CS202 - Algorithm Analysis

Balanced Tree Algorithms

Aravind Mohan

Allegheny College

April 26, 2021

Discussion Based On ...

Sedgewick 3.3 Red Black Trees

2-3 Tree Limitation

- **Too much** balancing, thereby certain insert takes longer.

RED BLACK Trees - An overview

- **Red-black** trees are a variation of binary search trees to ensure that the tree is balanced. A self balancing bst, that offers balancing in a much efficient time than other self balancing trees.
- **Height of the tree** is O(log(n)) and hence the search, insert, and delete operations could be done in O(log(n)) time complexity in the worst case.
- So what are we doing different from 2-3 Trees?

  We **aim** to **reduce** too much balancing act and thereby provide a better performance.

RED BLACK Trees - An overview

- **Where** is Red-black trees used?

  They are used to implement a data structure called finite maps.

  TreeMaps and SortedMaps are some examples of finite maps in Java Framework.

RB Trees - Properties

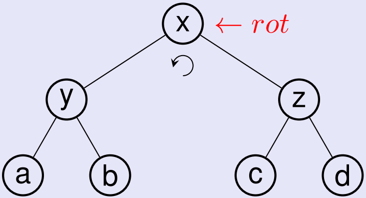
- Every node is either red or black.
- The **root** is black.
- Every leaf (nil) is black.
- If a node is red, then both its children are black.
- For each node, all paths from the node to descendant leaves contain the same number of **black** nodes.

RB Trees - Action Items

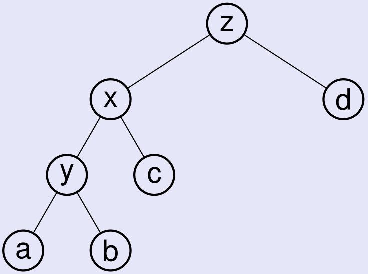
How does balancing work in RB Trees?

- Recoloring - change colors of the node
- Rotation - rotate left or rotate right to make the tree balanced. How does Rotation work?

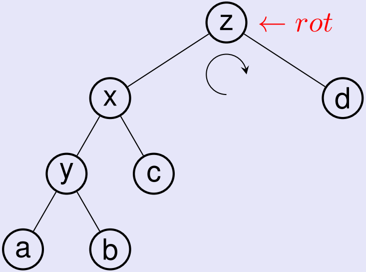
Left Rotation on x



After Left Rotation

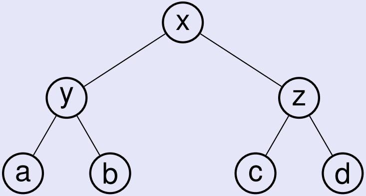


Right Rotation on z



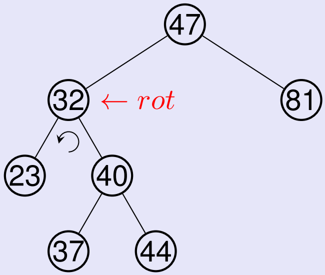
RB Trees - Rotation

After Right Rotation



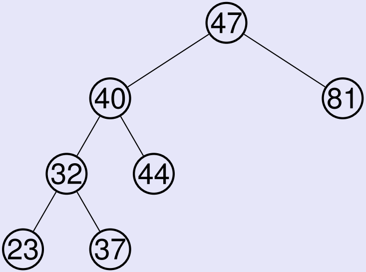
Left and right rotations are symmetrical.

