Data Analytics
CS301
Multiple Linear Regression
Understanding the Summary

Week 11: 10th November

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Up To Now in Regression

- We have discussed how one entity influences another.
- What about having two entities (independent) which may have some kind of influence on a dependent variable.
- Especially if a dependent variable has a high correlation with more multiple independent variable.

Main Idea

- GPA could be dependent on studying
- Student performance may be based on more than just one entity.
- GPA could be dependent on studying AND getting enough rest
- OR maybe even more variables are involved?
- GPA could be dependent on studying AND rest AND eating good food AND …

So, Multiple Linear Regression Is What … ?

- Multiple linear regression is the most common form of linear regression analysis.
- A predictive analysis, the multiple linear regression is used to explain the relationship between one continuous dependent variable and two or more independent variables.
- The independent variables can be continuous or categorical (dummy coded as appropriate).

Types of Questions Answered

- Do age and IQ scores effectively predict GPA?
- Do weight, height, and age explain the variance in cholesterol levels?
- Are video game sales explained by their exciting graphics and inexpensive costs?
- Is road safety a combination of relaxed and defensive driving?
- Are there more independent variables to be used to answer to these above dependents?

Equation of Multiple Independent Variables

- The model is now a multi-independent variable equation.

Dependent Variable

$$y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \beta_3 x_{i,3} + \epsilon_i$$

Independent Variables

Multiple Variables Equation and Assumptions

- A population model for a multiple regression model that relates a *y*-variable to $p - 1$ predictor variables is written as the following.

$$y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \ldots + \beta_{p-1} x_{i,p-1} + \varepsilon_i$$

- We assume that the ε_i have a normal distribution with mean 0 and constant variance σ^2. These are the same assumptions that we used in simple regression with one *x*-variable.

- The subscript *i* refers to the ith individual or unit in the population. In the notation for the *x*-variables, the subscript following *i* simply denotes which *x*-variable it is.

Same Hypothesis as Before...

As an example, to determine whether variable X_1 is a useful predictor variable in a model, we use the following hypothsis:

$$H_0 : \beta_1 = 0$$

$$H_A : \beta_1 \; != 0$$

(think slope values)

If the null hypothesis above were the case, then a change in the value of X_1 would not change Y, so Y and X_1 are not related.

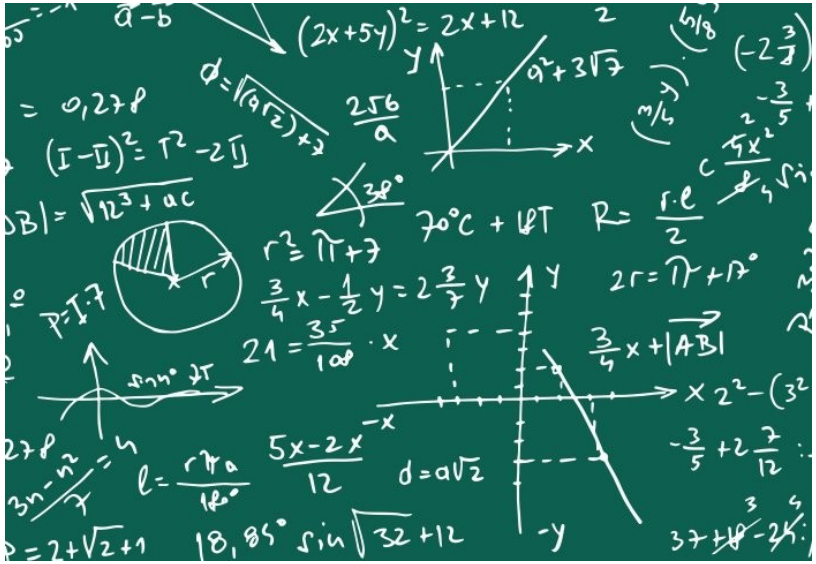
We would still be left with variables X_2 and X_3 being present in the model and so we could not reject the null hypothesis above. Instead we should say that we do not need variable X_1 in the model given that variables X_2 and X_3 will remain.

In general, the interpretation of a slope in multiple regression can be tricky. Correlations among the predictors can change the slope values dramatically from what they would be in separate simple regressions.

Analysis Question

- Two variables: Do *Age* and *Height* (both) influence the capacity of lungs (*LungCap*)?
- Asking actually, can we make a model that takes the following form?

$$LungCap = Age*b_1 + Height*b_2 + b_3$$



Lung Capacity Data

```
library(tidyverse)
# install.packages("psych")
library(psych)


#open lung capacity data
lc <-file.choose()
dataLungCap <- read.csv(lc)
View(dataLungCap)
```

Create the Multiple-Variable Regression Model

```
# model creation

mod <- lm(data = dataLungCap, LungCap ~ Age + Height)

# get a report of the model

summary(mod)
```

Summary

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

Intercept Value:
"When the age and height of zero"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
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```

The estimated mean lung capacity of someone having an age and height of zero. Is this *meaningful*?

Slope of Age:

