Data Science
CS301
Modeling: Formal Basics

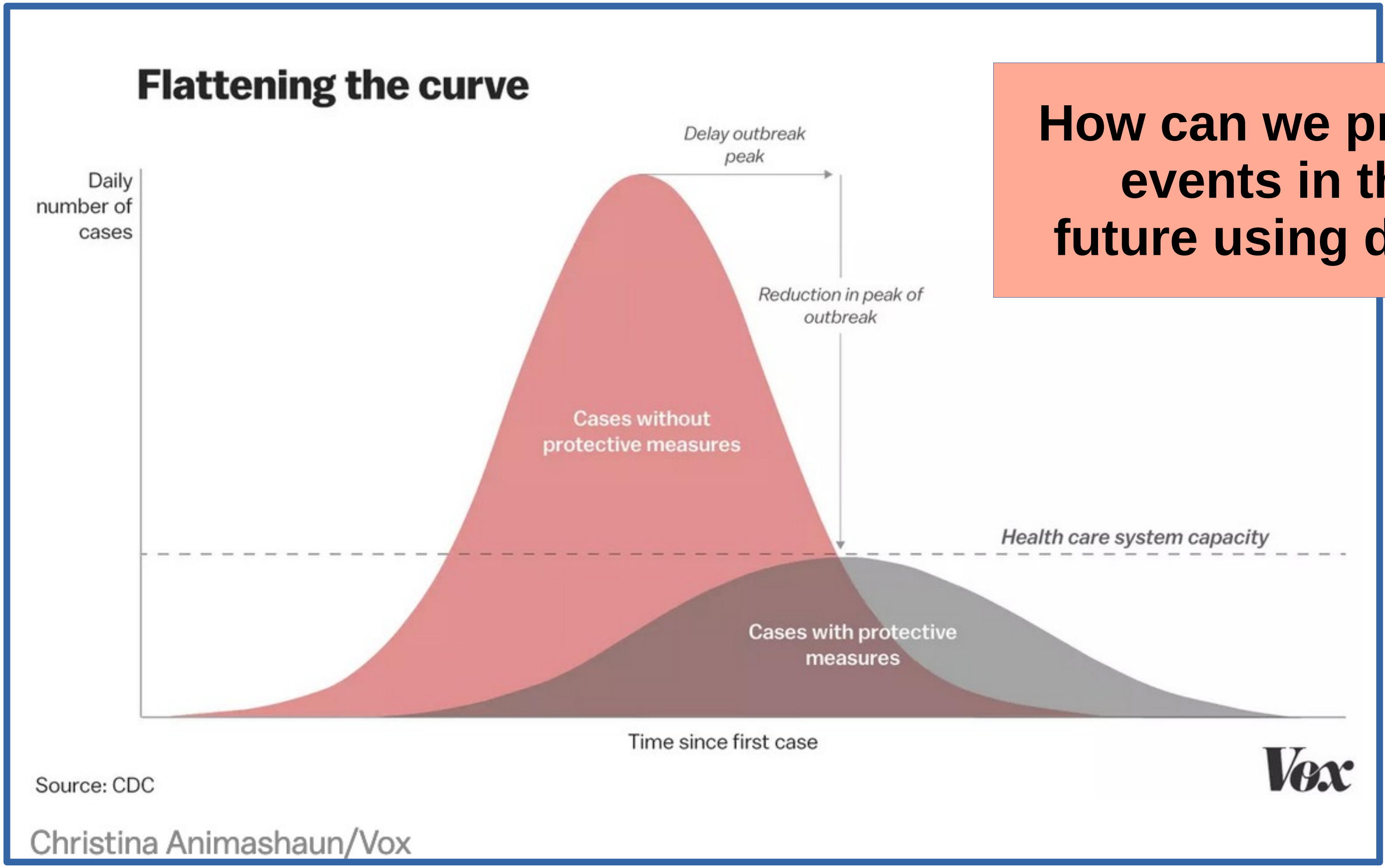
Week 08
Fall 2024
Oliver BONHAM-CARTER

Are you here today?!