Example: **Left Rotation on 32**

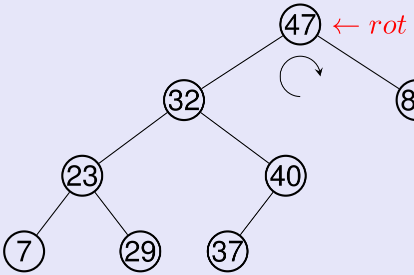


RB Trees - Rotation

Example: **After Left Rotation**

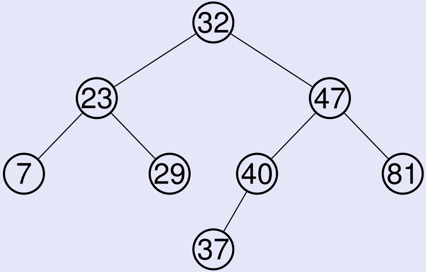


Example: **Right Rotation on 47**



RB Trees - Rotation

Example: After Right Rotation



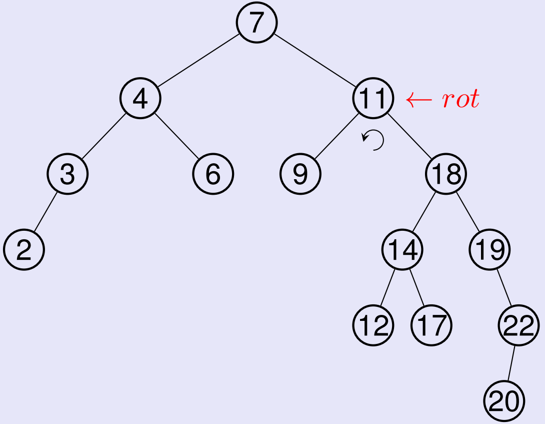
RB Trees - Exercise

Solve Try out on your own 1 and 2.

Try out 1: Left Rotate 11.

(by using the tree from the next slide.)

RB Trees - Exercise



Try out 2: Right Rotate 18

(by using the tree from your Tryout1 solution)

Important Point:

RB Trees Insert and Delete operations.

Get used to Rotation Process. We will need to do a lot of rotations in Insert and Delete!

RB Trees - Properties (Recall)

- Every node is either red or black.
- The **root** is black.
- Every leaf (nil) is black.
- If a node is red, then both its children are black.
- For each node, all paths from the node to descendant leaves contain the same number of **black** nodes.

How to insert a new node into the Red Black Tree?

- Insert node z in RB tree:
  1. As in ordinary Binary Search Tree
  2. Node z should be colored red

Violation of property of RB tree

- After insertion using the rules outlined in the previous slide, further examination is required to identify if the RB tree properties are preserved or not.
- The violation is categorized into two broad categories:
  1. Case 1: Node z is the root node
  2. Case 2: Node z is a child of a red parent

Violation of property of RB tree (contd)

Case 1: Node z is the root node and red node.

The fix for this case is:

- Change node z 's color to **Black**

Violation of property of RB tree (contd)

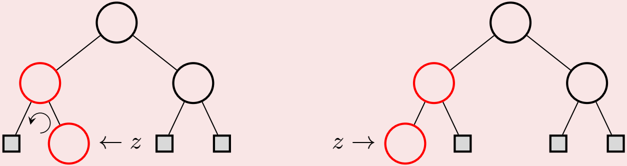
Case 2.1: Uncle of node z is red

The fix for this case is:

- Change node z 's grandparent color to **Red** and make this node the new z
- Change node z 's parent and uncle color to **Black**
- Elevate the grandparent of node z to be the new **node** z (FIX UP)

Violation of property of RB tree (contd)

Case 2.2: Uncle of node z is black and z is the right child

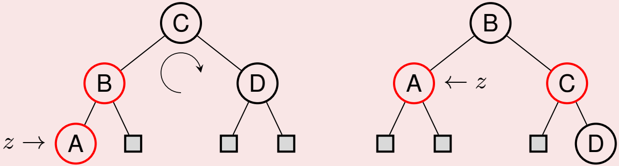


The fix for this case is:

- Left rotate node z 's parent node
- Change the position from right to left. This leads to Case 2.3

Violation of property of RB tree (contd)