"How is my *Age* variable related to *Height*?"

```
Call:
lm(formula = LungCap ~ Age

Residuals:
    Min      1Q  Median
-3.4080 -0.7097 -0.0078  0.71

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
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```

The effect of *Age* on *Lung Capacity* adjusting or controlling for *Height*. We may associate an increase of 1 year in *Age* with an increase of 0.126 in *Lung Capacity* adjusting or controlling for *Height*

Test Statistic:

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
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Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

The test statistic that we use to perform the hypothesis test that the slope for *Age* = 0.

Slope of Height:

"How is my *Height* variable related to A*ge*?"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

The estimated effect of *Height* on *Lung Capacity*, adjusted for *Age*.

Test Statistic:

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

The test statistic that we use to perform the hypothesis test that the slope for *Height* = 0.

R-squared Value:

"How do the independents explain the dependent?"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t valu     (>|t|)
(Intercept) -11.747065   0.476899 -24       < 2e-16 ***
Age           0.126368   0.017851        079 3.45e-12 ***
Height        0.278432   0.009926   28.051  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

Approximately 84% of the variation in *Lung Capacity* can be explained by our model (*Age* and *Height*)

F-Statistic of Test:

"What value do I look up in a table to check on significance?"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value
(Intercept) -11.747065   0.476899 -24.6
Age           0.126368   0.017851   7.0
Height        0.278432   0.009926  28
---
Signif. codes:  0 '***' 0.001 '**'  .01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.056 o  722 degrees of freedom
Multiple R-squared:  0.843,      Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

Null Hyp. Test:
The test of the null hypothesis that all model coefficients are zero.

Degrees of Freedom:
There are 725 rows in the data and three groups.
722 = 725 – 3

Used also for finding values in F-table for hypothesis testing

Our Test of The Null Hypothesis

- **Ho**: $\beta_1 = \beta_2 = \ldots = \beta_k$
- In our case,
  - **Ho**: $\beta_{age} = \beta_{height} = 0$ (slopes are zero)

Nothing is happening between the k-number of variables

$$y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \beta_3 x_{i,3} + \epsilon_i$$

The p-Value:
"Is this model statistically meaningful?"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q
-3.4080 -0.7097 -0.0078  0.7167  3.1

Coefficients:
             Estimate Std. Error t
(Intercept) -11.747065   0.476899 -24.
Age           0.126368   0.017851   7.079          2 ***
Height        0.278432   0.009926  28.051          5 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.      .' 0.1 ' ' 1

Residual standard error: 1.056 on 722 degrees of    reedom
Multiple R-squared:  0.843,     Adjusted R-squared    0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

The p-value is very close to zero and so we reject the Ho (i.e., all the model coefficients are zero (slope = 0).

Conclusion: There is something non-random happening in this model.

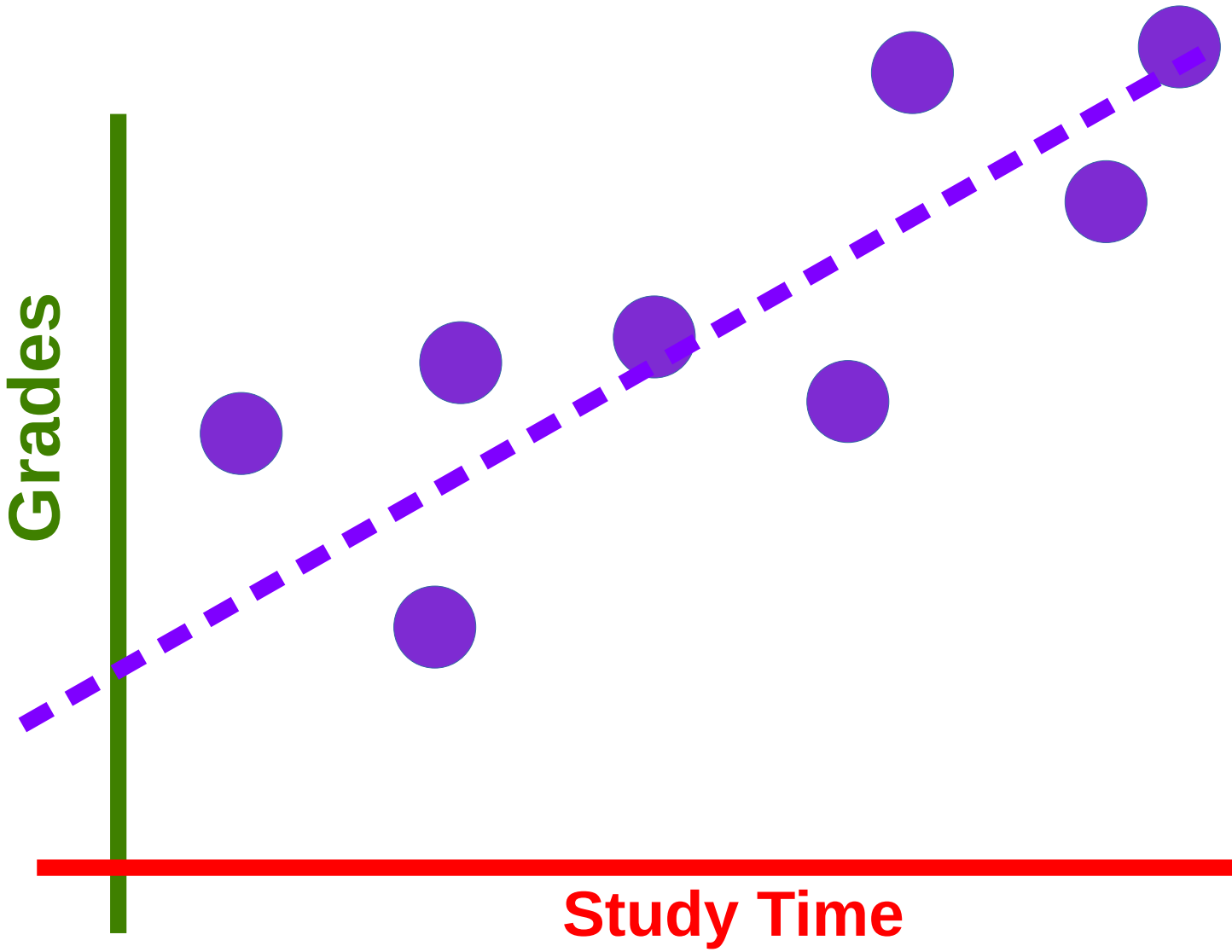
Residual Errors:

"What is the estimation of the difference between observed and predicted values?"

```
Call:
lm(formula = LungCap ~ Age + Height)

Residuals:
    Min      1Q  Median      3Q     Max
-3.4080 -0.7097 -0.0078  0.7167  3.1679