https://forms.gle/iaY7zBmxj8KvsDMa8

Good Reads

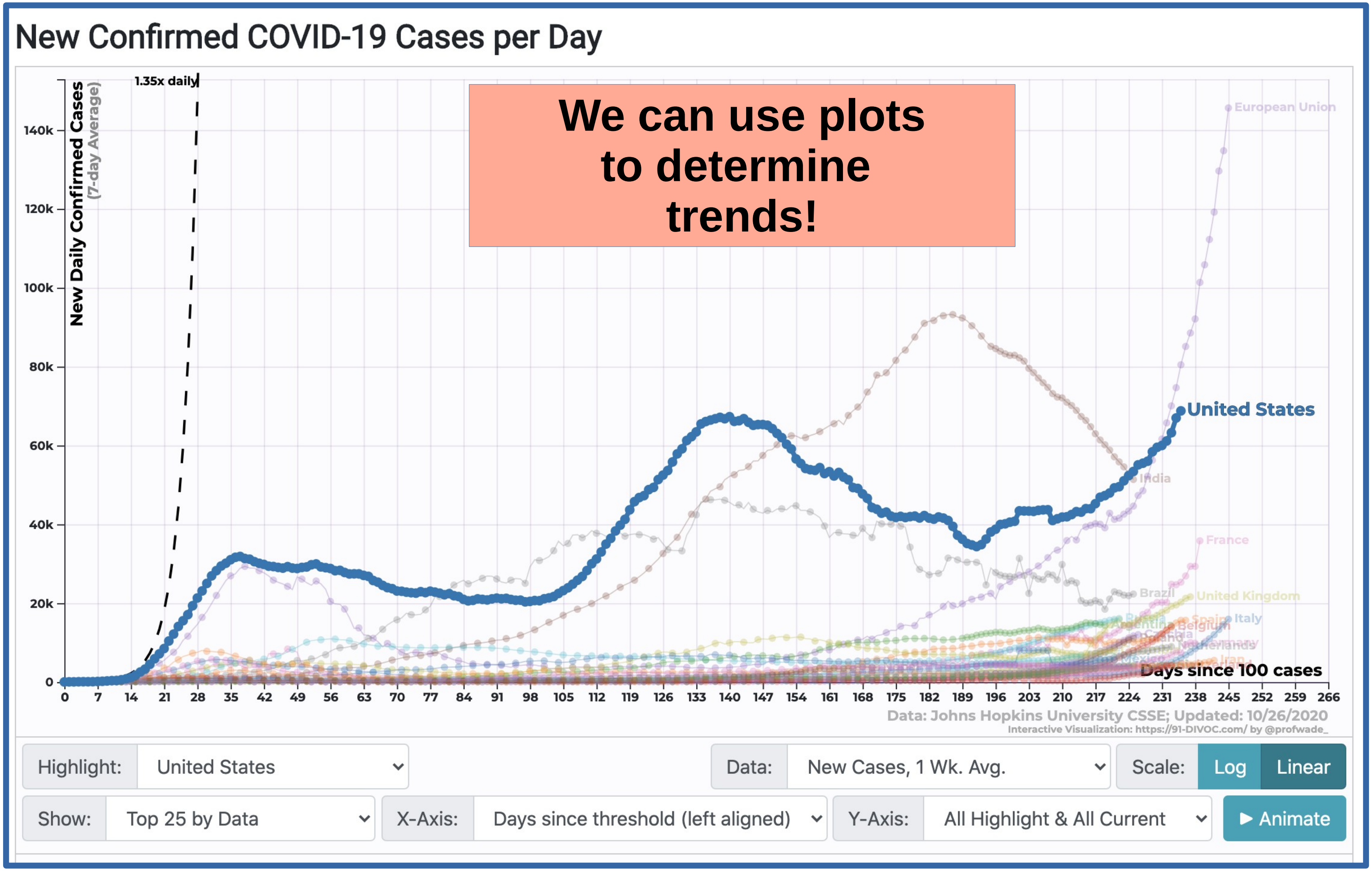
- How canceled events and self-quarantines save lives, in one chart



How can we predict events in the future using data?

https://www.vox.com/2020/3/10/21171481/coronavirus-us-cases-quarantine-cancellation