Case 2.3: Uncle of node z is black and z is the left child

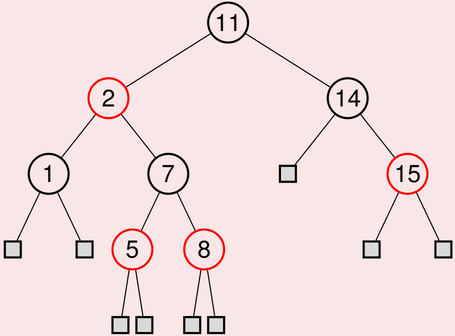


The fix for this case is:

- Change node z 's grandparent color to **Red** and parent color to **Black**
- Right rotate node z 's grandparent node

Okay, let's do an example?

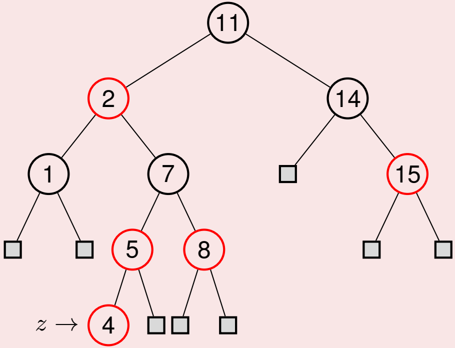
Insert a new node $z = 4$



So how to insert this new node?

Okay, let's do an example? [contd]

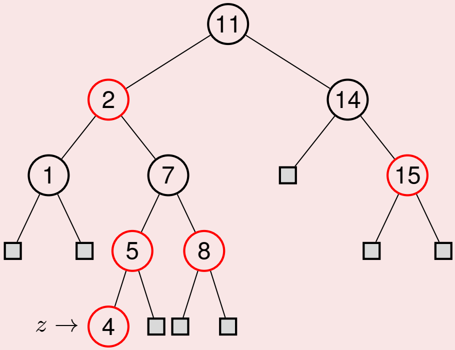
Step 1: Insert a new red color node $z = 4$



By using the same rule as BST

Okay, let's do an example? [contd]

Step 2: Violation at node $z = 4$



Okay, let's do an example? [contd]

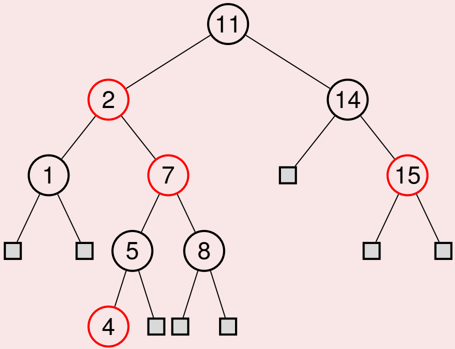
"Case 2.1 violation @ node z"

Step 2: Violation at node $z = 4$

- **Change node z 's grandparent color to Red**
- **Change node z 's parent and uncle color to Black**
- **Make grand parent of node z to be the new node z**

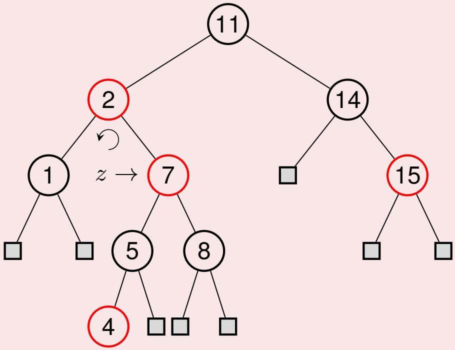
Okay, let's do an example? [contd]

Step 2: Violation at node $z = 4$



Okay, let's do an example? [contd]

Step 3: Violation at node $z = 7$



Okay, let's do an example? [contd]