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -11.747065   0.476899 -24.632  < 2e-16 ***
Age           0.126368   0.017851   7.079 3.45e-12 ***
Height        0.278432   0.009926  28.051  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.056 on 722 degrees of freedom
Multiple R-squared:  0.843,     Adjusted R-squared:  0.8425
F-statistic:  1938 on 2 and 722 DF,  p-value: < 2.2e-16
```

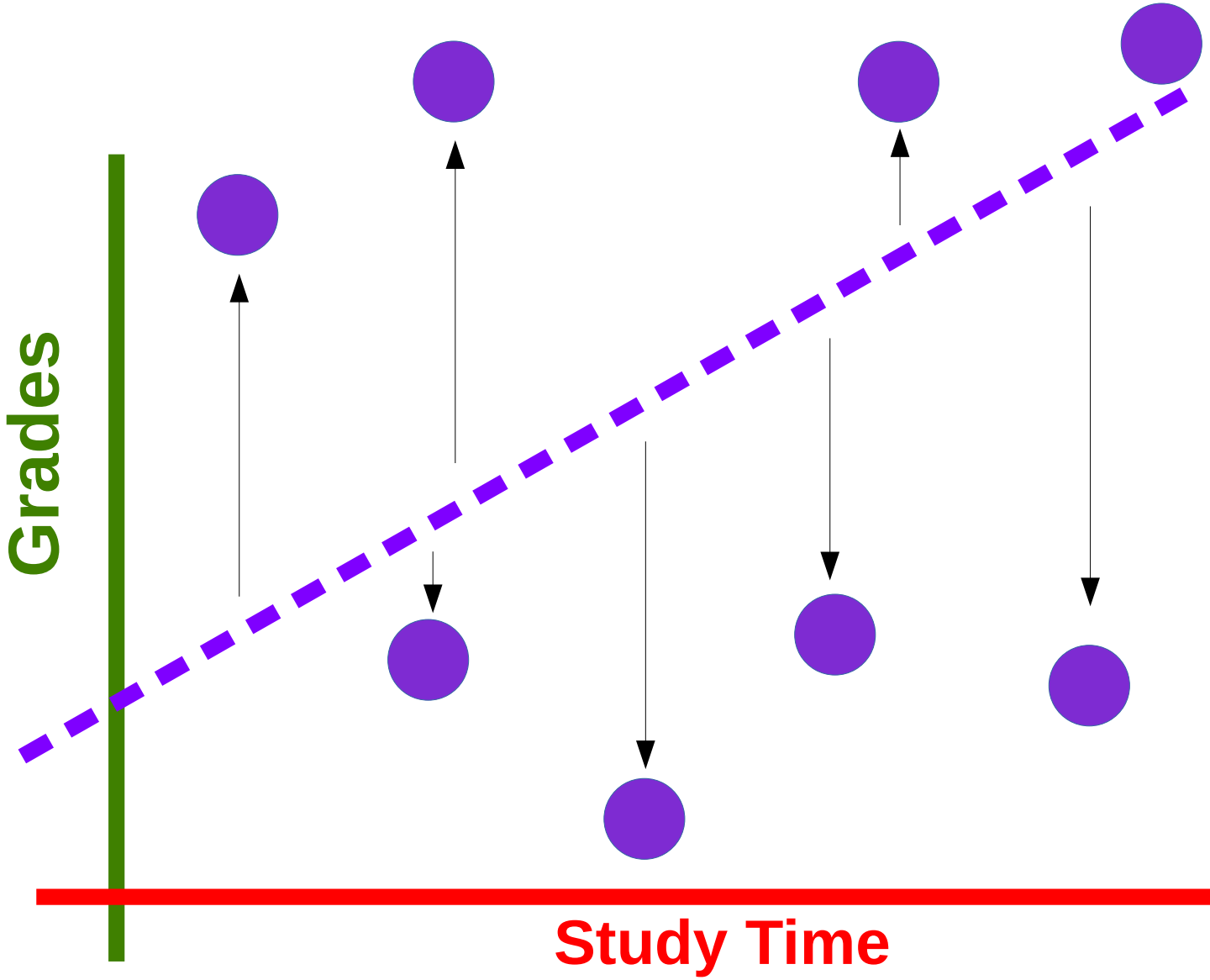
This error gives an idea about how far the observed *Lung Capacity* (dependent) values are from the predicted or fitted *Lung Capacity* (the "y-hats")

“Good” Residual Standard Error



Smaller standard error value

“Bad” Residual Standard Error



Bigger standard error value

Test for Correlation Between *All* Variables

```
library(psych)
pairs.panels(dataLungCap)
pairs.panels(dataLungCap, lm = TRUE)
```

Allows for a quick study of correlation across the variables of your study

pairs.panels {psych} R Documentation

SPLOM, histograms and correlations for a data matrix

Description

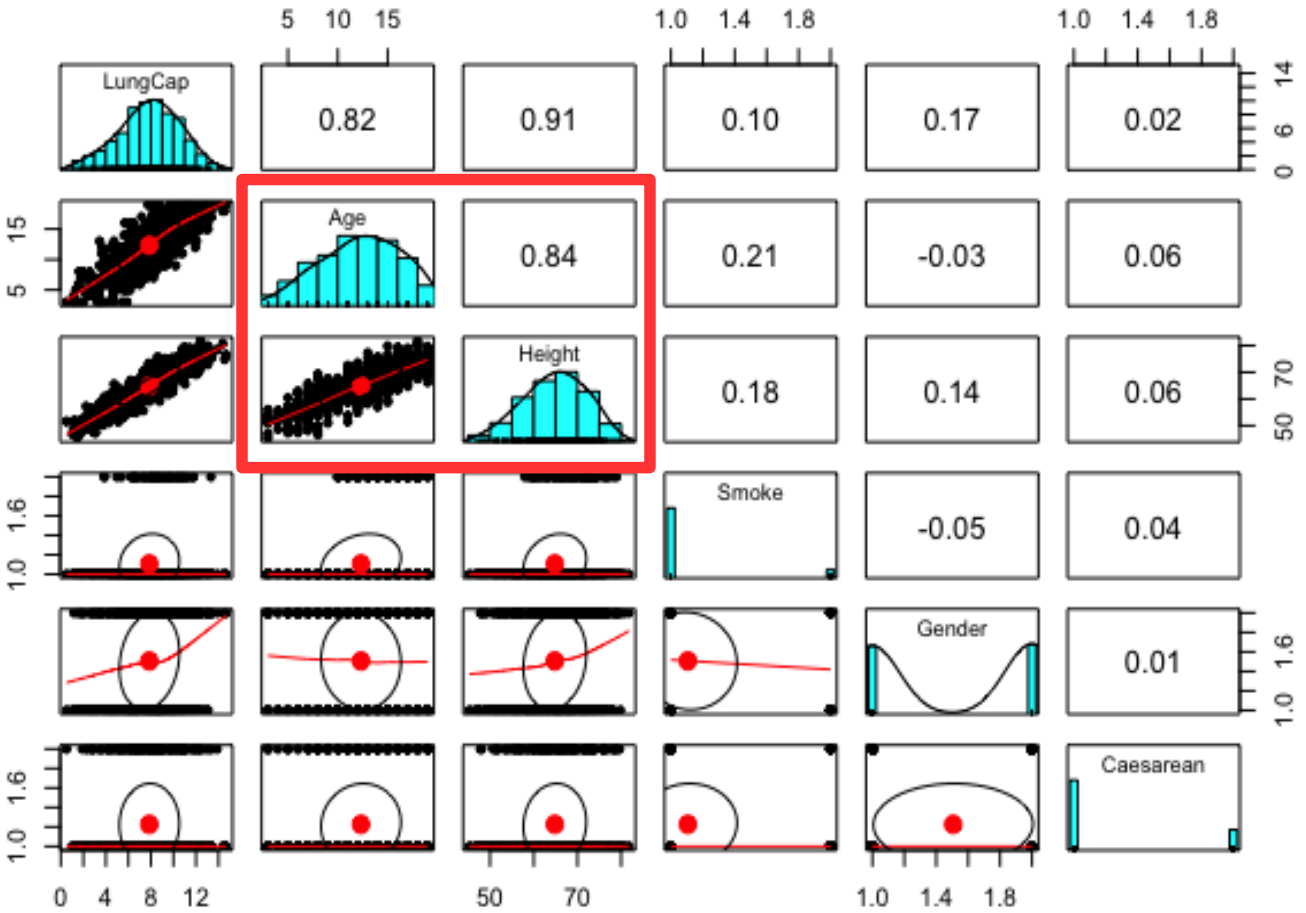
Adapted from the help page for pairs, pairs.panels shows a scatter plot of matrices (SPLOM), with bivariate scatter plots below the diagonal, histograms on the diagonal, and the Pearson correlation above the diagonal. Useful for descriptive statistics of small data sets. If lm=TRUE, linear regression fits are shown for both y by x and x by y. Correlation ellipses are also shown. Points may be given different colors depending upon some grouping variable. Robust fitting is done using lowess or loess regression. Confidence intervals of either the lm or loess are drawn if requested.

Correlation between *Age* and *Height*

- Pearson correlation between Age and Height = 0.84