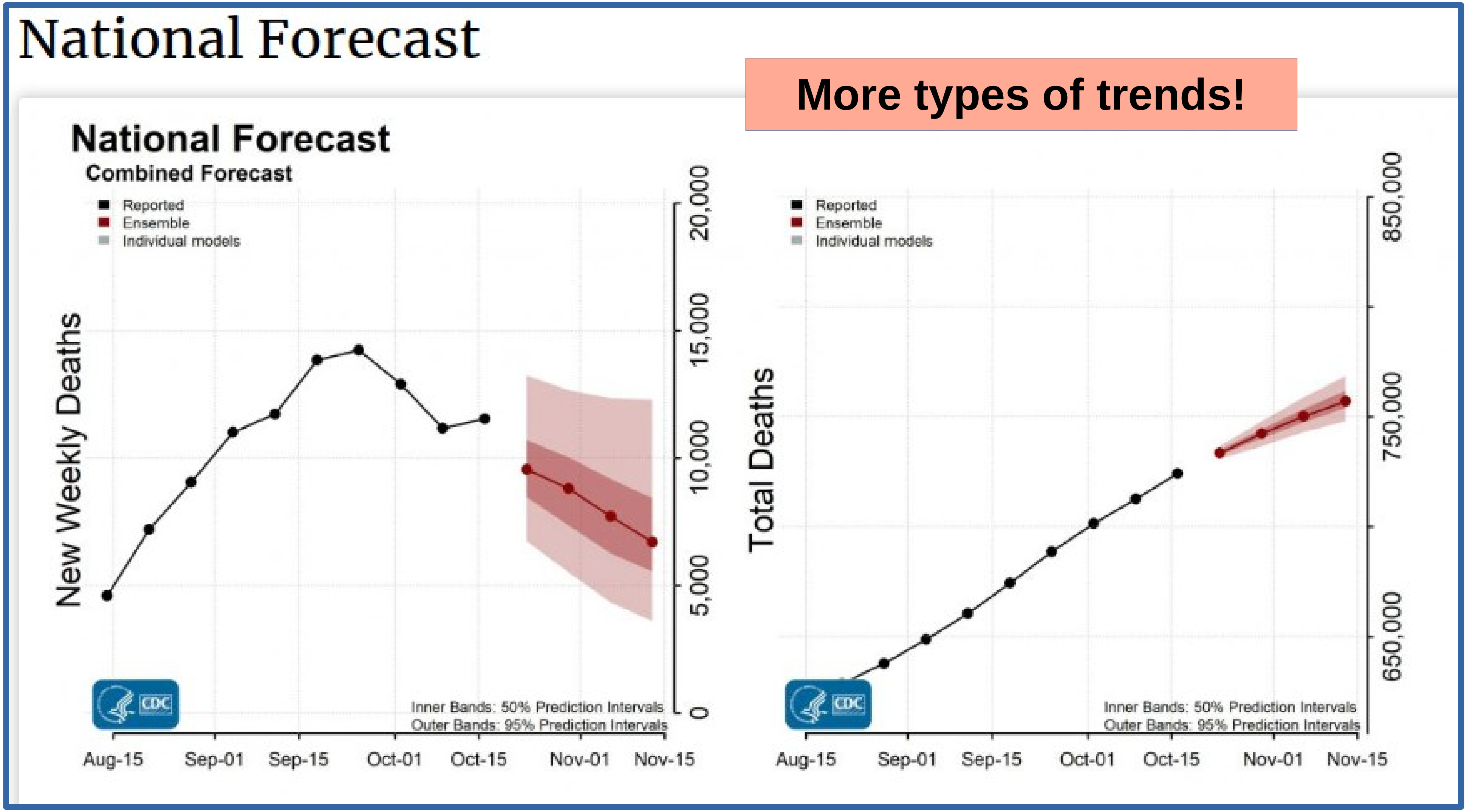
Interactive Plots: Covid-19 Cases



We can use plots to determine trends!

https://91-divoc.com/pages/covid-visualization/

Predicting With Plots



https://www.cdc.gov/covid/

The “Stuff” of Knowledge

- How do we use data to explain or argue something?
- How can we make types of predictions?

Modeling Basics

- What are models?
  - Data does not provide much insight unless something can be learned from it.
  - The ability to use data to extract meaning and extra value (the learning)
- Let’s talk about...
  - How to extract some meaning from your data
  - How to make predictions based on training by data

Types of Models (i)

- **Support Vector Machines**
  - Supervised learning models with associated learning algorithms that analyze data used for classification and regression analysis.
- **Generalized Linear Models**
  - Flexible generalization of ordinary linear regression that allows for response variables that have error distribution models other than a normal distribution
- **Generalized additive models**
  - Generalized linear model in which the linear predictor depends linearly on unknown smooth functions of some predictor variables, and interest focuses on inference about these smooth functions

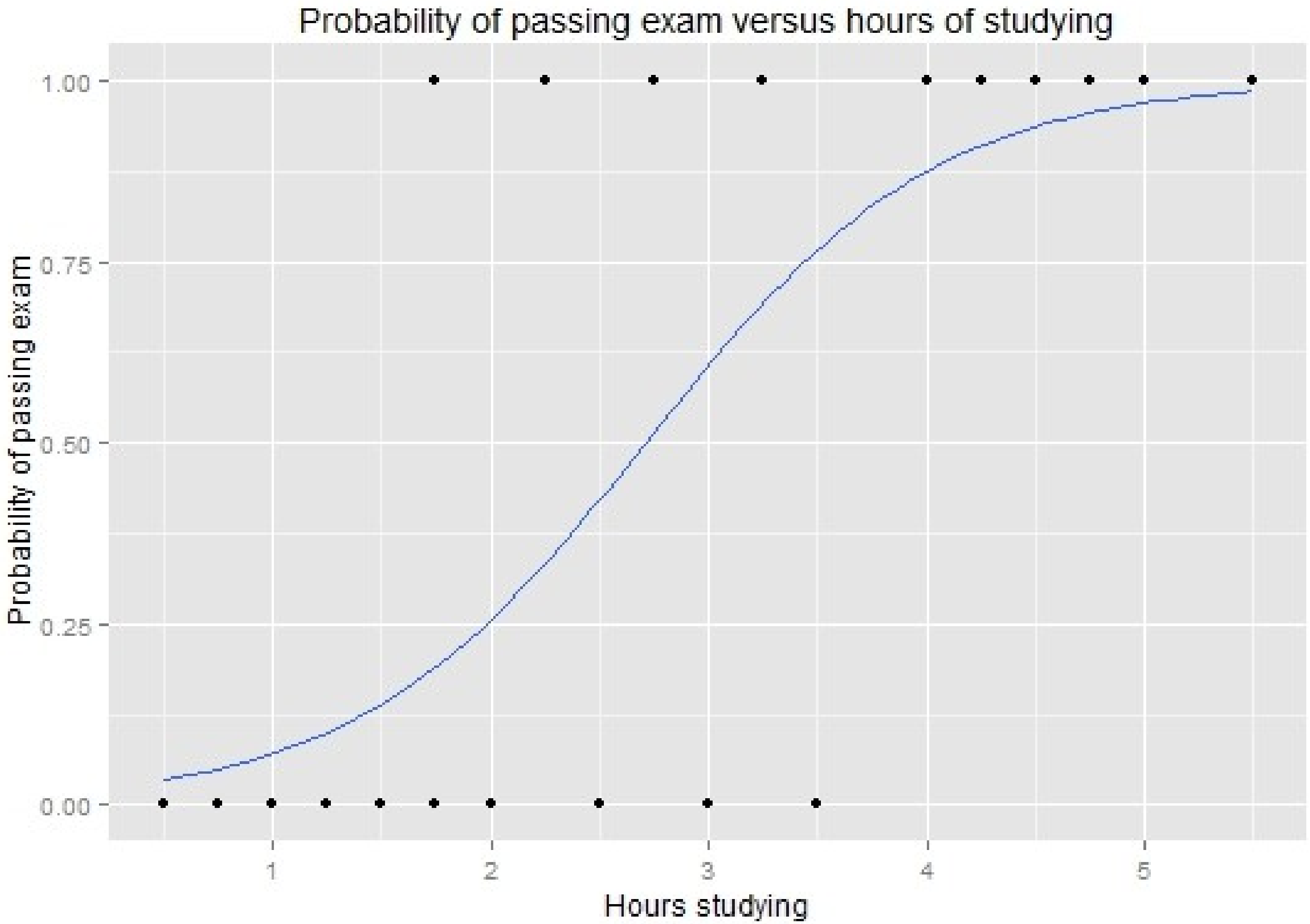
Types of Models (ii)

- **Linear Regression**
  - Linear approach for modeling the relationship between a scalar dependent variable y and one or more explanatory variables (or independent variables) denoted X
  - *(we have sort-of begun this study already: lines in scatter plots)*
- **LOESS Regression**
  - Combining much of the simplicity of linear least squares regression, but building with the flexibility of nonlinear regression.

