



Discrete Structures: CMPSC 102

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Week 5

Types of Sequences?

Types of Sequences

Sequences by the Math Elements

Properties of Sequences

Quiz 1

- Strings, which are sequences of characters.
- Files contain a sequence of lines and the lines are sequences of characters.
- Objects, over which the `range()` function, can iterate

Examples

```
for element in [1, 2, 3]: # lists
    print(element)
for element in (1, 2, 3): #sets
    print(element)
for key in {'one':1, 'two':2}: #dictionaries
    print(key)
for char in "123": #strings
    print(char)
for line in open("myfile.txt"): # open, read a file
    print(line, end='')
```

Building Tuples

To cover this again...

Types of
Sequences

Sequences by
the Math
Elements

Properties of
Sequences

Quiz 1

Building Tuples in Python

```
# Creating non-empty tuples  
myTuple = 'tea', 'coffee'  
print(myTuple)  
print(type(myTuple))
```

Or, Use Parenthesis to Build Tuples in Python

```
myOtherTuple = ('Bagels', 'Donuts')  
print(myOtherTuple)  
print(type(myOtherTuple))
```

Tuples and n -Tuples

Mathematically Speaking...

Types of
Sequences

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Properties of
Sequences

Quiz 1

- In mathematics, a *tuple* is a finite ordered list (sequence) of elements
- An n -tuple is a sequence (or ordered list) of n elements (n is a positive integer).
 - Ex: $(2, 7, 4, 1, 7)$ denotes a 5-tuple.

General Rule About Equality

- The general rule for the identity of two n -tuples is
$$(a_1, a_2, \dots, a_n) = (b_1, b_2, \dots, b_n) \text{ if and only if } a_1 = b_1, a_2 = b_2, \dots, a_n = b_n$$

$a = (1, 2, 3)$

$b = (1, 2, 3)$

$a == b$ # test?

$b = (1, 3, 2)$

$a == b$ # test?

Tuples and n -Tuples

Mathematically Speaking...

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Quiz 1

- 1 A tuple may contain multiple instances of the same element,
tuple $(1, 2, 2, 3) \neq (1, 2, 3)$ but,
set $\{1, 2, 2, 3\} = \{1, 2, 3\}$
- 2 Tuple elements are ordered,
tuple $(1, 2, 3) \neq (3, 2, 1)$ but,
set $\{1, 2, 3\} = \{3, 2, 1\}$
- 3 A **tuple** has a finite number of elements (also known as n -tuples), while a set or a **multiset** may have an infinite number of elements.

```
a = (1,2,3)
s = set({1,2,3})
a == s # test?
```

Elements of Tuples

Types of
Sequences

Sequences by
the Math

Elements

Properties of
Sequences

Quiz 1

- Sequences are not generic: they usually contain similar types of elements.
 - Ex: Lists contain same types of data structures, strings contain chars, files contain lines
- Sequences and n -tuples
 - n -tuples: An ordered set with n elements
 - Ex: File sequences are not n -tuples because they can contain any number of lines

Properties of Sequences

Commutative

Types of
Sequences

Properties of
Sequences

Commutative

Identity

Concatenation

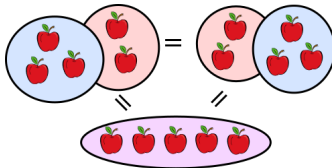
Associative

Monoids

Quiz 1

Commutative

- The term “commutative” is used in several related senses.
- A binary operation $*$ on a set S is called *commutative* if:
 $x * y = y * x$ for all $x, y \in S$
 - An operation that does not satisfy the above property is called *non-commutative*.
- One says that x *commutes* with y under $*$ if: $x * y = y * x$
- A binary function $f : A \times A \rightarrow B$ is called *commutative* if:
 $f(x, y) = f(y, x)$ for all $x, y \in A$



Properties of Sequences

Types of
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Quiz 1

Formal Definition of Identity

- **Identity:** There exists an element $e \in S$ such that for any $a \in M$, $e * a = a * e = a$

Properties of Sequences

Commutative's Identity Property

Types of
Sequences

Properties of
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Identity

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Quiz 1

Identity

- An identity is an equality relation $a = b$,
- Ex: a and b equal some numeric value.
 - $a + b == a + b$
 - $a + b == b + a$
 - $a * b == a * b$
 - $a * b == b * a$

- $a = a + e$
- $a + e = a$
- a is non-empty, contains some element
- e must be an empty sequence or is equal to 0
 - e has an *identity* property, meaning that it does not influence the operations
- $a * e = a$ or $a = a * e$, (what is e , the identity here?)

Properties of Sequences

Identity

Types of
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Quiz 1

Additive Identity

$$a + (0) = a$$

$$0 + (a) = a$$

Remember: Zero (0) preserves the Identity of every number during addition.

```
a = 1
```

```
b = 0
```

```
a == a + b #make a truth test
```

```
a + b == a #make another truth test
```

Properties of Sequences

Non-Commutative operations

Types of
Sequences

Properties of
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Quiz 1

- Washing and drying clothes resembles a noncommutative operation; washing and then drying produces a markedly different result to drying and then washing.
- Putting on left and then right socks on feet is commutative
- Putting on shirt and then sweater is not-commutative

Strings

```
a = "face"
b = "book"
a + b == b + a # run the test!
"facebook" != "bookface"
```

Properties of Sequences

Concatenation

Types of
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Properties of
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Quiz 1

- Definition: a series of interconnected things or events. The concatenation is to place one string after another. The order of placement is significant to the final product.