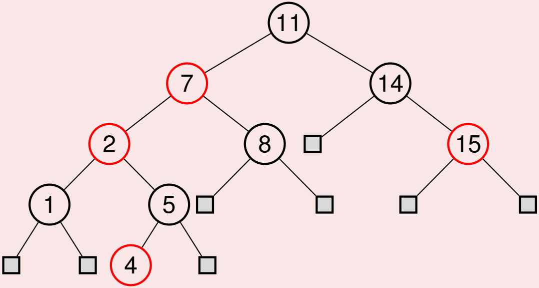
"Case 2.2 violation @ node z"

Step 3: Violation at node $z = 7$

- **Left rotate at node z 's parent**
- **Change the position from right to left. This leads to Case 2.3**

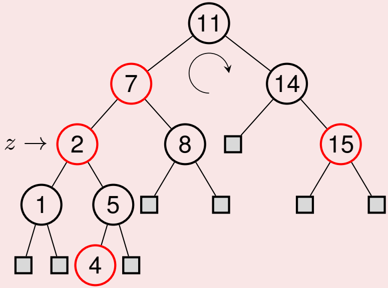
Okay, let's do an example? [contd]

Step 3: Violation at node $z = 7$



Okay, let's do an example? [contd]

Step 4: Violation at node $z = 2$



Okay, let's do an example? [contd]

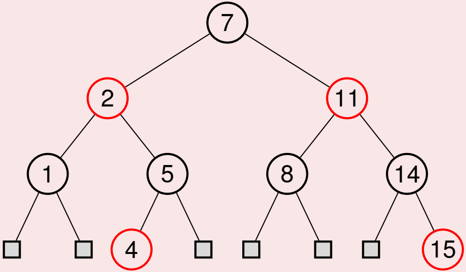
"Case 2.3 violation @ node z"

Step 4: Violation at node $z = 2$

- **Change node z 's grandparent color to Red and parent color to Black;**
- **Right rotate at node z 's grandparent**

Okay, let's do an example? [contd]

Step 4: Violation at node $z = 2$



- No more violation
- **All the path from the root node to the NIL nodes contains equal number of black nodes!**

RB Trees - Exercise

Solve Try out 1 and 2 on your own!

Try out 1: Insert 4, 3, 6 (step by step)

(by using the tree from the above slide.)

RB Trees - Exercise

Try out 2: Insert 9, 10, 13 (step by step)

(by using the tree from your Tryout1 solution)

How to delete a node in the Red Black Tree?

- **Step 1:** Perform BST deletion procedure.
- **Step 2:** If the node (z) to be deleted is red color, then just delete (z) from the tree.
- **Step 3:** If the node (z) to be deleted is black color, then color (z) as Double Black.

- **Double Black:** Apply a series of resolution rules to fix the violations. Cases are **Mirrored**.

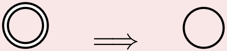
Double Black Node Resolution

Case 1:

- If (z) is a double black root node, then just convert the double black root to single black.

Double Black Node Resolution

Case 1:



Double Black Node Resolution

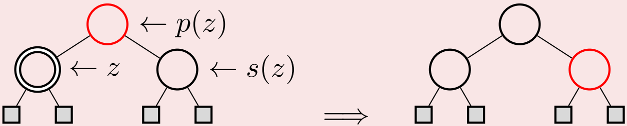
Case 2:

- If the sibling of the double black node s(z) is black and the children of s(z) are both black:
  1. **color** z as single **black**
  2. add **black** to the parent node p(z)
  3. **color** s(z) as **red**

add black

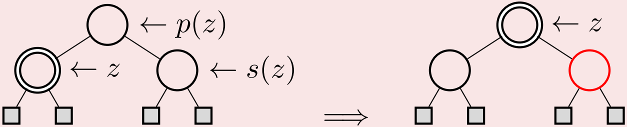
- if p(z) is originally black, then color p(z) as **double black** and recursively **fix** the violation upwards
- if p(z) is originally red, then **color** p(z) as **black**

Case 2 - (1):



Double Black Node Resolution

Case 2 - (2):

