motor activity will be used to illustrate proposed notions and evaluate their suitability. The choice of the learning process instead of rehabilitation or progressing disease imposes fewer restrictions

and allows concentrating attention on the proposed idea. The action of the ball throwing into the basket will be considered. The shape of the basket, weight of the ball and distance to the basket chosen to make learning (training) necessary. On the one hand it is easy to distinguish successful and unsuccessful trials and easy to capture. On the other hand, the action requires learning (or it is always possible to adjust conditions in a way that learning (training) is required). The learning process is organised into sessions separated by the equal time intervals. Each session consists of ten trials - attempts to throw the ball into the basket. Trainees are not allowed to practice between the sessions. Motion capture and necessary post processing of the captured data for each session will result ten numeric arrays describing motion trajectories of the joints of interest. Since trajectory (ordered set of points) is used to describe the movement of the joint, it is obvious that trajectory may be used to describe generalization of the joint movements observed during given session. In this case, one may compare observed trajectories to the trajectory representing generalized movement of the joint and determine variance at each particular point.

When talking about averaging sequential or (time)-series data Dynamic Time Warping (DTW) and its variations usually mentioned first. While DTW suits well as the measure of similarity for two trajectories, its application for the averaging trajectories does not satisfy the goals of the present paper. Also unlike the case of similarity computation there is no common agreement about DTW averaging, number of different approaches exists [8].

## 2.1 Motion Mass Parameters

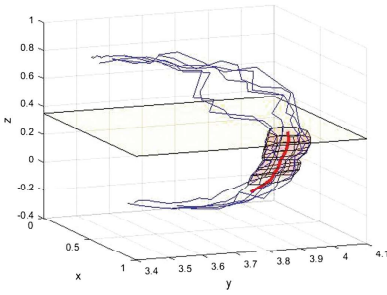
Motion Mass parameters is relatively new notion. In addition, here it is applied in slightly extended form. In order to make this paper self-sufficient let us remind the notion as it introduced in [6]. Let  $J$  be the set of human body joints, such that each point  $j_i$  represent one joint.  $J = \{j_1, \dots, j_n\}$ ,  $n$  is the number of the joints in consideration. With each joint  $j_i$  three following parameters are associated. The length of trajectory  $T_{j_i}$  observed during the movement. *Acceleration mass*  $A_{j_i}$  the sum of the absolute values of the accelerations observed at each observation point. Euclidean distance  $E_{j_i}$  computed between the locations of the joint in the beginning end ending of the movement.

$$T_J = \sum_{i=1}^n T_{j_i}; \quad A_J = \sum_{i=1}^n A_{j_i}; \quad E_J = \sum_{i=1}^n E_{j_i} \quad (1)$$

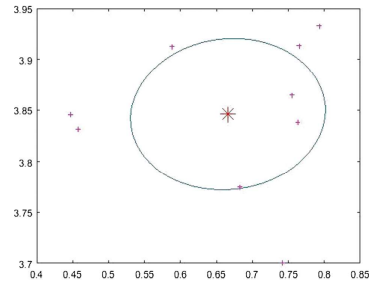
*Motion mass* of the set  $J$  is defined as tuple  $M_J = \{T_J, A_J, E_J, t\}$ . Unlike the original paper [6] present research also considers velocities mass (defined by analogy to the acceleration mass) and ratios of the trajectory length to the Euclidean distance and acceleration mass to the Euclidean distance. Within the frameworks of present contributions motion mass parameters used to demonstrate the difference between the states of the motor function in the beginning and in the ending of the training.

### 3 Average Trajectory

Let us suppose that there is  $n$  captures of the same motion. Without loss of generality, assume that motion takes place the most along one of the coordinates. This does not imply that there is no movement along the other coordinates. This coordinate will be referring as *leading coordinate*. Imagine normal plane sliding along the leading coordinate from the point where motion begins to the point marking ending of the motion. At each point of this interval there will be  $n$  points where trajectories  $T_i$  intersect with the normal plane. Averaging the coordinates of those points would provide corresponding point of the average trajectory as shown in Figs. 1 and 2.



**Fig. 1.**  $i$ th iteration of building average trajectory. (Color figure online)



**Fig. 2.** Intersection points and their average.

In the case of Fig. 1,  $z$  is chosen as the leading coordinate, thin blue lines represent captured trajectories, and yellow plane is the normal to  $z$ , finally bold red line represent average trajectory.