```
cor(dataLungCap$Age, dataLungCap$Height)
```

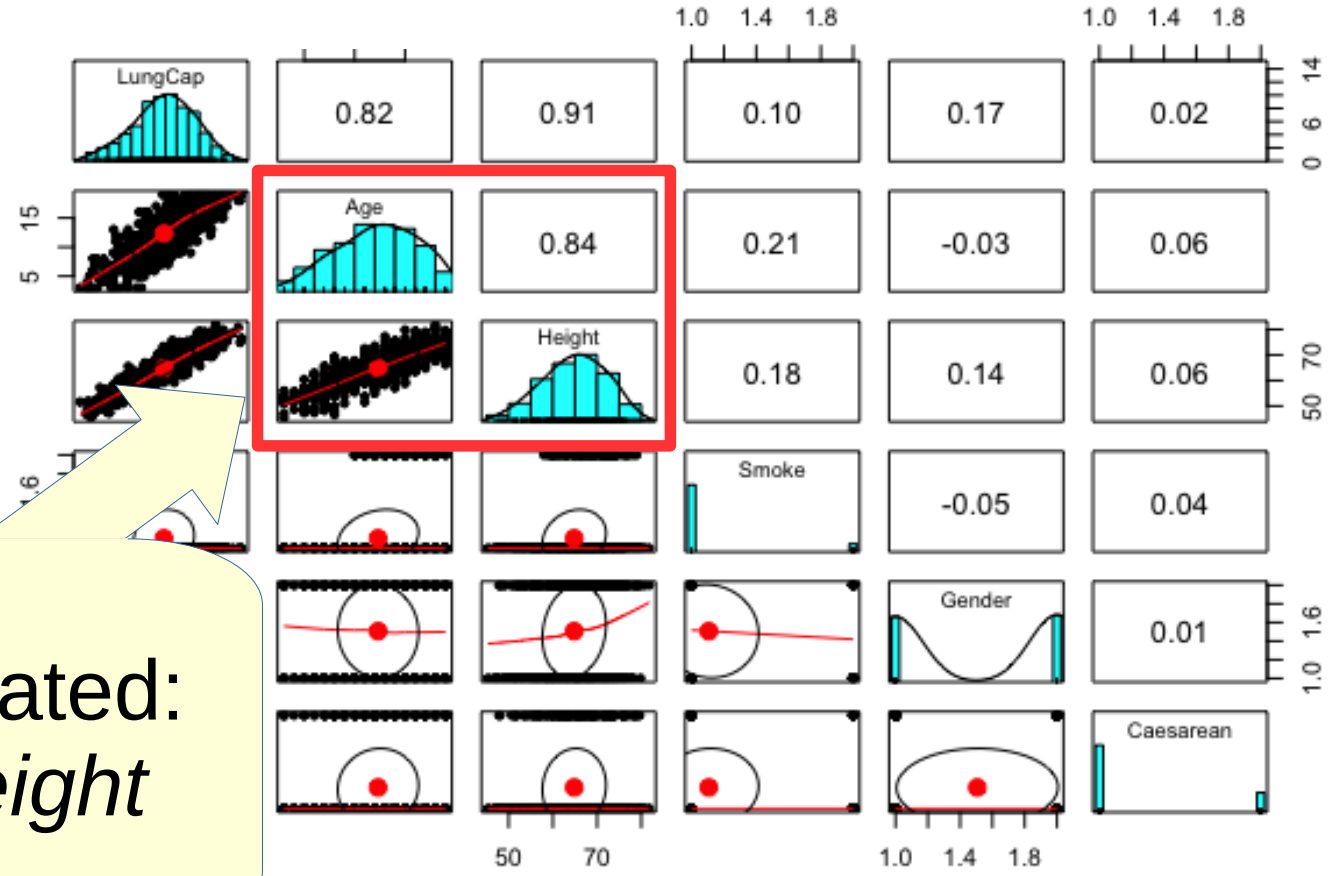
- The high correlation between *Age* and *Height* suggests that these two effects are related.



Correlation between *Age* and *Height*

- Pearson correlation between Age and Height = 0.84

```
> cor(Age, Height, method = "pearson")
[1] 0.8357368
```



Highly correlated: *age* and *height*

Correlation and Confidence