Ex: Concatenation of sequences

```
a = ("This", "Is")
type(a)
b = ('Loads', 'Of', 'Fun', ':-)')
type(b)
c = a + b
print(c)
( 'This', 'Is', 'Loads', 'Of', 'Fun', ':-)' )
type(c)
```

Properties of Sequences

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Quiz 1

Formal Definition of Associativity

- **Associativity Addition:** For any $a, b, c \in S$, $a + (b + c) = (a + b) + c$
- **Associativity Multiplication:** For any $a, b, c \in S$, $a * (b * c) = (a * b) * c$

Properties of Sequences

Associative Property

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Quiz 1

- Definition: The associative property states that you can add or multiply regardless of how the numbers are grouped.
- Concatenation of sequences with the associative property

- $(a + b) + c = a + (b + c)$ for any strings a , b and c .

$$a, b, c = 1, 2, 3$$

$$(a + b) + c == a + (b + c)$$

- $(a * b) * c = a * (b * c)$ for any strings a , b and c .

$$a, b, c = 1, 2, 3$$

$$(a * b) * c == a * (b * c)$$

Properties of Sequences

Associative Property

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Quiz 1

Generalized Associative Law: Keep variables in same order

- $((ab)c)d$
- $(ab)(cd)$
- $a(bc)d$
- $a((bc)d)$
- $a(b(cd))$

To Note:

- **Associative:** Variables kept in same order, operators may change order
- **Commutative:** Variables may change order, operators kept in same order.

Let's Apply Sequences

Modelling Interest Rates

Types of
Sequences

Properties of
Sequences

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Identity

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Associative

Monoids

Quiz 1

Problem:

Put x_0 money in a bank at year 0. What is the value after N years if the interest rate is p percent per year?

Solution:

The fundamental information relates the value at year n , x_n to the value of the previous year, x_{n-1} .

$$x_n = x_{n-1} + \frac{p}{100} * x_{n-1}$$

Start with x_0 and then calculate x_1 , then x_2 , and onward...

The output of the program?

Types of
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Quiz 1

- $x_0 = 100000$ # initial amount
- $p = 3.92$ # interest rate
- $N = 6$ # number of years

```
At year = 0 Current value is = 103920.0
At year = 1 Current value is = 107993.664
At year = 2 Current value is = 112227.0156288
At year = 3 Current value is = 116626.31464144896
At year = 4 Current value is = 121198.06617539376
At year = 5 Current value is = 125949.0303694692
```

Test these values online

- For example: http://www.moneychimp.com/calculator/compound_interest_calculator.htm

Monoid: Stavely's Definition, Section 6.2, pp. 59

Both strings with concatenation and integers with addition are examples of the mathematical structure called a monoid. A monoid is a set that has an associative binary operator and an identity element.

More formally, a *monoid* is an ordered pair (S, \otimes) such that S is a set and \otimes is some **binary operator**, satisfying these conditions:

- ① For all a and b in S , $a \otimes b$ is defined and is also in S
- ② For all a, b and c in S , $(a \otimes b) \otimes c = a \otimes (b \otimes c)$
- ③ There is an element e in S such that, for all a in S ,
 $e \otimes a = a \otimes e = a$
- ④ Then we also say that S is a *monoid* under \otimes , with identity e

Rounding errors

- The Python floating-point numbers are not quite a monoid under addition: for floating-point operands, $(a + b) + c$ is often not exactly equal to $a + (b + c)$ because of the error during round-off processes
- The same is true of multiplication.

Values round to 1, or do they?

```
a = 0.9999999999999999
b = 0.9999999999999999
a == b #False
```

Min Function

- The Min function, $\min(x, y)$, is defined to be x if $x \leq y$ and y otherwise.
- We treat \min as an operator and so, $x \min y$ is an operator (like the \max operator)
- Here, \min is both associative and commutative, and the identity value is obtained from `float("inf")` (an inferior value)

Max Function

- The Max function, $\max(x, y)$, is defined to be x if $x \geq y$ and y otherwise.
- We treat \max as an operator and so, $x \max y$ is an operator
- Here, \max is both associative and commutative, and the non-negative integers are a monoid under \max , with identity 0

Monoids

Big Plus and Big Times

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Quiz 1

$$\begin{array}{ccccccc}
 s_0 & s_1 & s_2 & \dots & s_n & & s_0 & s_1 & s_2 & \dots & s_n \\
 \underbrace{\hspace{10em}} & & & & & & \underbrace{\hspace{10em}} \\
 + & & & & & & \otimes
 \end{array}$$

Figure: The operator '+' is associative, the operator \otimes behaves as an associative

- Knowing that one type of calculation is monoid allows us to use monoid-type code on it.



- Given on Friday 28th during class time (11am)
- Online format
- One hour to complete
- Ten questions: Multi-choice, True/False, Matching and Short Essay
- Picking out bugs of code or determining output

What to study

Types of
Sequences

Properties of
Sequences

Quiz 1

- **Slides, notes, with chapters to add detail to class material**
- Main ideas behind mathematical subjects in class (again, study your slides)
- Python basics and code
 - Study the code from the practicals and covered in class to understand the how programs worked.
 - Mathematical operators: using doing calculations on in the interpreter with Python
 - for loops using `range()`
 - Iterations over sequences
 - Strings, characters, integers, floats
 - Sequences, sets, lists, dictionaries, tuples
 - Conditional statements