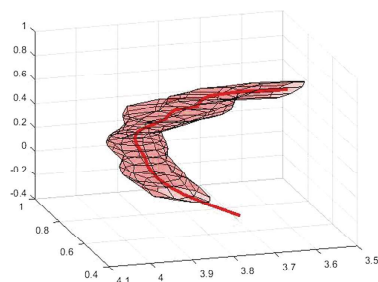
More formally:

- **Step 1.** For each captured motion determine the interval on the leading axis where trajectory is present. Find intersection of these intervals, (in other words find the interval where all the trajectories are present).
- **Step 2.** begin cycle: for each consequent point of the interval defined in Step 1
  - Define the normal plane to the leading coordinate.
  - For each plane find points where trajectories intersect with this plane. Compute an “average point” (coordinate wise).
  - Ellipse is used to represent variability of the points with respect to their average. Compute the covariance matrix (of the intersection points) and perform its eigendecomposition. Eigenvalues of the covariance matrix represent squared radiuses of the ellipses and the columns of the eigenmatrix describes orientation of the radiuses Fig. 2.

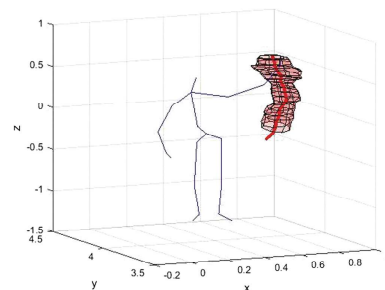
- **Step 3.** Average points (computed for each plane) define the average trajectory. The curve depicts generalized shape of the motion. Pipe-shaped neighbourhood is defined by the boundaries of the ellipses and depicts variability between the motions.

In Fig. 2 ‘+’ represent the intersection points of the individual trajectories with the normal plane. ‘\*’ represents the point of intersection between the average trajectory and the normal plane. Ellipse represent variability of the intersection points. One may argue about the choice of the ellipse to describe variability of the intersection points in Step 2. On the viewpoint of the authors, this is most natural and intuitive choice. The arguments to support this choice, its drawbacks and other possible choices are discussed in Sect. 5. Here and after the pair consisting of the trajectory derived by the algorithm and its enclosing pipe-shaped neighborhood would be referred as the *average trajectory* of the motion.

Proposed algorithm leads to the purely geometric representation that totally excludes time from the consideration. On the first view, such approach may seem strange, since timing is a natural way of motion parametrization. Also time frequently used as the parameter of the motion both in sportive training and in medical exercises. The latest is the main reason to consider time-less representation of the motion and demonstrate other properties, which vary significantly during learning of new motor activity or medical therapy.



**Fig. 3.** General structure of the *average trajectory*



**Fig. 4.** Average trajectory of the human wrist during the ball throwing exercise.

Average trajectory derived on the basis of the trajectories representing successful ball throwing in the beginning of the training (learning) process is demonstrated in Fig. 4. Figure 3 depicts position of the average trajectory with respect to the standing human. There are two numeric parameters, which may be naturally associated with the average trajectory. The first one is the length of the average trajectory and the second one volume of the pipe-like neighborhood. While the length of the trajectory will not necessarily change much during the training, the volume should decrease. This is because goal-directed movements to visual targets

consist of an initial impulse towards the target and a later corrective adjustment near the target to compensate for initial trajectory errors [9]. We hypothesized that during learning the primary movement needs less and less adjustment and thus the overall volume of the movements decreases. One of the possible ways to validate the proposed definition is to demonstrate that volume decrease during the training.

## 4 Illustrative Examples

Let us now consider the learning process of the ball throwing explained in Sect. 2. The process divided into ten sessions whereas each session consists of ten attempts (ball throwing). In the beginning of the training motion mass parameters computed for the trajectories of successful trials are indistinguishable from those of failed trials, (corresponding parameters of the motion mass are available from the authors upon request). In the end of training process, three of the motion mass parameters of the successful trials were distinguishable from those of the failed. Testing results are presented in Table 1.