```
# Pearson correlation test
cor(dataLungCap$Age, dataLungCap$Height)
# output: 0.8357368
# Examine the 95 percent confidence level
confint(mod, conf.level = 0.95)
```

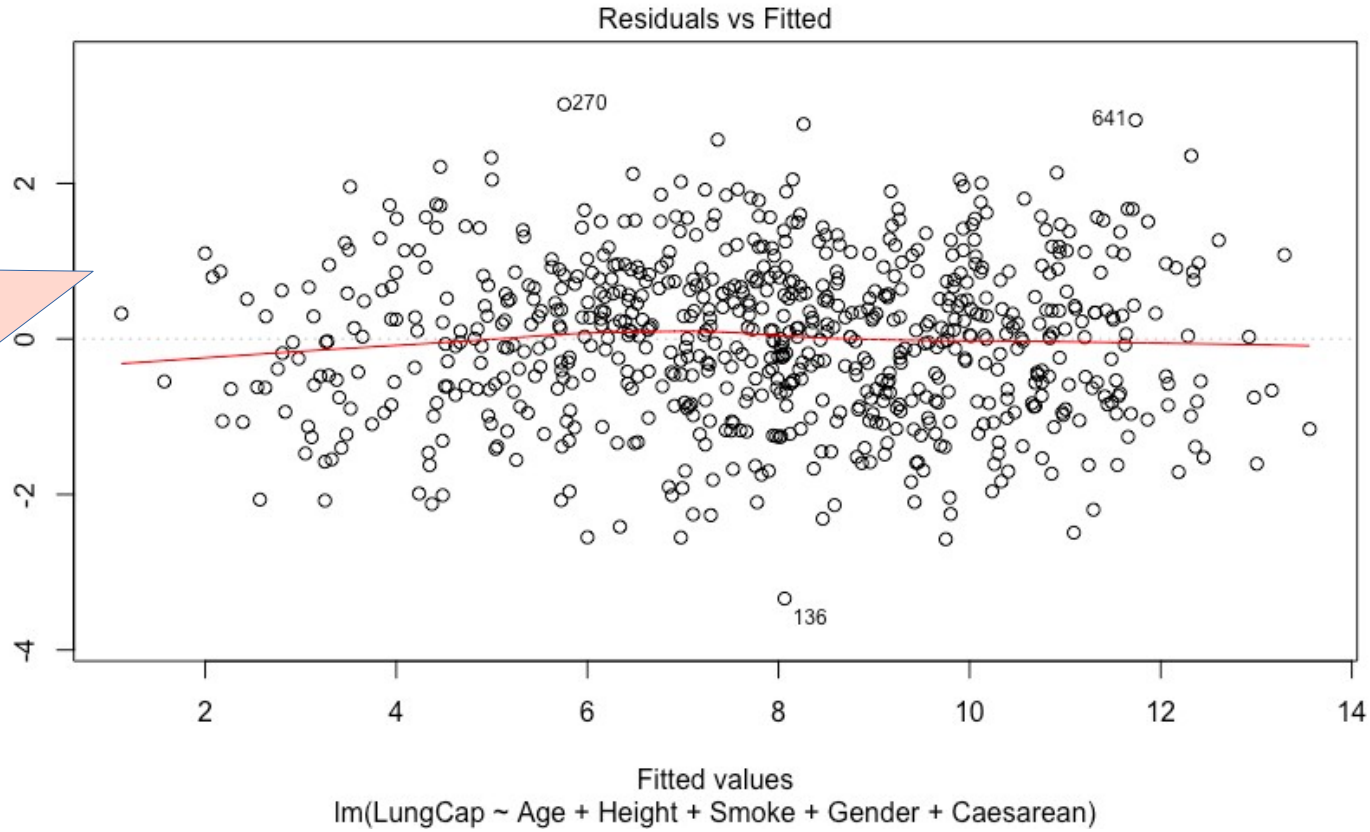
The **estimated slope** for Age is 0.126 and we are 95 percent sure that the **true slope** of *Age* is between 0.09 and 0.16.

