

CS200 - Computer Organization

Data Internals - Part1

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Motivation to learn data representation

- How is data represented internally?
- Examine how data is referenced inside a Program?

How is Data represented internally?

- **Binary:** is readable by hardware.
- **Decimal:** is readable by human beings.
- **HexaDecimal:** is readable by storage devices.



Binary

Decimal

HexaDecimal

Binary Vs Decimal



- What does the number $(123)_{10}$ mean?
 - $100 + 20 + 3$
 - $1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$
- What does the number $(1001010)_2$ mean?
 - $1000000 + 1000 + 10$
 - $1 \times 2^6 + 1 \times 2^3 + 1 \times 2^1$
 - $2^6 + 2^3 + 2^1 = 64 + 8 + 2 = 74$

Data referencing in a C Program

```
1  #include <stdio.h>
2  int main(){
3      int alpha = 100;
4      printf("%d", alpha);
5  }
```

Q₁: What happens when line 3 is executed?

Q₂: What happens when line 4 is executed?

Q₃: How is data referenced in Memory when lines 3 and 4 are executed?

Decimal to Binary

Q₁: What happens when line 3 is executed?

- Divide repeatedly by 2 and retain the remainders. Continue until the quotient = 0.
- For example, 245_{10}

- | | |
|----------------------|-----------|
| - $245 \div 2 = 122$ | R = 1 LSB |
| - $122 \div 2 = 61$ | R = 0 |
| - $61 \div 2 = 30$ | R = 1 |
| - $30 \div 2 = 15$ | R = 0 |
| - $15 \div 2 = 7$ | R = 1 |
| - $7 \div 2 = 3$ | R = 1 |
| - $3 \div 2 = 1$ | R = 1 |
| - $1 \div 2 = 0$ | R = 1 MSB |
-
- Solution is 11110101_2

Q₂: What happens when line 4 is executed?

- Starting with the most significant bit (left to right), repeatedly multiply by 2, adding each bit as we move along.
- For example, 1010111_2
 - $(0 + 1) \times 2 = 2$
 - $(2 + 0) \times 2 = 4$
 - $(4 + 1) \times 2 = 10$
 - $(10 + 0) \times 2 = 20$
 - $(20 + 1) \times 2 = 42$
 - $(42 + 1) \times 2 = 86$
 - $(86 + 1) = 87$
 - Solution is 87_{10}

Hexa Decimal To Binary

Q₃: How is data referenced in Memory when lines 3 and 4 are executed?

- Expand each hexadecimal digit into the corresponding 4 binary digits:
- For example: $(1234AF0C)_{16}$

- 0001 0010 0011 0100 1010 1111
0000 1100

- Solution: $1234AF0C_{16} =$
 $00010010001101001010111100001100_2$

Binary to Hexa Decimal

Q₃: How is data referenced in Memory when lines 3 and 4 are executed?

- Create groups of 4 bits (LSB to MSB), and translate each group to its corresponding Hex:
- For example: 11001011101_2

- 110	0101	1101 ₂
- 6	5	D ₁₆

- Solution: $11001011101_2 = 65D_{16}$

Hexa Decimal to Decimal

Q₃: How is data referenced in Memory when lines 3 and 4 are executed?

- Starting with the most significant digit, repeatedly multiply by 16, adding each digit as we move along.
- For example, $24E_{16}$
 - $(0 + 2) \times 16 = 32$
 - $(32 + 4) \times 16 = 576$
 - $(576 + 14(E)) = 590$
 - Solution is 590_{10}

Decimal to Hexa Decimal

Q₃: How is data referenced in Memory when lines 3 and 4 are executed?

- Divide repeatedly by 16 and retain the remainders. Continue until the quotient = 0.
- For example, 53241_{10}

- $53241 \div 16 = 3327$	R = 9	LSB
- $3327 \div 16 = 207$	R = 15(F)	
- $207 \div 16 = 12$	R = 15(F)	
- $12 \div 16 = 0$	R = 12(C)	MSB
- Solution is CFF9 ₁₆		

Try out!

