CMPSC 102
Discrete Structures
Fall 2018

Practical 8: The Fibonacci Sequence at Arm's Length

Refer to your notes, slides and sample Python code from this week and other weeks. In particular, follow the python code that we created in class or check on line for interesting pieces of code to help you in your programming.

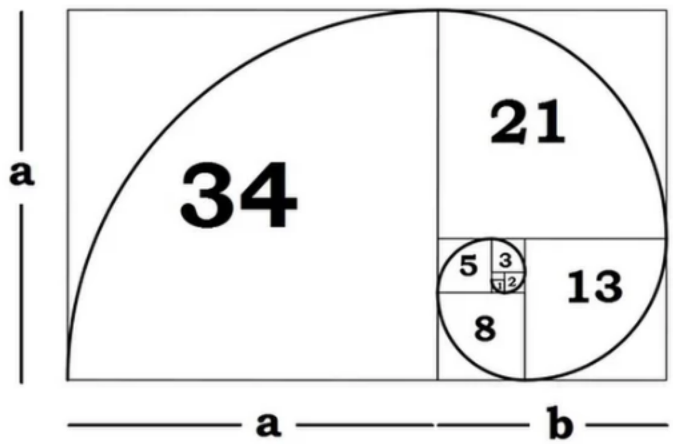

Figure 1: The *Golden Ratio* (ϕ) is an irrational number (1.61803398875) that can be found all over nature, including in the progression of values in the Fibonacci sequence. For example, ϕ may be found in the quotient of tangent Fibonacci sequence pairs, f_m and f_n by the following calculation $\left(\frac{f_{n-2}}{f_{n-1}}\right)$. This ratio becomes more accurate (i.e., a better approximation to the real value) as the sequence pairs in this fraction become increasingly large.

GitHub Starter Link

`https://classroom.github.com/a/nlaIHCMJ`

To use this link, please follow the steps below.

- Click on the link and accept the assignment.
- Once the importing task has completed, click on the created assignment link which will take you to your newly created GitHub repository for this lab.
- Clone this repository (bearing your name) and work on the practical locally.
- As you are working on your practical, you are to commit and push regularly. You can use the following commands to add a single file, you must be in the directory where the file is located (or add the path to the file in the command):

- `git commit <nameOfFile> -m ''Your notes about commit here''`
- `git push`

Alternatively, you can use the following commands to add multiple files from your repository:

- `git add -A`
- `git commit -m ''Your notes about commit here''`
- `git push`

Summary

In this practical, you be studying yet another fascinating way to determine the Fibonacci sequence using a special relationship between the numbers, shown in Figure 1. For this task, this your approach will be to determine the *Golden Ratio* (ϕ), that will be used to grow all terms in the sequence, using mere measurements from your arm, as shown in Figure 2. The value of the length of your shoulder to your fingers, over the value of the length of your elbow to your fingers, is (coincidentally) an approximation of the same ratio of tangent Fibonacci numbers, discussed below.

Fibonacci's Sequence and ϕ

For example, the 18_{th}, 19_{th} and 20_{th} values for the Fibonacci sequence are the following.

$$\{\texttt{2584, 4181, 6765}\}$$

Using these numbers, we apply Python to note the following quotients.

$$\phi \sim \frac{6765}{4181} = 1.6180339631667064$$

and

$$\phi \sim \frac{4181}{2584} = 1.618034055727554$$

The larger the value of the Fibonacci sequence pairs, the better the approximation to the actual ϕ value you will find. Although these values are not exactly equal to the value of ϕ (which is 1.61803398875, according to Google), we note that the above quotients are still very close and should work well for the task of producing a following sequence term after an initial value.

An Equation to Note

The equation to grow a sequence term (f_n) from a previous and tangent term (f_n) is the following.

For the sequence,

$$\{..., f_m, f_n, ...\},$$

where f_n and f_m are sequential terms of the Fibonacci sequence, and ϕ is the *Golden Ratio*,

$$f_n = \phi * f_m.$$

You may find that you will have to use the `math.ceil()` function to round your value up to the nearest integer to reduce the errors that will naturally arise in your calculations.

$$f_n = \texttt{math.ceil}(\phi * f_m).$$

The File `src/fibArm_starter.py`, that you will modify, already imports the `math` library for this function to be available in your code.