```
> confint(mod, conf.level = 0.95)
                    2.5 %      97.5 %
(Intercept) -12.68333877 -10.8107918
Age           0.09132215   0.1614142
Height        0.25894454   0.2979192
```

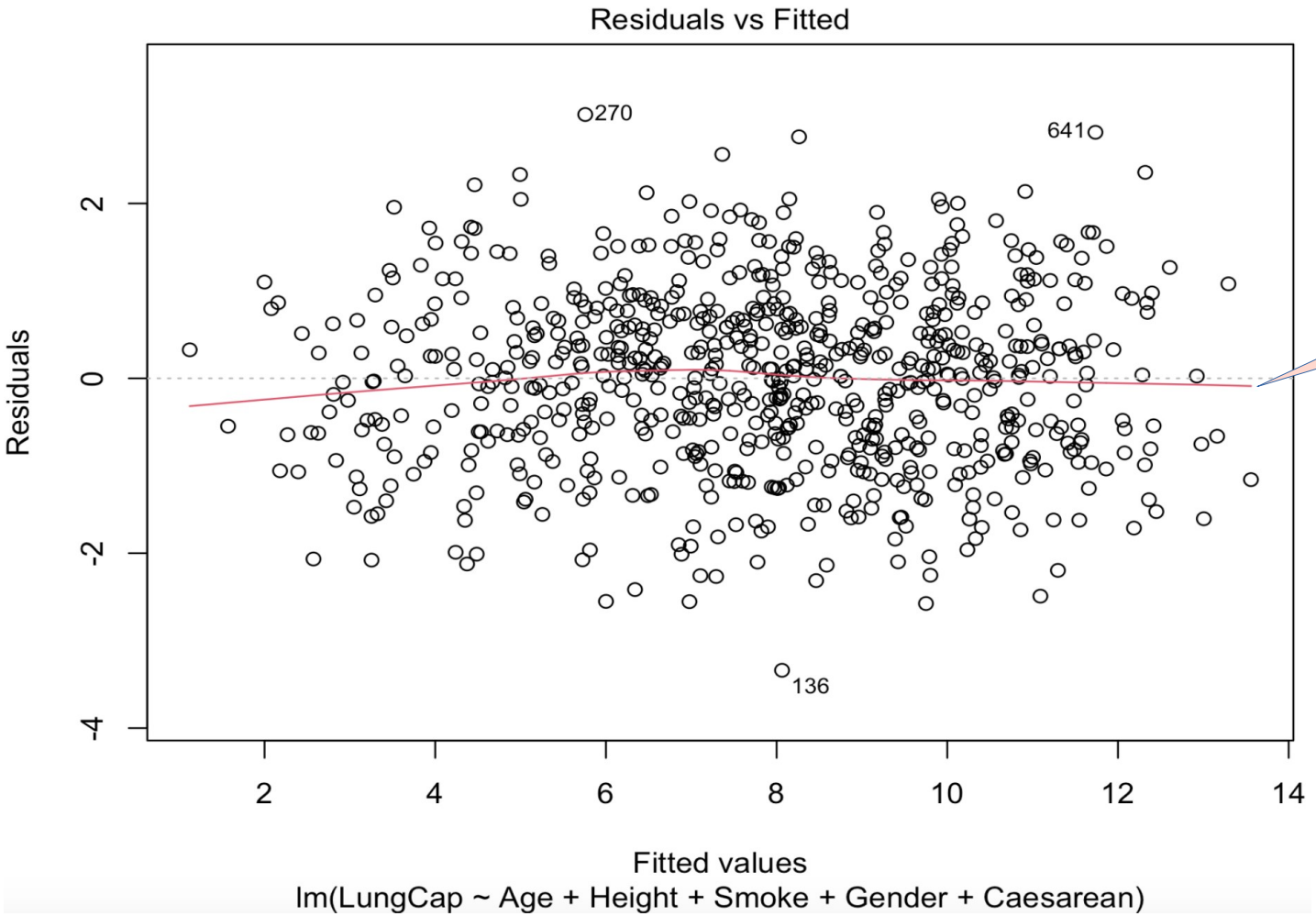
Create Bigger Model!!

- `mod2 <- lm(data = dataLungCap, LungCap ~ Age + Height + Smoke + Gender + Caesarean)`
- `summary(mod2)`
- `plot(mod2) # check the four plots!`

Residuals Vs. Fitted: The relationship between *Age*, *Height* and *Lung Capacity* is approx linear.



Residuals Vs. Fitted

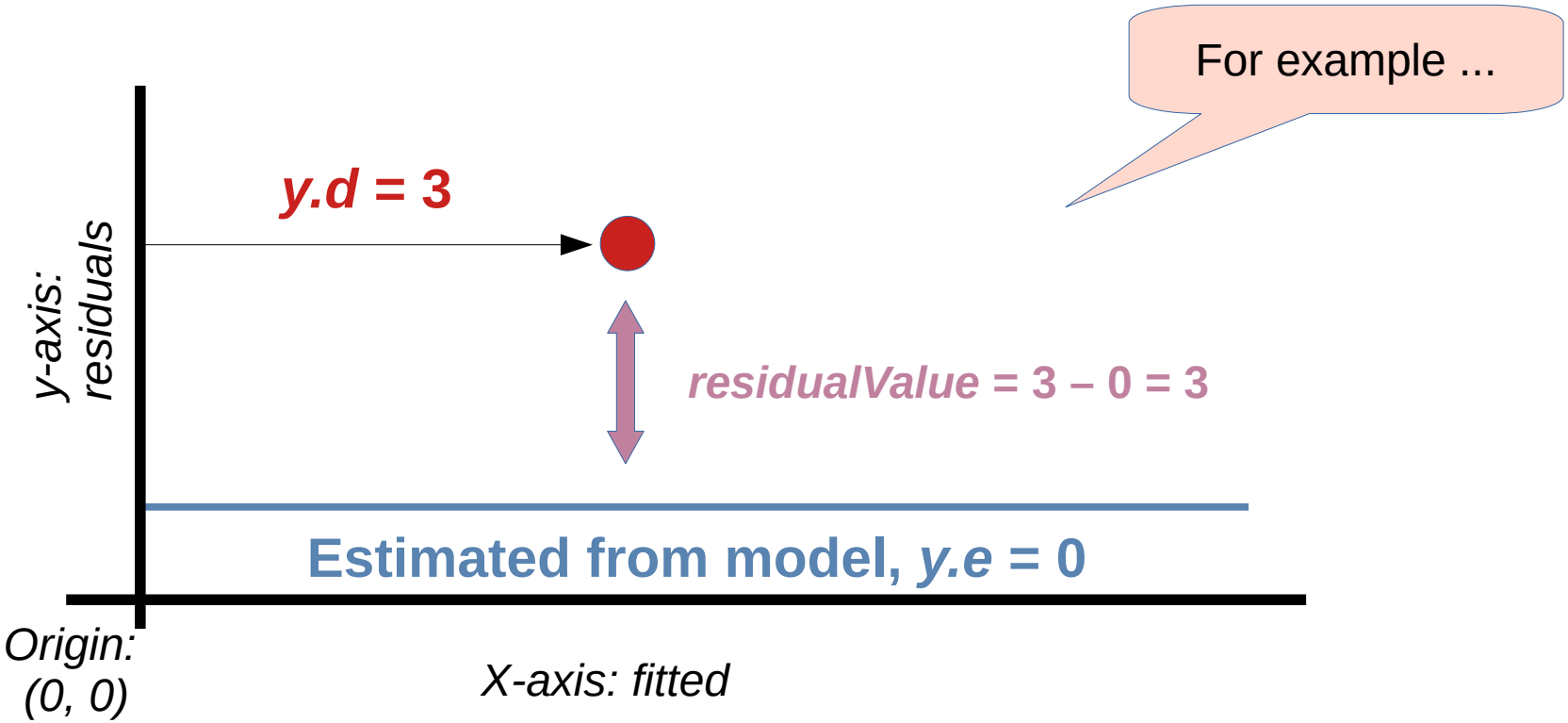


Expected

- Points are vertical distances between *y.d* (i.e.,datapoint) and *y.e* (i.e.,estimated)