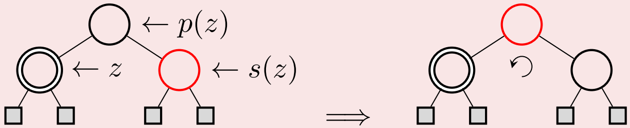


Double Black Node Resolution

Case 3:

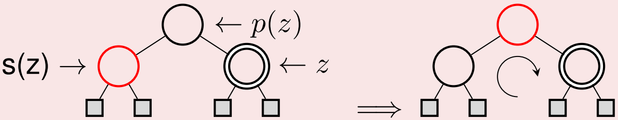
- If the sibling of the double black node s(z) is red:
  1. **swap** colors of parent p(z) and s(z)
  2. **rotate** p(z) towards the direction of z
  3. **reapply** cases

Case 3 - (1):



Double Black Node Resolution

Case 3 - (2):



MIRRORED Case 3 - (1)

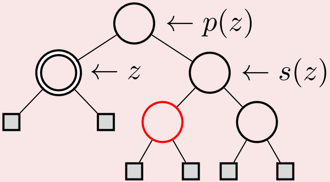
Double Black Node Resolution

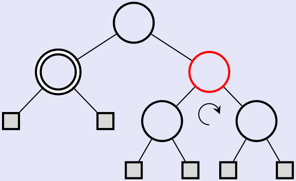
Case 4:

- If the sibling of the double black node s(z) is black and the closest child of s(z) from z is red, farthest child of s(z) from z is black.
  1. **swap** colors of sibling s(z) and the closest child of s(z) from z
  2. **rotate** sibling s(z) towards the opposite direction of z
  3. **apply** Case 5

Double Black Node Resolution

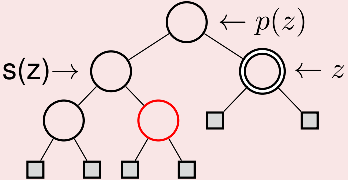
Case 4 - (1):

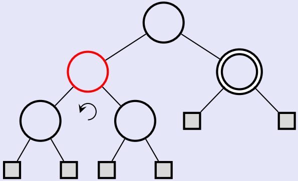




Case 4 - (2):

MIRRORED Case 4 - (1)



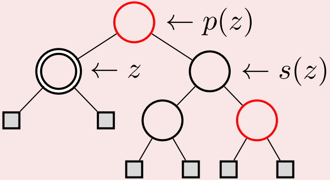


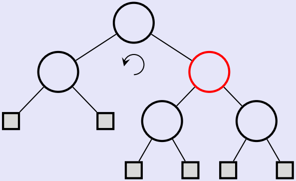
Double Black Node Resolution

Case 5:

- If the sibling of the double black node s(z) is black and the farthest child of s(z) from z is red:
  1. **swap** colors of parent p(z) and sibling s(z)
  2. **change** color of the farthest child of s(z) from z, to black
  3. **rotate** parent p(z) towards the direction of z
  4. **color** z as single black.

Case 5 - (1):

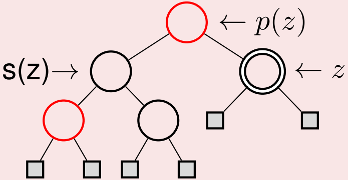


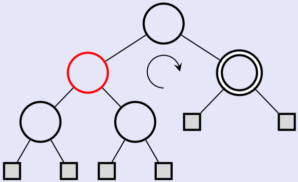


Double Black Node Resolution

Case 5 - (2):

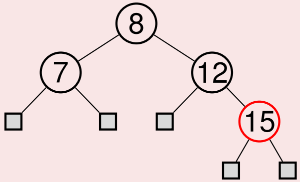
MIRRORED Case 5 - (1)