The Steps to Complete

1. Locate the incomplete Python source file in `src/fibArm_starter.py` to modify. The code already produces the correct Fibonacci sequence that you can use for comparison to your own sequence values.
2. To make your own Fibonacci sequence values, you are to modify the file Python code to accept the user inputs of the arm (i.e., shoulder to fingers and elbow to fingers, shown in Figure 2) to calculate the ratio. Then you will write a function (called, `fibByArm()`) to return the product of an inputted sequence value and ϕ. This value (the product you just calculated) will be the next term of your own Fibonacci sequence.
3. You will also add code to ask the user how many iterations of the sequence to produce. This could be another user input or could be simply hard-coded into the source file.
4. Please note that due to natural errors in determining the necessary arm measurements, your ϕ value is likely to be prone to error and your sequence values will consequently be slightly different to the actual Fibonacci sequence values. In this case, your sequence may be called a Lucas Sequence which grow similarly to Fibonacci sequences (read more about Lucas numbers at `https://en.wikipedia.org/wiki/Lucas_number`.)

Output

The output from the source code will look like the following. Note, the sequence may start at 2, or the third term of the Fibonacci sequence.

```
 Length of shoulder tip of longest finger :
 Length of elbow to tip of longest finger :
  From these inputs, the calculated Golden Ratio is :  1.6181229773462784
Index :: Sequence Term
   0  ::  2
   1  ::  4
   2  ::  7
   3  ::  12
   4  ::  20
   5  ::  33
   6  ::  54
 The Real Fibonacci Sequence:
[1, 1, 2, 3, 5, 8, 13, 21, 34, 55]
```

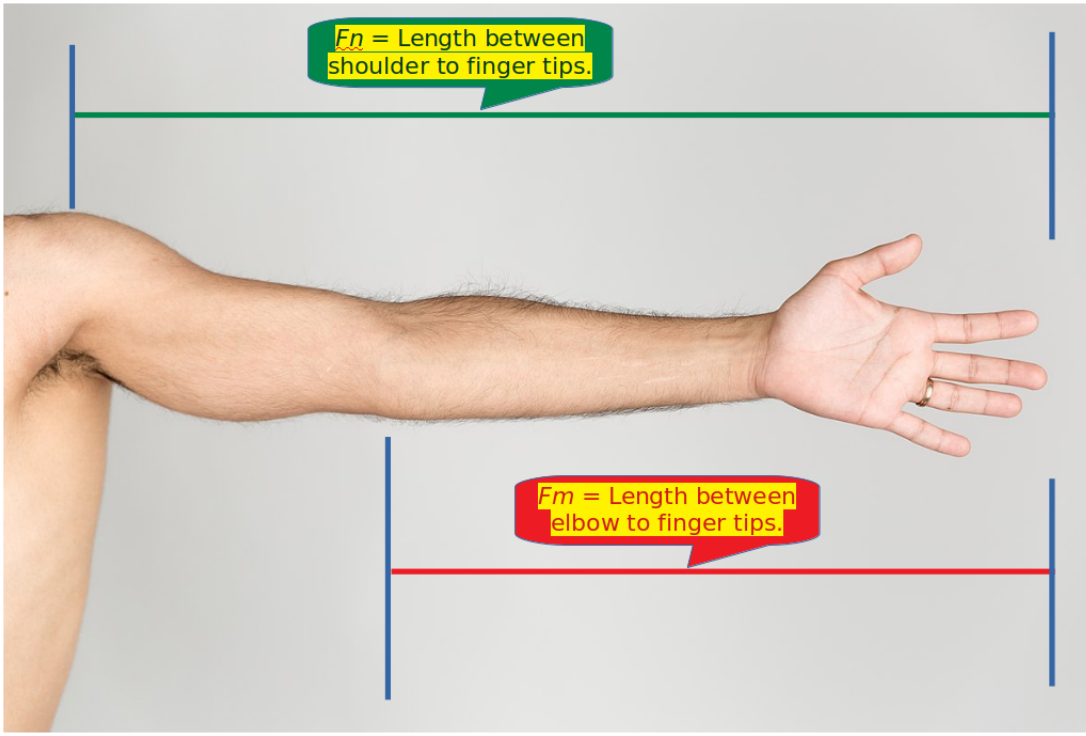

Figure 2: Your measurements to determine the *Golden Ratio* are extracted by measuring the distance between your longest finger and your shoulder (f_n) and the distance between your elbow to your longest finger (f_m). The *Golden Ratio* can be found here by finding the quotient of $\frac{f_n}{f_m}$. Yes, this is my arm. :-)

My value of ϕ above is already off by a small amount. You will note that even with fairly accurate measurements, approximation errors happen and prevent the your sequence from being exactly the same as the real Fibonacci sequence. Perhaps your measurements will be more accurate and your sequence will look better than the one above.

Deliverables

1. Your completed (and working) Python code (`src/fibArm_starter.py`)

```
--. --- / --. . - / .----. . --
```