Discrete Structures: CMPSC 102

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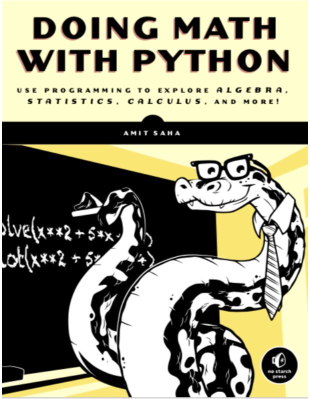
Fall 2018
Week 14

Where We Are?



Saha, Chapter 5: Playing with sets and probability

- Using Sets with Sympy
- Containing probabilities in sets.

Using Sympy

Clone the GitHub Repository

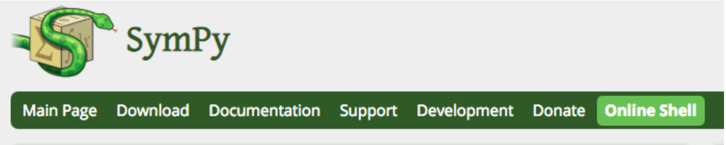
```
git clone git://github.com/sympy/sympy.git
```

Install locally

```
python3 setup.py install
```

Or use the Interactive shell online

https://live.sympy.org/



Remember *Sets*

- Sets have no order and all members are unique.
- Sets are also symbolically manipulated in SymPy

Sets, The old way

```
set([1,2,2,2,2,2,3]) == set([3,2,1])
```

Note: If the libraries do not exist, try using python (version 2) or the SymPy website's interactive interpretor, `https://live.sympy.org/`

Sets, Working with Sympy

```
from sympy import FiniteSet
FiniteSet(1,2,2,3,3,3,3) == FiniteSet(1,3,2)
```

Construction and Membership

Let's build a bigger set

```
from sympy import FiniteSet
from fractions import Fraction
s = FiniteSet(1, 1.5, Fraction(1, 5))
print(s) #{1/5, 1, 1.5}
```

What's in the set

```
print(" Number of elements :",len(s))
for i in s: print(i)
1 in s # does this value exist in the set
Fraction(1,5) in s
```

Converting Tuples to FiniteSets

Converting a list to a FiniteSet

```
m = [1,2,3] # list
s = FiniteSet(*m)
print(s) #{1, 2, 3}
type(s) #<class 'sympy.sets.sets.FiniteSet'>
```

```
m = [1, 2, 3, 2] # list
s = FiniteSet(*m)
type(s) # <class 'sympy.sets.sets.FiniteSet'>
```

Iterating through set

```
s = FiniteSet(1, 2, 3)
for member in s:
  print(member)
```

Unions and Intersections

Union of sets

```
from sympy import FiniteSet
s = FiniteSet(1, 2, 3)
t = FiniteSet(2, 4, 6)
u = FiniteSet(3, 5, 7)
s.union(t).union(u)
```

Intersection of sets

```
s = FiniteSet(1, 2, 3)
t = FiniteSet(2, 4, 6)
u = FiniteSet(3, 5, 7)
s.intersect(t).intersect(u) #EmptySet() why?
```

$$\emptyset = \{\}$$

The empty set contains the *element* of nothing

- In set theory, *nothing* is actually *something* to note: here we imply that there are no members in the set
- The empty set contains nothing, and is denoted by the symbol: $\emptyset$

Creating an empty set

```
from sympy import FiniteSet
s = FiniteSet()
print(s) #EmptySet()
```

Finite and Infinite

$$A = \{-\infty, \cdots, \infty\}$$

The set of all real numbers, $A_i \in A$

Defining sets

- Set of even numbers: E $= \{\cdots, -4, -2, 0, 2, 4, \cdots\}$
- Set of odd numbers: O $= \{\cdots, -3, -1, 1, 3, \cdots\}$
- Set of prime numbers: P $= \{2, 3, 5, 7, 11, 13, 17, \cdots\}$
- Positive multiples of 3 that are less than 10: L $= \{\cdots, 3, 6, 9\}$
- Set of the first five letters: F $= \{a, b, c, d, e\}$

Members in a set (True? False?)