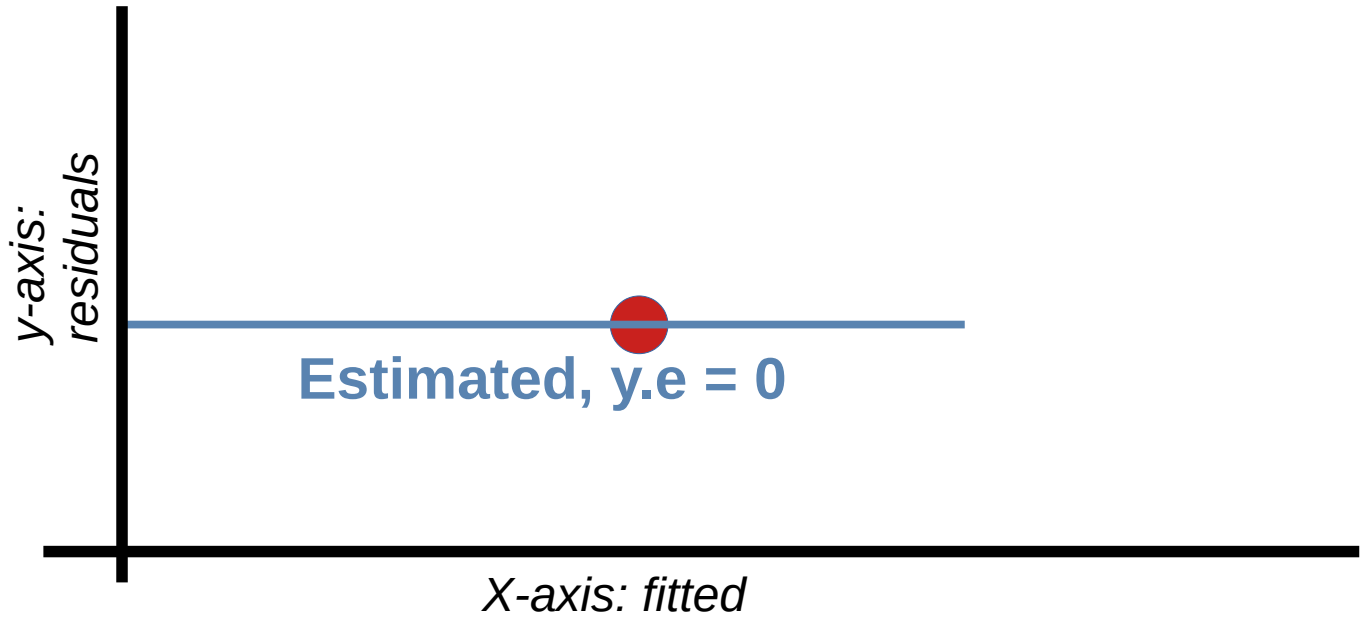
Residuals Vs. Fitted

- **Residual value**: a vertical distance between any one data point *y.d* (i.e.,datapoint) and the estimated value *y.e* (i.e.,estimated)
- *residualValue* = *y.d* – *y.e*

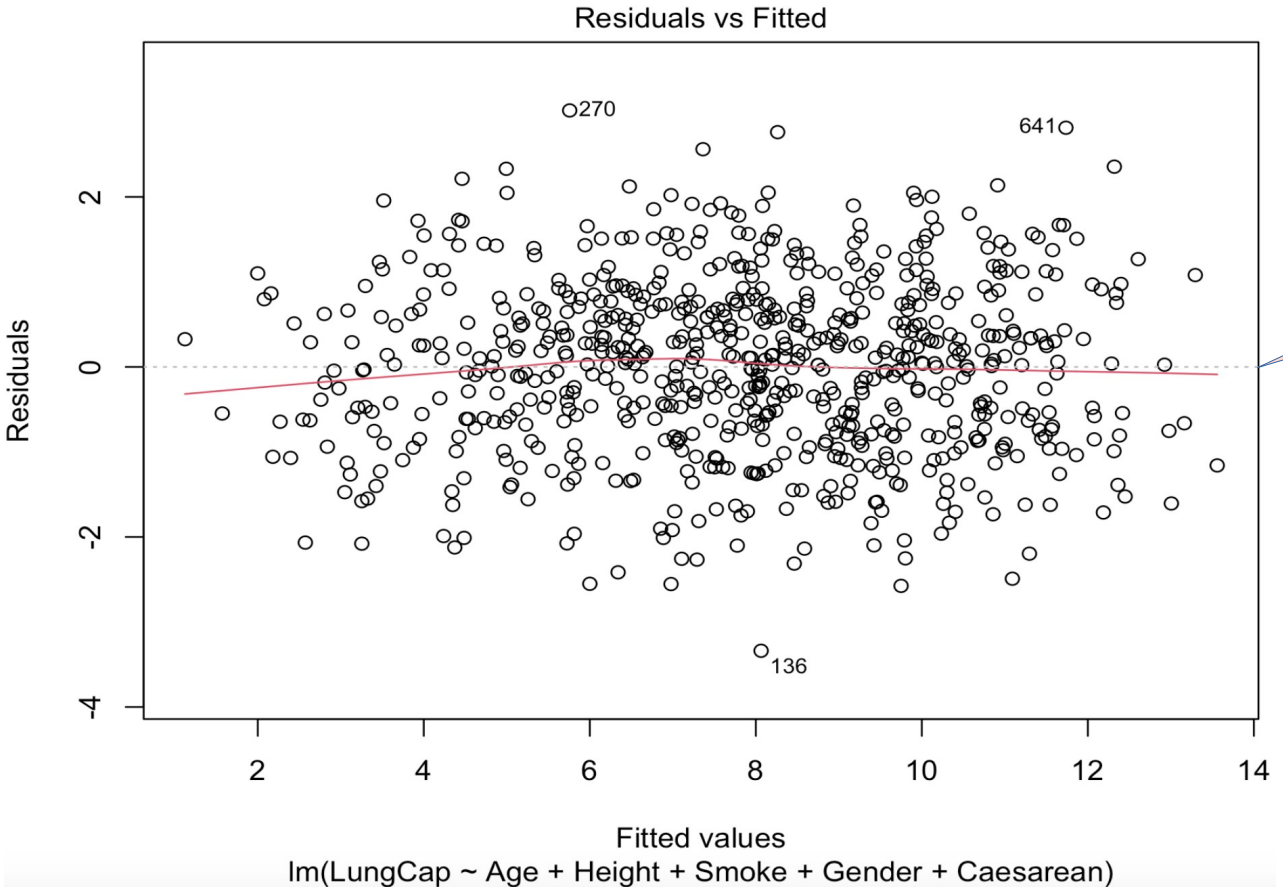


Residuals Vs. Fitted

- Any data point found (directly) on the estimated regression line has a residual of 0.
- The residual = 0 line corresponds to the estimated regression line.
- Suggestions for the *Appropriateness* of linear regression model



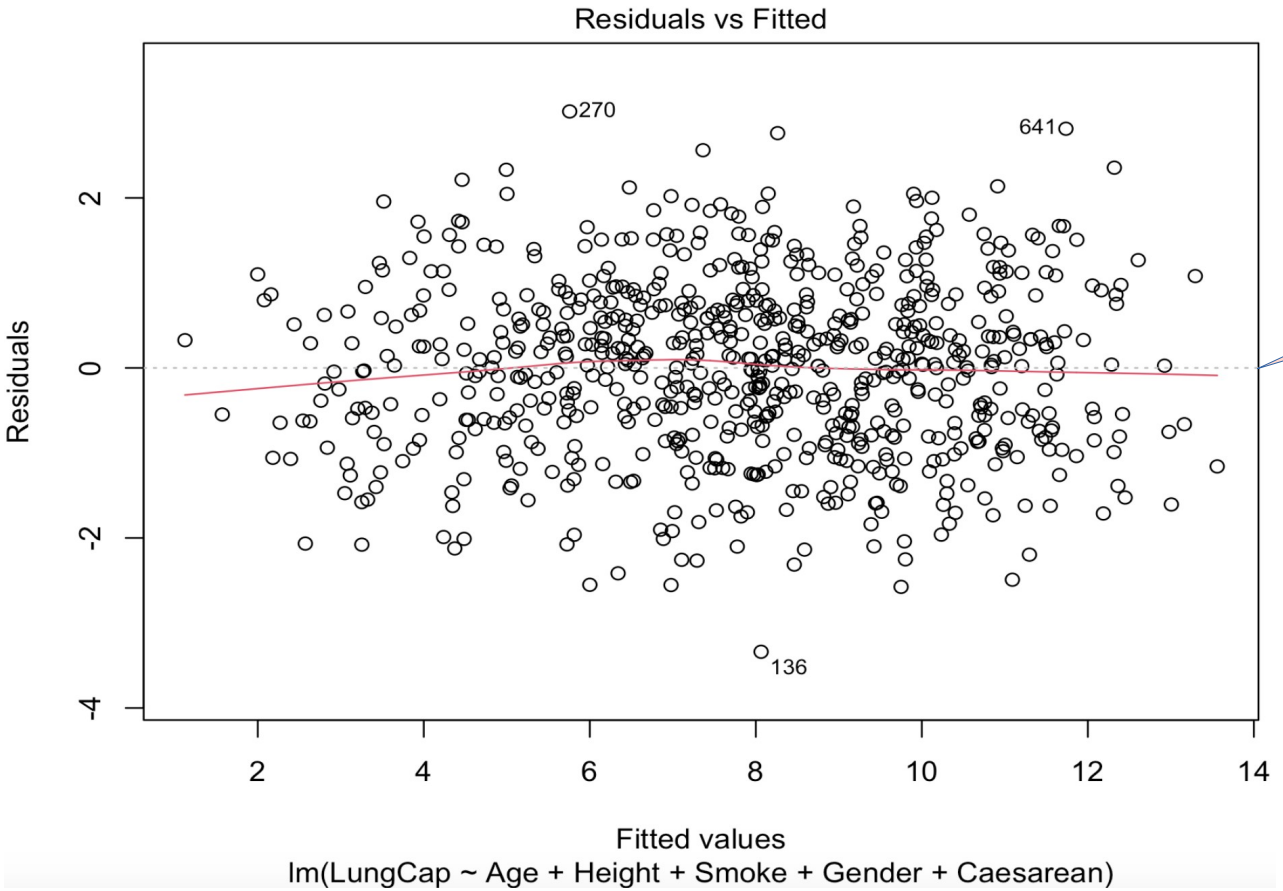
Suggestion: 1



Expected

- The residuals are generally (randomly) situated around the 0 line.
