Discrete Structures: CMPSC 102

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Week 6

- Water is *blue*, the shirt is also *blue*, therefore this shirt is made up of water?

Logic is ...

a **truth-preserving** **system** of **inference**

- **Truth-preserving**: If the initial statements are True, the inferred statements will also be true
- **System**: A set of mechanistic transformations which are based on syntax alone
- **Inference**: The process of deriving (inferring) new statements from old statements (i.e., finding conclusions based on previous observations)

Prepositional Logic

- A *proposition* is a statement that is either *true* or *false*
- Every proposition is true or false, but its truth value (true or false) may be unknown
- Examples:
  - You are in Discrete Structures (CMPSC102) $\rightarrow$ True
  - Today is Sunday $\rightarrow$ False
  - $1 == 2 \rightarrow$ False
  - It is currently raining in Paris $\rightarrow$ who can say?
  - In Alice's pocket, she has exactly 58 cents $\rightarrow$ who can say?

Different from philosophical assessments of truth...

- Philosopher Ludwig Wittgenstein observed, structures have spatial locations, but facts do not
- The Eiffel Tower can be moved from Paris to Rome, but there is a fact that it is currently in Paris (and nowhere else)
- "Truth" is something of a system of beliefs (i.e., I believe that it is in Paris, because I have seen it there)

Prepositional Logic

- A proposition statement:
  - Denoted by a capital letter (i.e., "A")
  - A negation of a proposition statement
  - $\sim A$: "**not** A"
  - Two proposition statements joined by a *connective*
  - $A \wedge B$: "A **and** B"
  - $A \vee B$: "A **or** B"
  - If a connective joins complex statements, parenthesis are added
  - $A \wedge (B \vee C)$: "A and (B or C)"

Prepositional Logic

- $A =$"It is raining"
  - $\sim A$: "not A"
  - "It is **not** raining"
- $A =$ "I will have peanut butter"
- $B =$ "I will have jelly"
  - Sandwich order: $A \wedge B$: "A and B"
  - I will have a sandwich composed of peanut butter **and** Jelly
- $A =$ "I will wear a white t-shirt"
- $B =$ "I will wear a blue t-shirt"
  - I will wear a t-shirt: $A \vee B$: "A or B"
  - I will wear the white **or** the blue t-shirt

Truth Tables

- Compound propositional statements are built out of simple statements using logical operations: negations, conjugations (using *AND*'s) and disjunctions (using *OR*'s)
- Determined by truth tables
  - A table of a combinations of truths using the connectives
  - Truth tables define the truth value of a connective for every possible truth value of its terms

Negation of Proposition

- Negation of proposition A is $\sim A$
- A: It is sunny.
- $\sim A$: It is not sunny.
- A: Newton often drank tea.
- $\sim A$: Newton did not often drink tea.
- A: I am from the planet Zogitron.
- $\sim A$: I am not from the planet Zogitron. (regretfully)

Negation of Proposition

Let A be a propositional statement ...

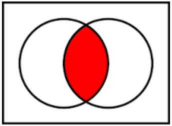
A	$\sim A$
True	False
False	True

In Python

```
True # True
not True # False
not False # True

A = True
not(A) # False
```

Logical AND (Conjunction)



- Conjunction of A and B is: $A \wedge B$
  - A: We are learning logical expressions
  - B: We are learning Python code for logical expressions
  - $A \wedge B$: We are learning logical expressions and python code for logical expressions

In Python

```
A = True
B = True
A & B # True
A and B # True
```

Logical AND (Conjunction)

- Conjunction of A and B is: $A \wedge B$
  - A: I like art
  - B: I like this work of art
  - $A \wedge \sim B$: I like art but I do not like this work of art

In Python

```
A = True
B = True
A and not(B) # False
```

Logical AND (Conjunction)

Table for Conjunction

A	B	$A \wedge B$
False	False	False
False	True	False
True	False	False
True	True	True

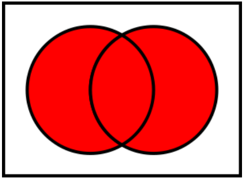
Logical OR (Disjunction)

- Also known as the inclusive OR
- Conjunction of A or B is: $A \vee B$
  - A: Today is sunny
  - B: It is Monday
  - $A \vee B$: Today is sunny or it is Monday
- This statement is true if any of the following hold:
  - Today is sunny
  - It is Monday
  - Both
- Otherwise it is false