- Is $0 \in E$? Is $5 \in O$? Is $13 \in P$?
- Is $90 \in E$? Is $4 \in P$? Is $f \in F$?

Who is in Which Set?

Members of a set

- $0 \in E$: 0 is a member of E
- $4 \notin O$: 4 is not a member of O
- $4000 \in E$
- $8 \notin P$
- $59 \notin P$
- $5 \in P$
- $3 \notin F$
- $\diamondsuit \notin F$

Who is in Which Set?

Members of a set

- $0 \in E$: 0 is a member of E
- $4 \notin O$: 4 is not a member of O
- $4000 \in E$

The set of real numbers between 0 and 1

```
from sympy import Interval
Interval(0, 1).contains(0.5)
0.5 in Interval(0,10)
```

Proper Subsets

What is a *Proper Subset*?

- A proper subset of a set A is a subset that cannot be equal to A
- If B is a proper subset of A, then all elements of B are also in A but A contains at least one element that is not in B.

- Ex: Let $A = \{1, 3, 5\}$ then $B = \{1, 5\}$ is a proper subset of A. The set $C = \{1, 3, 5\}$ is a subset of A, but it is not a proper subset of A since $C = A$. The set $D = \{1, 4\}$ is not even a subset of A, since 4 is not an element of A.

The sets of A and B

```
A = FiniteSet(1,3,5)
B = FiniteSet(1,5)
for i in B: i in A # is each element in A?
for i in A: i in B # is each also element in B?
len(A) == len(B) # same cardinality?
```

Proper Subsets

- **Example**: $A =$ 1, 2, 3, 4, 5
  - Subsets of A: {1, 2, 3}, {3, 4} and {1}
  - Written: $\{1, 2, 3\} \subset A$,
  - $\{3, 4\} \subset A$,
  - $\{1\} \subset A$
  - Note: {1, 6} is not a subset, since it has an element (6) which is not in the parent set.

Is B a subset of A?

```
A = FiniteSet(1, 2, 3, 4, 5)
B = FiniteSet(1, 6) #potential subset?
for i in B: i in A # is each element in A?
for i in A: i in B # is each also element in B?
len(A) == len(B) # same cardinality?
```

Proper Subsets

- $A = \{1, 2, 3, 4, 5\}$
- $B = \{1, 2, 3\}$

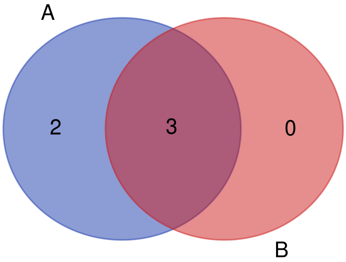


Figure: $B \subset A$ since there are other elements in A that are not in B

Proper Subsets

- **Another Example**: Is A a proper subset of B?
- Let $A = \{1, 3, 4\}$ and let $B = \{1, 4, 3, 2\}$?
  - 1 is in A, and 1 is in B as well. (good, so far!)
  - 3 is in A and 3 is also in B.
  - 4 is in A, and 4 is in B.
  - We have covered all elements of A, and each is in B and so we stop here.
- Yes, A is a proper subset of B since the sets cannot be equal (more in B than in A)
- $A \subset B$

Subsets

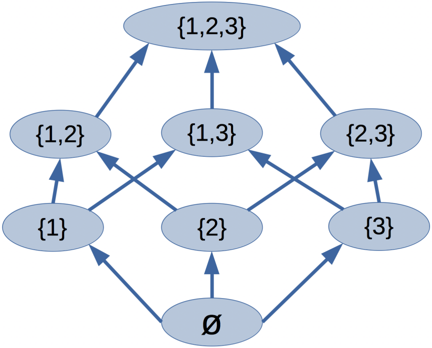
- A set, s, is a subset of another set, t, if all the members of s are also members of t.
- For example, the set 1 is a subset of the set 1, 2. You can check whether a set is a subset of another set using the `is_subset()` method:

Subsets

```
s = FiniteSet(1)
t = FiniteSet(1,2)
s.is_subset(t) #True
t.is_subset(s) #False
```

Subsets Make up Powersets

A Power Set is a set of all the subsets of a set.