Types of Models (iii)

Logistic Regression

- Models where the dependent variable is categorical (i.e., 0's or 1's as factors)



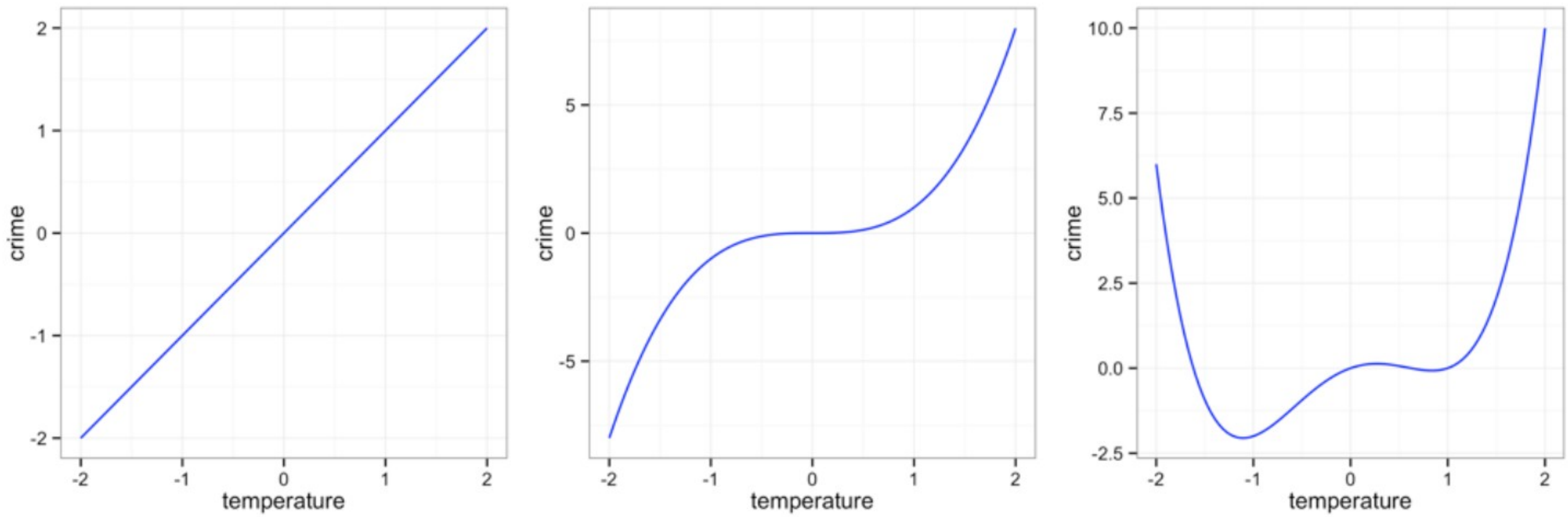
How Do We Answer Complex Questions?

- **Modeling**: We employ a computational framework based-on historical data trends.
- **Prediction**: We play with the framework to see what happens when we apply changes a variable to see what happens ...

What if...

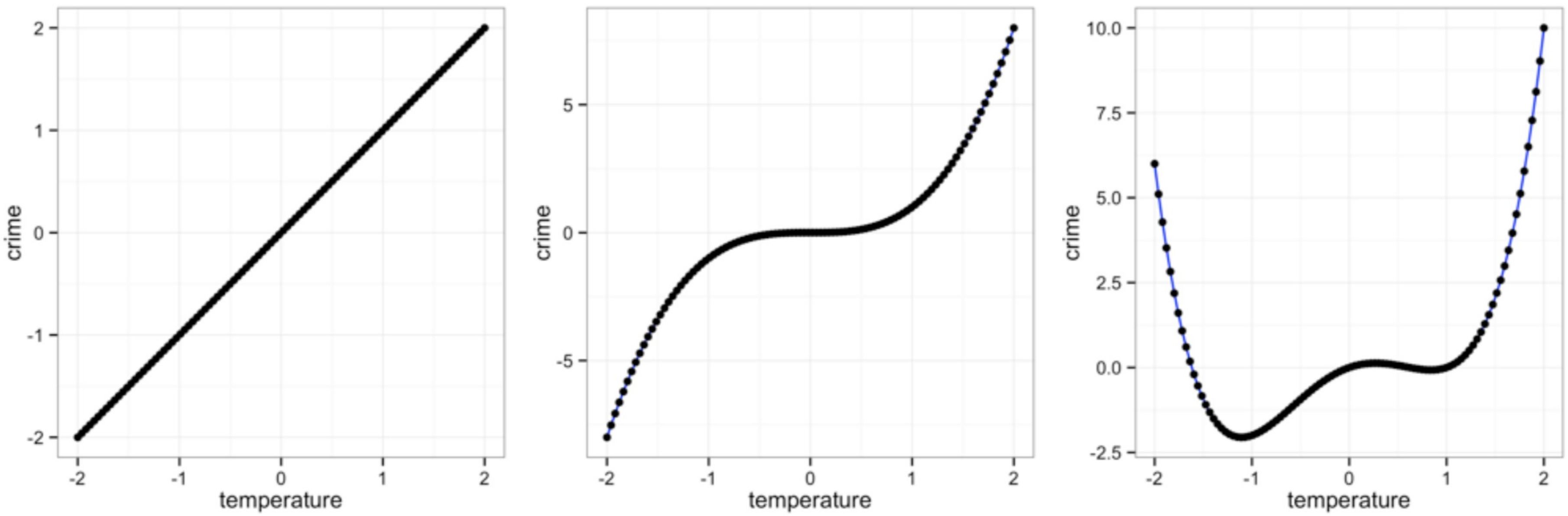
Functions:
the *stuff* behind the models