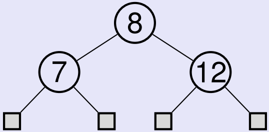




RB Delete Examples

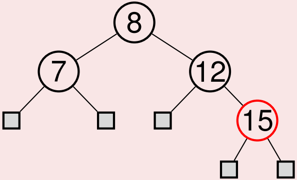
Example 1: Delete 15

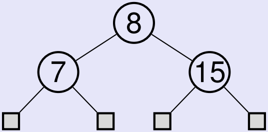




RB Delete Examples

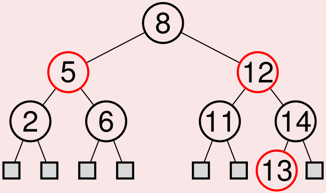
Example 2: Delete 12

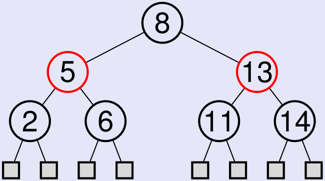




RB Delete Examples

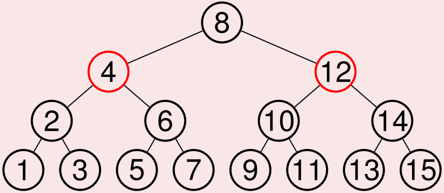
Example 3: Delete 12

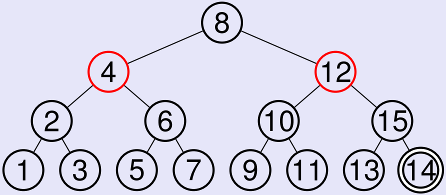




RB Delete Examples

Example 4: Delete 14

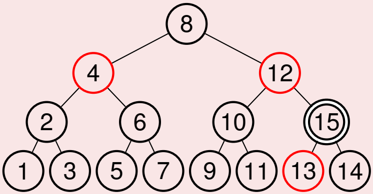




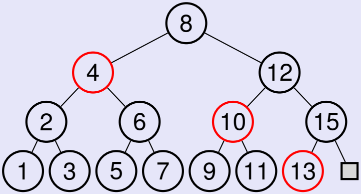
RB Delete Examples

Example 4: Delete 14 (contd)

Case 2

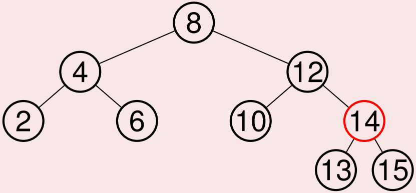


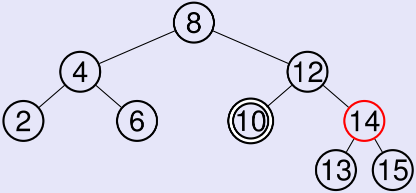
Case 2



RB Delete Examples

Example 5: Delete 10

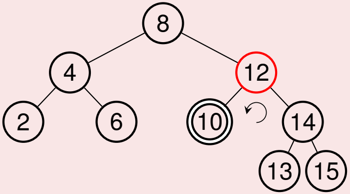




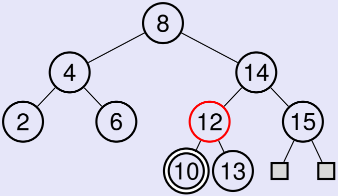
RB Delete Examples

Example 5: Delete 10 (contd)

Case 3



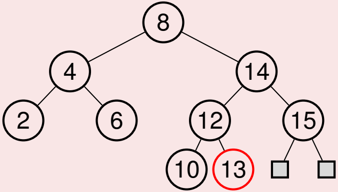
Case 3



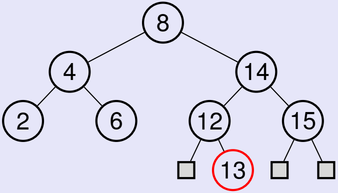
RB Delete Examples

Example 5: Delete 10 (contd)