Subsets

A Power Set is a set of all the subsets of a set.

Putting it together

- The set $\{1, 2, 3\}$:
- Contains The empty set ($\emptyset$) $\{\ \}$ and is a subset
- Contains subsets: $\{1\}$, $\{2\}$ and $\{3\}$
- Contains subsets: $\{1, 2\}$, $\{1, 3\}$ and $\{2, 3\}$
- Contains $\{1, 2, 3\}$ and is a subset of self

The Subsets of a Powerset

```
s = FiniteSet(1,2,3)
print(s) #{1, 2, 3}
ps = s.powerset()
print(ps)
# {EmptySet(), {1}, {2}, {3},
# {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}
len(ps) # set cardinality: number of elements
```

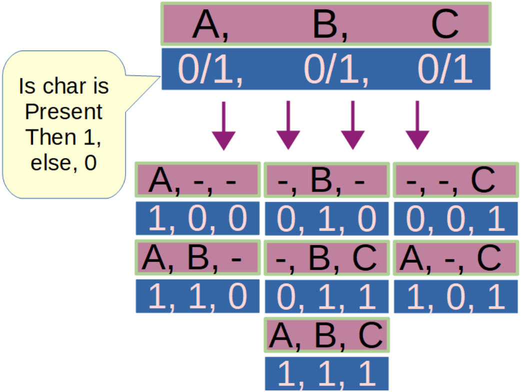
A Simple Application

Create a coding system

We have three characters that we wish to use to create a coding system to send binary signals over a channel. We want as many unique (binary) codes as possible from these three chars. How many codes can we create and what are these codes?

A Simple Application
Use the Powerset


```
b = FiniteSet('a','b','c')
b.powerset()
#{EmptySet(), {a}, {b}, {c}, {a, b}, {a, c},
# {b, c}, {a, b, c}}
```



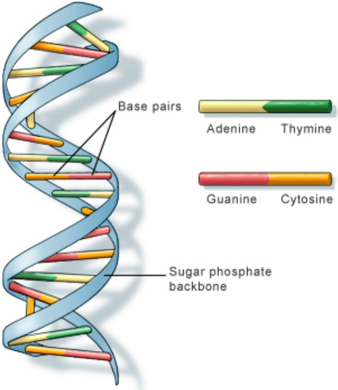
A Simple Application
Use the Powerset

Subset	Sequence of digits	Binary interpretation	Decimal equivalent
{ }	0, 0, 0	000_2	0_{10}
{ a }	0, 0, 1	001_2	1_{10}
{ b }	0, 1, 0	010_2	2_{10}
{ a, b }	0, 1, 1	011_2	3_{10}
{ c }	1, 0, 0	100_2	4_{10}
{ a, c }	1, 0, 1	101_2	5_{10}
{ b, c }	1, 1, 0	110_2	6_{10}
{ a, b, c }	1, 1, 1	111_2	7_{10}

Another Application

How many ways to arrange four characters?

We wish to know how many possible *words* we can make from an alphabet of four characters (i.e., a permutation)



U.S. National Library of Medicine

Another Application

Use Quadruple Coding! (Only kidding, use the powerset function again)

powerset stuff

```
d = FiniteSet('a','c','g','t')
dddd = d**4 #Cartesian crossproduct
#{a, c, g, t} x {a, c, g, t}
# x {a, c, g, t} x {a, c, g, t}
len(dddd)
for i in dddd: print(i) #word combinations
```

Some sample words

```
(a, a, a, a), (a, a, t, g), (a, a, t, t),
(a, t, t, c), (c, a, a, a), (c, a, t, a),
(c, a, t, c), (g, c, t, a), (c, g, t, a), and etc
```

What Solutions Belong in the Set?