- This suggests that the assumption that the relationship is linear is reasonable.

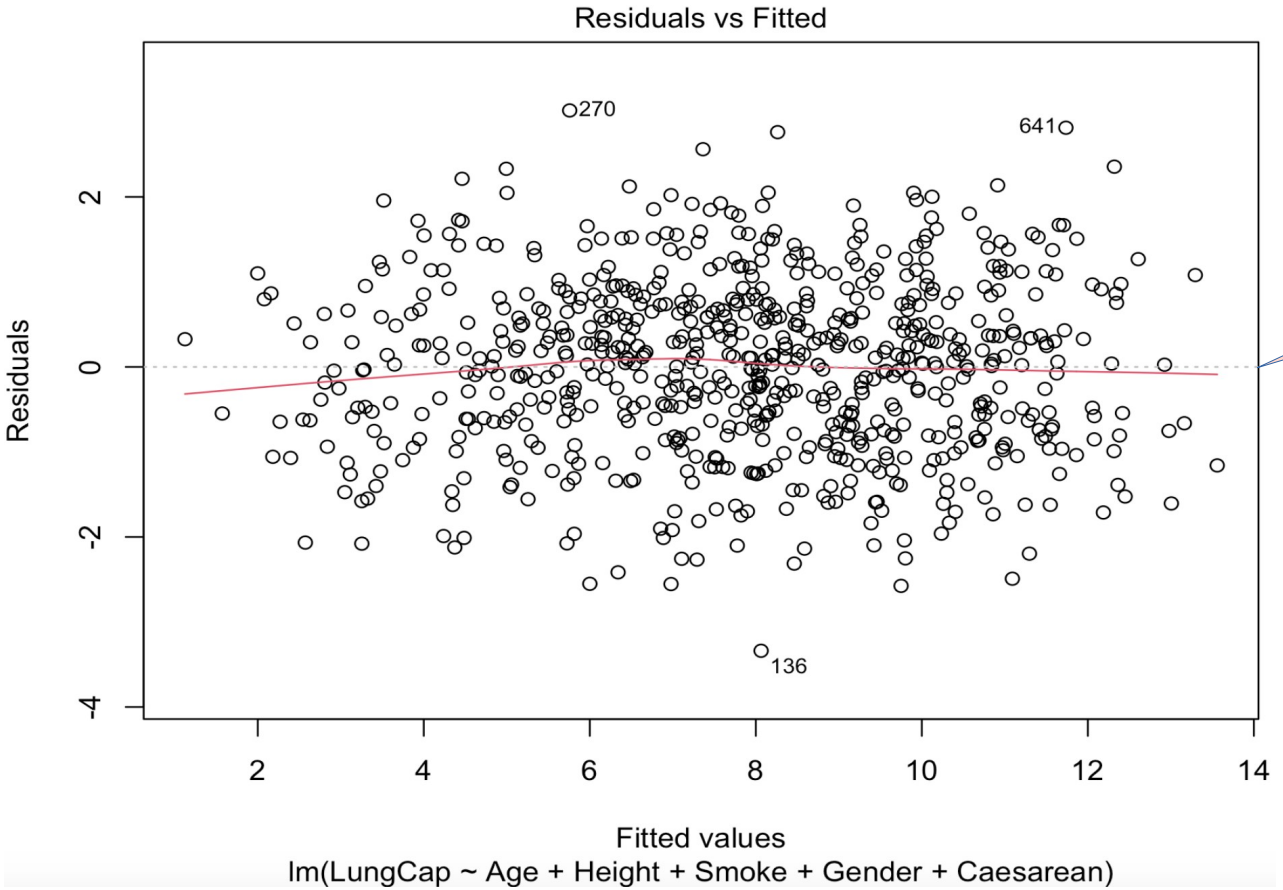
Suggestion: 2



Expected

- The residuals roughly form a "horizontal band" around the 0 line.
- This suggests that the variances of the error terms are equal.

Suggestion: 3



Expected

- Few residuals are "standing out" from the basic random pattern of residuals.
- This suggests that there are few outliers
- Is further study necessary to explain?

Go Create a Bigger Model!!

- Use this data set to make a bigger model.
- Fit a linear model using ALL *x* variables.

  Or try a different data set and do more model fitting!