In Python

```
A = True
B = False
A | B # True
A or B # True
```

Logical OR (Disjunction)



Truth Table for Disjunction (Inclusive OR)

A	B	$A \vee B$
False	False	False
False	True	True
True	False	True
True	True	True

Exclusive OR

- The inclusive OR is true if either or both arguments are true
- The exclusive OR ($\otimes$) is true if both arguments are not the same

Truth Table for Exclusive OR (*either or*)

A	B	A $\otimes$ B
False	False	False
False	True	True
True	False	True
True	True	False

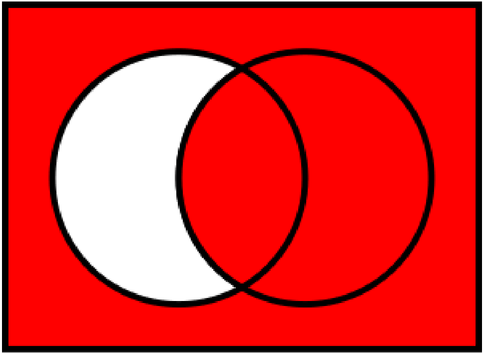
Implication $\rightarrow$

- The conditional implication connective is $\rightarrow$
  - $A \rightarrow B$: A implies B
  - *If A is true, then B is also true such that the statement $A \rightarrow B$ is False only when A is True and B is False*
- Logically equivalent to $\sim (A \wedge \sim B) == (\sim A \vee B)$

Truth Table for the Conditional Implication

A	B	A $\rightarrow$ B
False	False	True
False	True	True
True	False	False
True	True	True

Implication $\rightarrow$



Implication $\rightarrow$

Truth Table for the Conditional Implication

```
A = False
B = False
not(A) or B # True

A = False
B = True
not(A) or B # True

A = True
B = False
not(A) or B # False

A = True
B = True
not(A) or B # True
```

Meaning of Conditional Implication ($\rightarrow$)

- A: Homework is due
- B: It is Monday
- $A \rightarrow B$
  - If homework is due, then it must be Monday.
- Can we also conclude...?
  - If it is Monday, then a homework is due.

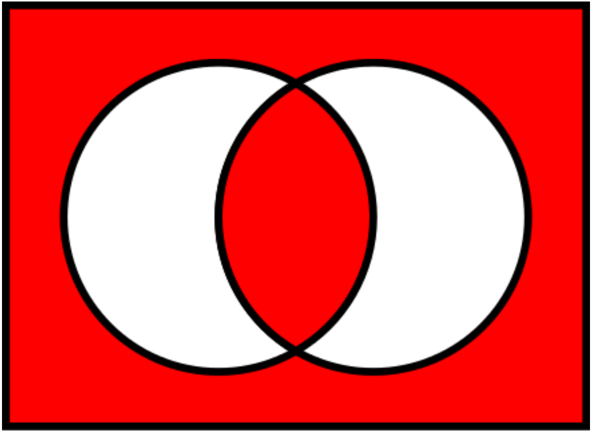
Implication $\leftrightarrow$

- The conditional implication connective is $\leftrightarrow$
  - $A \leftrightarrow B$: "A if and only if B",
- Logically equivalent to $(A \rightarrow B) \wedge (B \rightarrow A) == (A \wedge B) \vee (\sim A \wedge \sim B)$

Truth Table for the Conditional Implication

A	B	A $\leftrightarrow$ B
False	False	True
False	True	False
True	False	False
True	True	True

Implication $\leftrightarrow$



Implication $\leftrightarrow$

Truth Table for the Conditional Implication

```
A = False
B = False
((A and B) or (not(A) and not(B))) # True

A = False
B = True
((A and B) or (not(A) and not(B))) # False

A = True
B = False
((A and B) or (not(A) and not(B))) # False

A = True
B = True
((A and B) or (not(A) and not(B))) # True
```

Meaning of Conditional Implication ($\leftrightarrow$)

- A: You can buy the shirt
- B: You have enough money
- $A \leftrightarrow B$
  - You can buy the shirt, if and only if, you have enough money (and *vice versa*)
- Can we also conclude...?
  - If you have enough money, then you can buy the shirt.

Consider this

Try coding these...

- Use Python to test the following conditions:
  - $\sim A \vee B$, for A = True and B = False
  - $\sim A \vee \sim B$, for A = True and B = False
  - $\sim A \rightarrow B$, for A = True and B = False