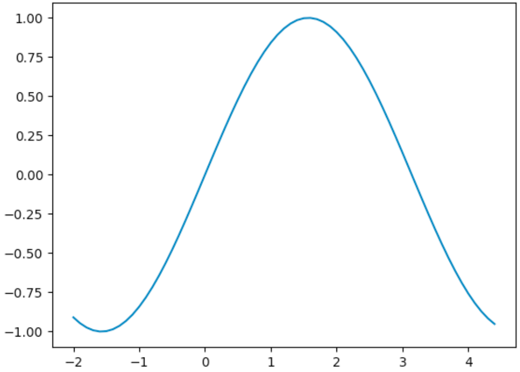
- How can we build a set using mathematics?
- Let's use the sin(x) function to design a set for all solution values between 0 and Pi.
- First: What points of the x-axis are we talking about?

```
from sympy import Interval, Symbol, solve, solve_univariate_inequality, sin
from sympy.plotting import plot
from pylab import plot, show
import math

#plot: where are these solutions?
x = [i*.1 for i in range(-20,45)]
y = [math.sin(i*.1) for i in range(-20,45)]
plot(x,y)
show()
```

What Solutions Belong in the Set?

- Domain: Sin(x) > 0 is defined for all x in the x to Pi interval.
- `[0, Pi)`



Create the Set, Mathematically

- We build the interval for all solutions (x) that satisfy Sin(x) $>$ 0

```
# define the variables
x = Symbol('x')
# define equation
ineq_obj = sin(x) > 0
# solve an equation
s = solve_univariate_inequality(ineq_obj,
x, relational=False)
#Interval.open(0, pi)
```

Intervals

- How can we build a set using mathematics?
- Let's use the sin(x) function to design a set for all solution values between 0 and Pi.
- First: What solutions values are in the set?

```
from sympy import Interval, Symbol, solve, solve_univariate_inequality, sin
from sympy.plotting import plot
from pylab import plot, show
import math

# plot: where are these solutions?
x = [i*.1 for i in range(-20,45)]
y = [math.sin(i*.1) for i in range(-20,45)]
plot(x,y)
show()
```

Intervals

- Test the interval, s, to see what values can be found in it.

```
# Determine whether a value is in my_set
-0.9 in s
0 in s
0.1 in s
-0.1 in s
3.14 in s
3.15 in s
```

Sets in Pendulum Motion

- Sets can be created from natural laws of gravity and pendulum motion.
- Here we use the equation for a pendulum's periodic motion to create another set.

The amount of time to create one swing of a pendulum

$$T = 2 * \pi \sqrt{\frac{L}{g}},$$

for L and g, pendulum of length and gravity acceleration coefficient, respectively

Pendulum Motion

- A simple scenario where all possible combinations of the elements of multiple sets (or a group of numbers) are required. We use the Cartesian product.

gravity.py

```
from sympy import FiniteSet, pi
def time_period(length, g):
    T = 2*pi*(length/g)**0.5
    return T
#end of time_period()

if __name__ == '__main__':
    L = FiniteSet(15, 18, 21, 22.5, 25) #define the lengths as sets
    g_values = FiniteSet(9.8, 9.78, 9.83) #define three gravity values in a set
    print('{0:^15}{1:^15}{2:^15}'.format('Length(cm)', 'Gravity(m/s^2)', 'Time Period(s)'))
    for elem in L*g_values: # the cartesian product
        l = elem[0]
        g = elem[1] #take defined number
        t = time_period(l/100, g)
#        print("\n",elem)
        # output the gravity values in triplicate
        print('{0:^15}{1:^15}{2:^15.3f}'.format(float(l), float(g), float(t)))
```

Rolling Dice Probabilities With Sets

Defining the terms of an experiment

- **Experiment**: Use algorithms to roll a dice
- **Sample Space**: All the possible outcomes of rolling a six-sided dice. One of the numbers in $\{1, 2, 3, 4, 5, 6\}$ will result. We define $S = \{1, 2, 3, 4, 5, 6\}$, the set of possible outcomes. Written, $N(S)$ is the space of all these possible outcomes.
- **Event**: The set of outcomes of an experiment. We define this set, E, the actual outcomes of rolls.
- **Probability of an event**: Written, $P(E)$ is a calculation of the event in light of the possible outcomes: $P(E) = \frac{singleEvent}{allEvents}$
- **Uniform Distribution**: This defines what the odds are for a particular roll being made. By, *Uniform*, we imply that all outcomes are equally likely to happen.