**Table 1.** Test results for the mean values of MM parameters comparing successful and failed trials in the end of the training

Parameter	Reject $H_0$	$p$ -value	$t$ -statistic
$V_J$	1	0.0338	-2.1842
$T_J$	1	0.0355	-2.1629
$A_J$	0	0.6922	-0.3981
$E_J$	0	0.6791	-0.4162
$T_J/E_J$	0	0.0732	1.8315
$A_J/E_J$	0	0.5779	0.5601
$t$	1	0.0356	-2.1622

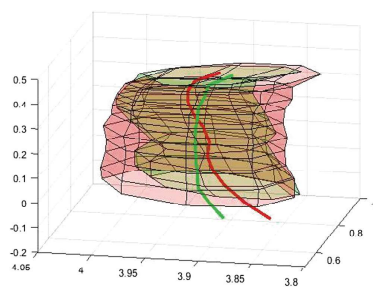
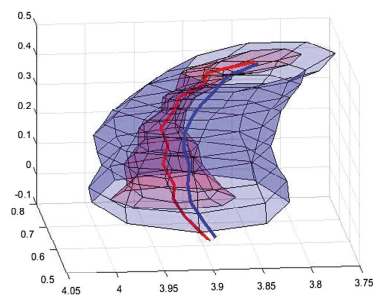
In addition, statistical hypothesis testing has demonstrated that four motion mass parameters computed for the successful trials in the end of the training were clearly distinguishable from those computed in the beginning. See Table 2 for details. This demonstrate that learning of the new motor activity caused changes in the way motions are performed.

Let us now turn attention to the average trajectories computed for the successful and failed attempts in the beginning and in the end of the training. In the beginning, there were no visible difference between the average trajectories computed for the successful and failed attempts Figs. 5 and 6.

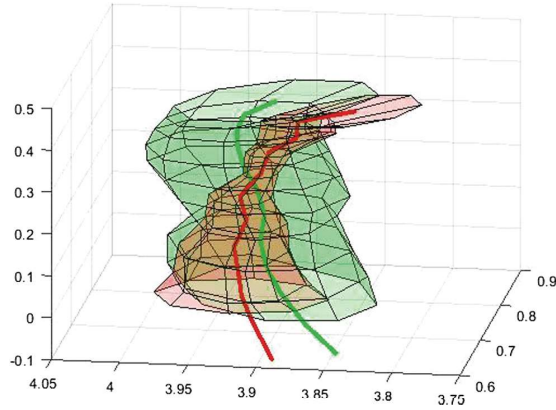
The difference between the average trajectories corresponding to the successful trials in the beginning and the ending of the training depicted in Fig. 7, where narrow (red) pipe corresponds to the end of the training and wider (green) pipe to the beginning.

**Table 2.** Test results for the mean values of MM parameters comparing beginning and ending of the training.

Parameter	Reject $H_0$	$p$ -value	$t$ -statistic
$V_J$	1	0.0125	-3.2068
$T_J$	1	0.0134	-3.1581
$A_J$	0	0.2492	-1.2423
$E_J$	0	0.7886	0.2772
$T_J/E_J$	1	0.0212	2.8570
$A_J/E_J$	0	0.2069	1.3733
$t$	1	2.2007e-04	-6.3522

**Fig. 5.** Average trajectories corresponding to the successful and failed trials in the beginning of the training process.**Fig. 6.** Average trajectories corresponding to the successful and failed trials on the end of the training process.

In the beginning of the learning, trainee was not able to make more than two successful attempts, which is not enough to generate pipe-shaped neighbourhood of the average trajectory. In order to overcome this problem the session results considered pair wise. Evolution of the volumes of average trajectories during the learning process is summarized in Table 3. One may see that in course of the training, volume of the average trajectory computed for the successful attempts decreases as training progresses. At the same time the volume of the average trajectories computed for the failed attempts does not demonstrate any trend in its behavior. The other participants of the pilot research group demonstrated similar results.



**Fig. 7.** Average trajectories of the successful trials in the beginning and ending of the training. (Color figure online)

**Table 3.** Evolution of the volumes and success rates.

	Success rate	Volume of the average trajectory of:	
		Success attempts	Failed attempts
Trials 1–2	3	0.0042	0.0049
Trials 3–4	5	0.0085	0.0315
Trials 5–6	5	0.0051	0.0054
Trials 7–8	6	0.0011	0.0064
Trials 9–10	3	0.0027	0.0064

## 5 Discussion

In Sect. 3 variability of the intersection points between the normal plane and captured trajectories represented by means of the ellipse. On the one hand, this is common way to describe deviation from the average in two-dimensional space. On the other hand, this method may lead to undesirable results. If the motion was repeated just a few times. For example, some patients could not perform the exercise more than three times. Specific distribution of the trajectories also may cause that ellipses will become degenerate and would not produce any pipe-shaped neighbourhood. In those cases ellipses may be replace by a circles. Whereas the radius may be chosen as the maximum of ellipsoid radiuses or as the average or maximal distance between the average point and intersection points. Most probably, the way to describe variability is not universal and depends on the particular application.