Class Activity

- Convert $(10101010)_2$ to Decimal
- Convert $(87)_{10}$ to Binary
- Convert $(DECAF)_{16}$ to Decimal
- Convert $(1234567)_{10}$ to HexaDecimal
- Convert $(3A8D2)_{16}$ to Binary
- Convert $(11101001010010100101)_2$ to HexaDecimal

Fractions: Binary Vs Decimal



- Starting with the least significant bit, divide the value by 2 and add the next bit. Continue to the binary point.
- For example, 0.01101_2
 - $(0 + 1)/2 = 1/2$
 - $(1/2 + 0)/2 = 1/4$
 - $(1/4 + 1)/2 = 5/8$
 - $(5/8 + 1)/2 = 13/16$
 - $(13/16 + 0)/2 = 13/32$
- Solution: $0.01101_2 = 13/32$

Fractions: Decimal Vs Binary



- Multiply repeatedly by 2 and subtract the whole numbers until the multiplicand = 0.
- For example, 0.6875_{10}

- $0.6875 \times 2 = 1.375$

Most Significant Bit

- $0.375 \times 2 = 0.75$

- $0.75 \times 2 = 1.5$

- $0.5 \times 2 = 1.0$

Least Significant Bit

- Solution is $0.6875_{10} = 0.1011_2$

Signed Binary

```
1  #include <stdio.h>
2  int main(){
3      int alpha = 10;
4      int beta = 3;
5      int gamma = 5;
6      alpha += beta;
7      alpha -= gamma;
8      printf("%d\n", alpha);
9  }
```

Q₁: What happens when lines 6, 7, and 8 are executed?

How does Binary (2 bit) Add Work?

Rules:

- 1 $0 + 0 = 0$
- 2 $0 + 1 = 1$
- 3 $1 + 0 = 1$
- 4 $1 + 1 = 0$ with carry (1)

a	b	output	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

How does Binary (3 bit) Add Work?

Rules:

- 1 $0 + 0 + 0 = 0$
- 2 $0 + 0 + 1 = 1$
- 3 $0 + 1 + 0 = 1$
- 4 $0 + 1 + 1 = 0$ with carry (1)
- 5 $1 + 0 + 0 = 1$
- 6 $1 + 0 + 1 = 0$ with carry (1)
- 7 $1 + 1 + 0 = 0$ with carry (1)
- 8 $1 + 1 + 1 = 1$ with carry (1)

How does Binary (3 bit) Add Work?

a	b	carry in	output	carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Try out

How does computer's execute:

- $(10 + 3)$
- $(8 + 7)$
- $(5 + 6)$

How does Binary Subtraction Work?

- What does $(5 - 2)$ mean to computers?
- $(5 - 2) = (5 + (-2))$

How do we represent (-2) in binary?

How is Signed Data represented internally?

- In decimal we are quite familiar with placing a "-" sign in front of a number to denote that it is negative.
- The same is true for binary numbers a computer won't understand that.
- What happens in memory then?

Binary Negative Numbers



- There are several representations
 - Signed magnitude
 - One's complement
 - Two's complement
- Two's complement is the system used in microprocessors
- Most significant bit becomes important

Signed Magnitude



- Represent the decimal number as binary.
- Left bit (MSB) used as the sign bit.
- Only have 7 bits to express the number.

$$12_{10} = 00001100$$

$$-12_{10} = 10001100$$

Signed Magnitude Limitation



- What is -7 in signed magnitude? (duplicates)
- How does computer's execute $(5 - 2)$ using signed magnitude?

One's Complement



- Method: Invert the ones and zeros

$$11_{10} = 00001011$$

$$-11_{10} = 11110100$$

- 0 in MSB implies positive
- 1 in MSB implies negative

One's Complement Limitation



- What is 1111 in one's complement? (duplicate)

Two's Complement



- Method: Take the one's complement and add 1 to the result. **most stable**

$$11_{10} = 00001011$$

$$-11_{10} = 11110100 \quad \text{one's comp}$$

$$-11_{10} = 11110101 \quad \text{two's comp}$$

Reading Assignment

Section 1.10 in **PH**

Questions

Do you have any questions from this class discussion?