Rolling Dice

Probability Equation

$$P(E) = \frac{n(E)}{n(S)}$$

The probability is calculated given by a particular event $n(E)$, divided unto the total number of possible events, $n(S)$. The fraction is made up of the cardinalities of each set, events and all possible events.

Probability Equation

- The changes of an event happening:
  - **Not likely** $\rightarrow 0 \leq P(E) \leq 1 \leftarrow$ **Certain to happen**

Rolling Dice

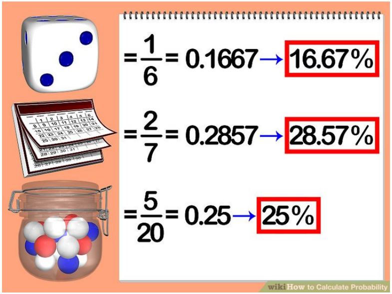
Probability Equation

$$P(E) = \frac{n(E)}{n(S)}$$

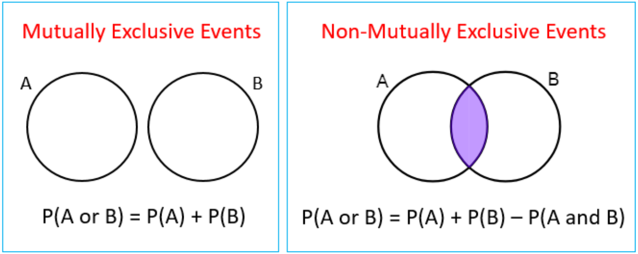
Recap

- $S = \{1, 2, 3, 4, 5, 6\}$ (all possible outcomes)
- $E = \{3\}$ (A particular event)
- $n(S) = 6$ (all possible events)
- $n(E) = 1$ (a single event)
- $P(E) = \frac{1}{6}$ (the equation to calculate frequency)

Rolling Dice



Mutually Exclusive



- Mutually Exclusive events cannot happen at the same time.
  - Ex: A single coin landing on `Heads` and `Tails` at the same time.
- Non-Mutually Exclusive events are able to happen at the same time.
  - Ex: Flipping a dice and having a odd number which is also a prime number.

Dice Rolling a Prime

diceProbability.py

```
# Saha, page 132

def probability(space, event):
         return (1.0 * len(event))/len(space)
         # the 1.0 is used to convert floats in python2
#end of probability()

def check_prime(number):
    if number != 1:
        for factor in range(2, number):
            if number % factor == 0:
                return False
    else:
        return False
    return True
#end of check_prime()

from sympy import FiniteSet

if __name__ == '__main__':
    space = FiniteSet(*range(1, 21)) # store sample space
    primes = []
    for num in space:
        if check_prime(num): # is this number a prime
            primes.append(num) # make a list of the primes
    event = FiniteSet(*primes)
    p = probability(space, event) # calculate probability
    print('Sample space: {0}'.format(space))
    print('Event: {0}'.format(event))
    print('Probability of rolling a prime: {0:.5f}'.format(p))
```

Participation 6

Search for this repository and push work to it

Place work in:
`cs102-participation-starters/06_part_starter/`
and push it

- In your repository: `mkdir 06_part_starter/`
- Note: Participation checks are given only for **work done while you are in class**.
- Time limit: Push your work by the end of class (12pm) for credit.

- Details on next slide...

Participation 6

Explore!

- You are to use the `FiniteSet()` function in a Python program that determines how many telephone numbers may be generated a 7 digit number (i.e., numbers that look like: 555-1234). Then, determine how many numbers are possible when using an area code (i.e., numbers that look like: 814-555-1234).
- Use the interactive interpreter to work with the code or use the python version 2 interpreter on your machine. Save your work in a source file `/06_part_starter/telephone.py`
- Note: Your program is to ask the user how many digits long the program is and then to output a number of possible telephone numbers that can be generated from the length.