In order to illustrate the notion of the average trajectory relatively short, in time, learning process was observed. On the one hand, small number of observation points does not allow validating proposed notion in a stronger way. For example by demonstrating significant correlation between the success rate and



volume of the average trajectory. On the other hand, it allows avoiding effects caused by fatigue and other complications of lengthy processes. Another tuple of the parameters, which may be used for comparison of different trajectories, is the sequence constitute by the areas of the ellipses on the secant planes. In order to perform comparison one should build the trajectories within the same limits. This will result in the same number of secants for both cases. Which leads the possibility to compare pairwise areas of the corresponding ellipses. While this option seems attractive on the first view its results are not strait forward interpretable and therefore left of future studies. One may suggest using principal component analysis (PCA) to find the direction of maximum variance. Application of PCA means coordinate rotation, which would tangle interpretation of the results. Therefore, the choice of the leading coordinate is left to the coach or practitioner.

## 6 Conclusions

The notion of the *average trajectory* of the repeating motion is introduced in the present paper. The pair of elements describe average trajectory. The first one represents the shape of the motion and the second one represents variability. Being strictly geometric, the notion allows visualizing the results of the training or therapy session, in an easily understandable form. Volume of the average trajectory is the most natural numeric value to be associated with average trajectory. It was demonstrated that during the learning process volume of the average trajectory (computed for the successful trials (attempts)) is tend to decrease. Based on results of this pilot research one may conclude that type of activity is a defining factor to choose the way to describe variability of the trajectories. This will be first main problem for further studies.

## References

1. Ahmadi, A., Destelle, F., Monaghan, D., O'Connor, N., Richter, C., Moran, K.: A framework for comprehensive analysis of a swing in sports using low-cost inertial sensors. In: 2014 IEEE SENSORS, pp. 2211–2214, November 2014
2. Alexiadis, D., Daras, P.: Quaternionic signal processing techniques for automatic evaluation of dance performances from MoCap data. *IEEE Trans. Multimed.* **16**(5), 1391–1406 (2014)
3. Estrada, S., O'Malley, M., Duran, C., Schulz, D., Bismuth, J.: On the development of objective metrics for surgical skills evaluation based on tool motion. In: 2014 IEEE International Conference on Systems, Man and Cybernetics (SMC), pp. 3144–3149, October 2014
4. Kwak, Y.S., Jung, S.K.: Recognition of visual signals and firing positions for virtual military training systems. In: 2013 The 6th International Conference on Human System Interaction (HSI), pp. 656–658, June 2013
5. Lin, Z., Uemura, M., Zecca, M., Sessa, S., Ishii, H., Tomikawa, M., Hashizume, M., Takanishi, A.: Objective skill evaluation for laparoscopic training based on motion analysis. *IEEE Trans. Biomed. Eng.* **60**(4), 977–985 (2013)

6. Nõmm, S., Toomela, A.: An alternative approach to measure quantity and smoothness of the human limb motions. *Est. J. Eng.* **19**(4), 298–308 (2013)
7. Payeur, P., Nascimento, G., Beacon, J., Comeau, G., Cretu, A.M., D'Aoust, V., Charpentier, M.A.: Human gesture quantification: an evaluation tool for somatic training and piano performance. In: 2014 IEEE International Symposium on Haptic, Audio and Visual Environments and Games (HAVE), pp. 100–105, October 2014
8. Petitjean, F., GanSarski, P.: Summarizing a set of time series by averaging: from Steiner sequence to compact multiple alignment. *Theoret. Comput. Sci.* **414**(1), 76–91 (2012)
9. Suzuki, M., Kirimoto, H., Sugawara, K., Kasahara, Y., Kawaguchi, T., Ishizaka, I., Yamada, S., Matsunaga, A., Fukuda, M., Onishi, H.: Time course of change in movement structure during learning of goal-directed movement. *J. Med. Biol. Eng.* **35**(1), 113–124 (2015)
10. Takai, M.: Extracting method of characteristic posture from human behavior for surveillance camera. In: ICCAS-SICE 2009, The International Joint Conference on Instrumentation, Control and Information Technology, Fukuoka, Japan, pp. 159–164 (2009)
11. Takai, M.: Production of body model for education of dance by measurement active quantity. In: The 1st IEEE Global Conference on Consumer Electronics 2012, pp. 212–216, October 2012