- Ideally, functions are mathematical descriptions of relationships



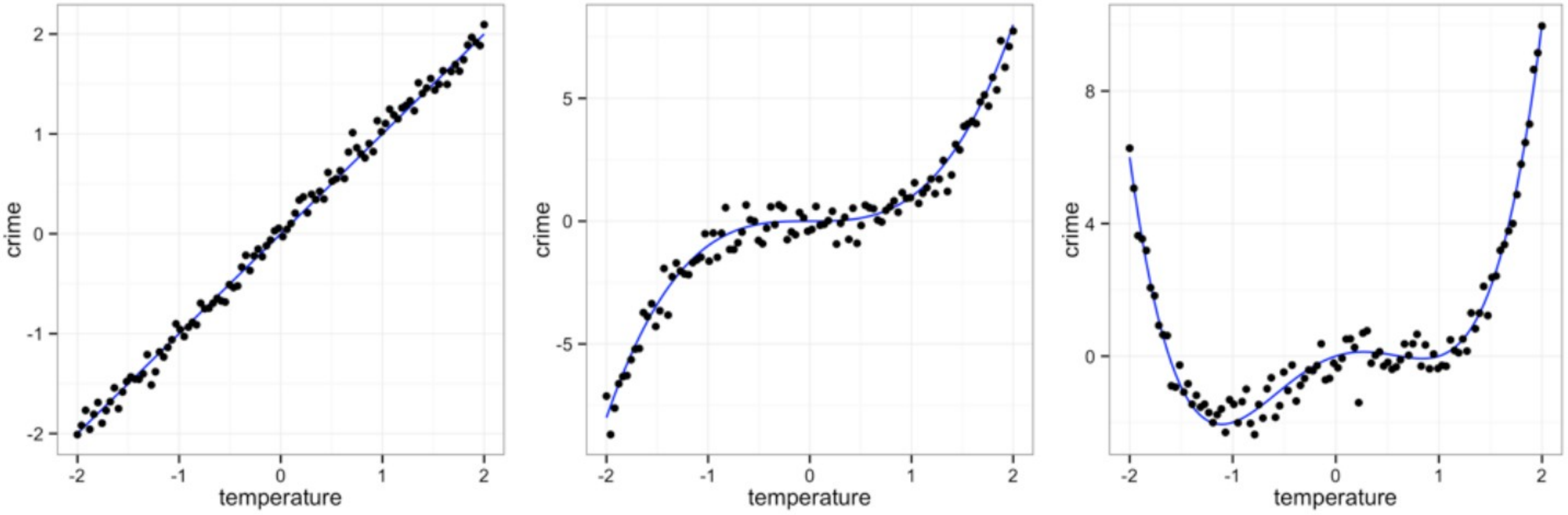
Functions: the *stuff* behind the models

- If one variable *completely* determines another, every (x, y) data point will fall on the function's line.



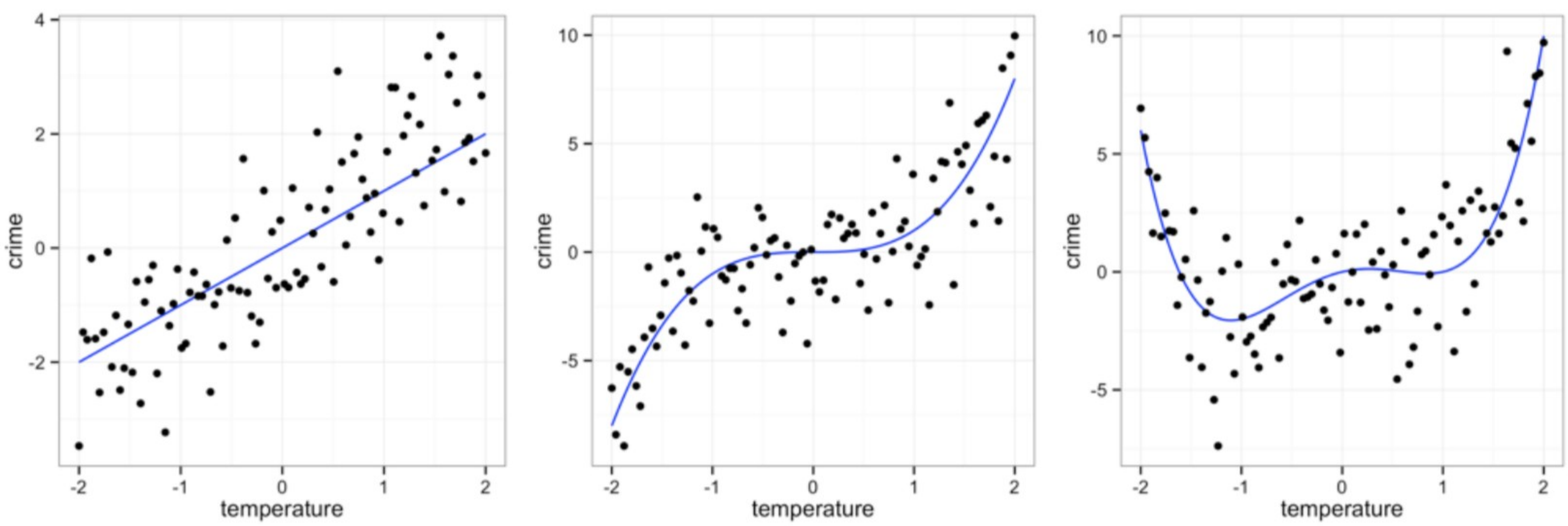
Relationships Between Variables are Messy

- This is what real data looks like on a good day!