Case 2

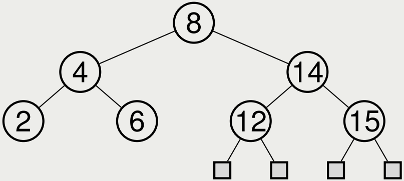


Case 3



RB Trees - Exercise

Try out 1: Delete 15 from the tree below:

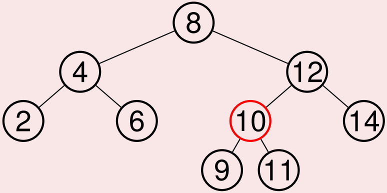


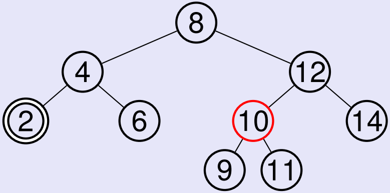
RB Trees Deletion

Continue with more examples.

RB Delete Examples

Example 6: Delete 2

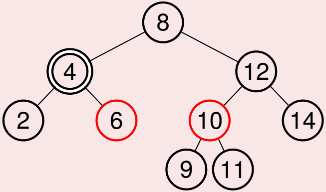




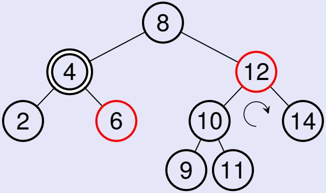
RB Delete Examples

Example 6: Delete 2 (contd)

Case 2

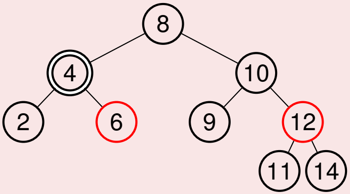


Case 4

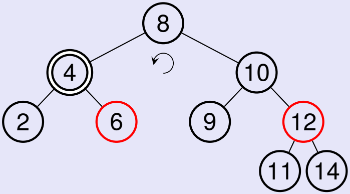


Example 6: Delete 2 (contd)

Case 4



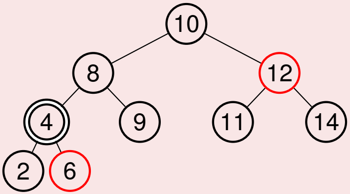
Case 5



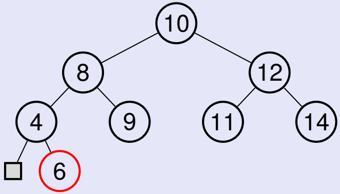
RB Delete Examples

Example 6: Delete 2 (contd)

Case 5



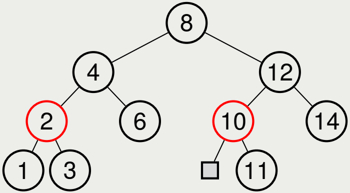
Case 5



Try out 1 - delete from the tree below:

- **First** Delete 2
- **Second** Delete 3

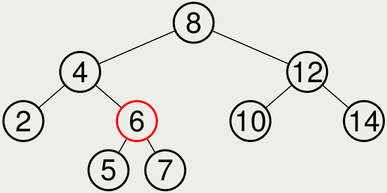
  from the tree produced after deleting 2



RB Trees - Exercise

Try out (optional) - do you want more?

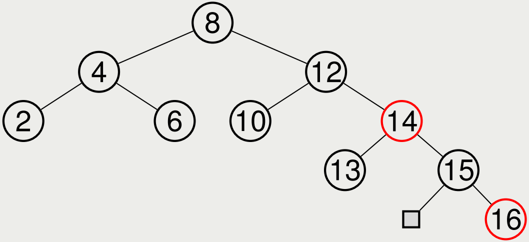
- **Delete 14**



Next:

- **Graph based algorithms**

RB Trees - Exercise



Delete 10, 4, 16, 15, 8, 6, 2

Reading Assignment

Sedgewick 3.3 RB Trees

Questions?

Please ask if there are any Questions!