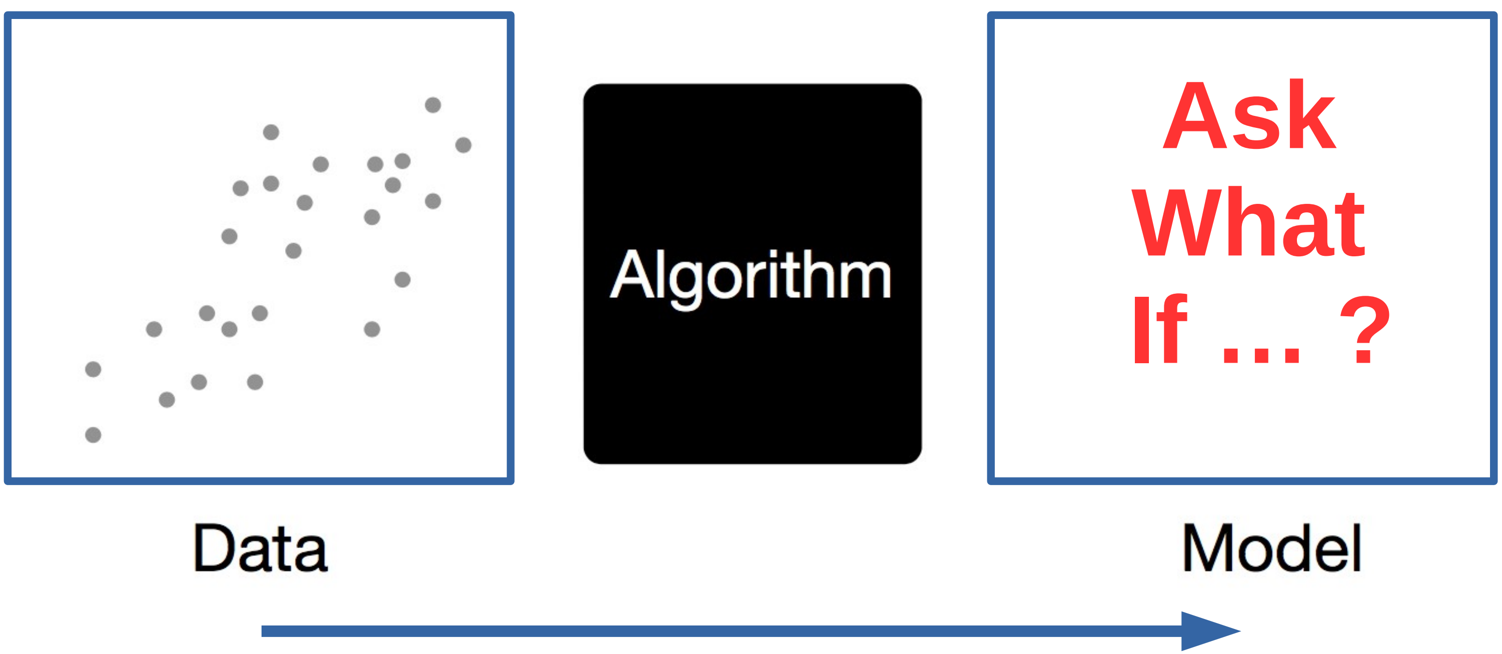
Relationships Between Variables

- If the actual relationship is affected by other variables, data points may not fall directly on the function-line.
- **Noise**: The greater the effect of other variables, the weaker the relationship. This is normally the situation with real data.



So, A Model, Then?

- Noise is what we get in data when not every point does *what it is supposed to do.*
- Modeling *attempts* to *more*-correctly identify functional relationships in noisy data.

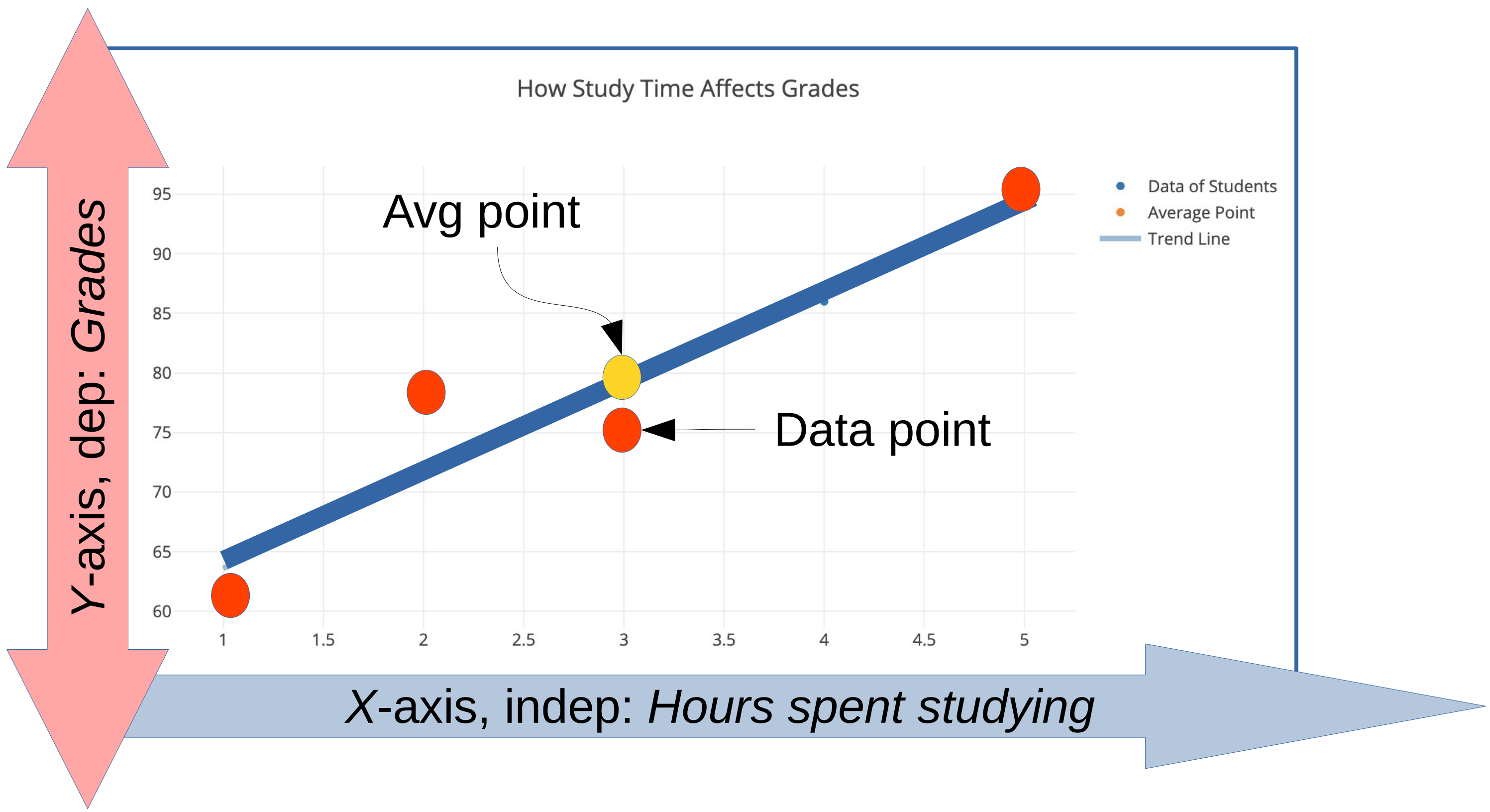


Let’s Talk Linear Models

- **Linear regression**: How much do/does my **independent variable(s)** influence my **dependent variables**?
- As one variable climbs, does the other also climb (decline) at some *predicable* rate?
- Can I impose some value into my model to determine a ***what-if*** type of question which is firmly based on my data?

Variables?

- **Independent variable:** a variable (often denoted by *x*) whose variation does not depend on that of another (i.e., time).
- **Dependent variables:** a variables (often denoted by *y*) that depends, by some law or rule (e.g., by a mathematical function), on the values of other variables (i.e., grades).
- Example: https://chart-studio.plotly.com/~bchapman27/73.embed



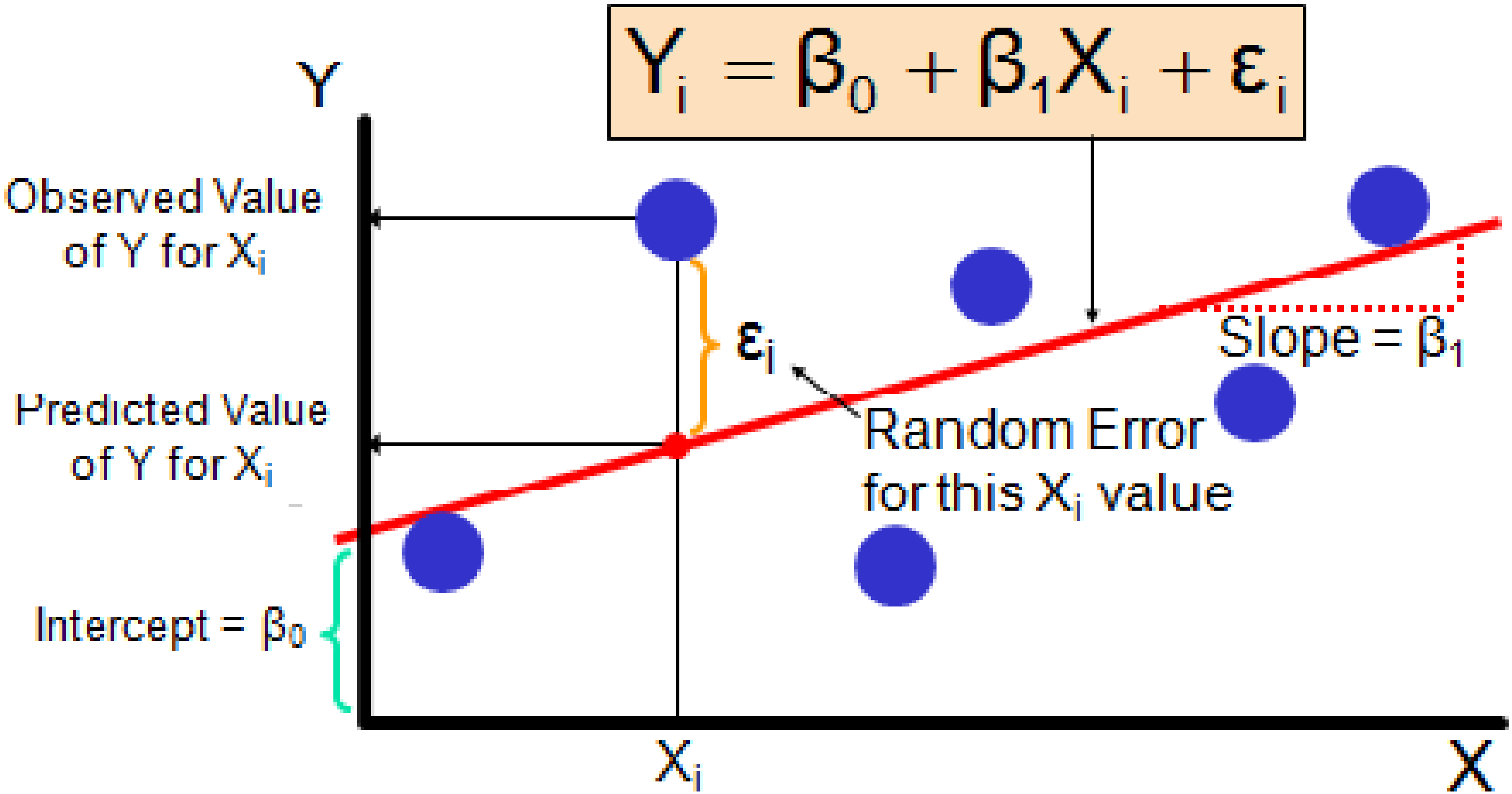
Let's Talk **Linear Models**

- Linear regression (formally) is a function that draws a line from data
- The function f(*x*) has the form:

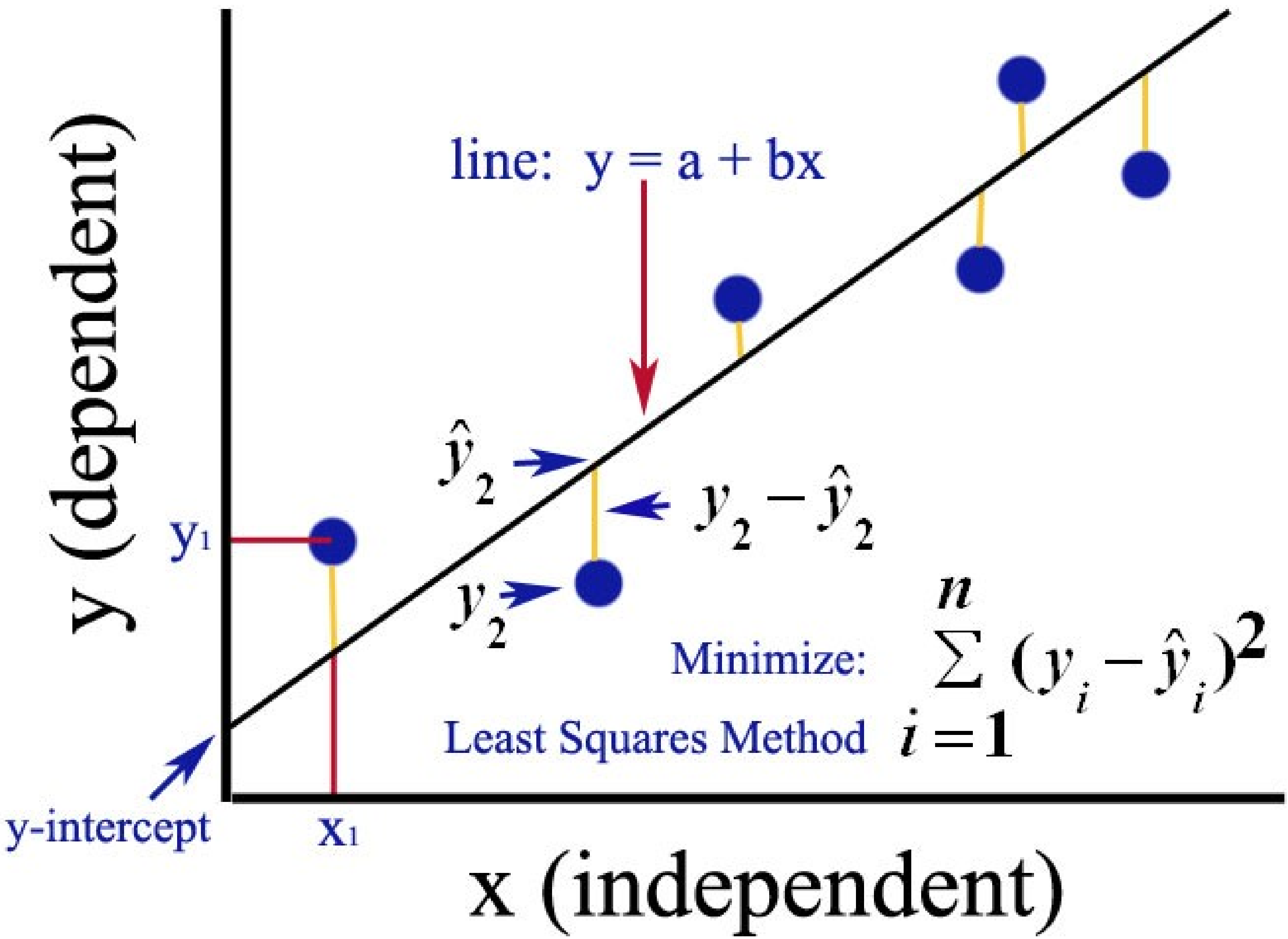
$$f(x) = \alpha + \beta x + \epsilon$$

  - alpha: intercept
  - Beta: a weighted slope
  - Epsilon: account for the error
- *Note: This f(x)* will be a straight line for *x*

Just a Formula For a Line!

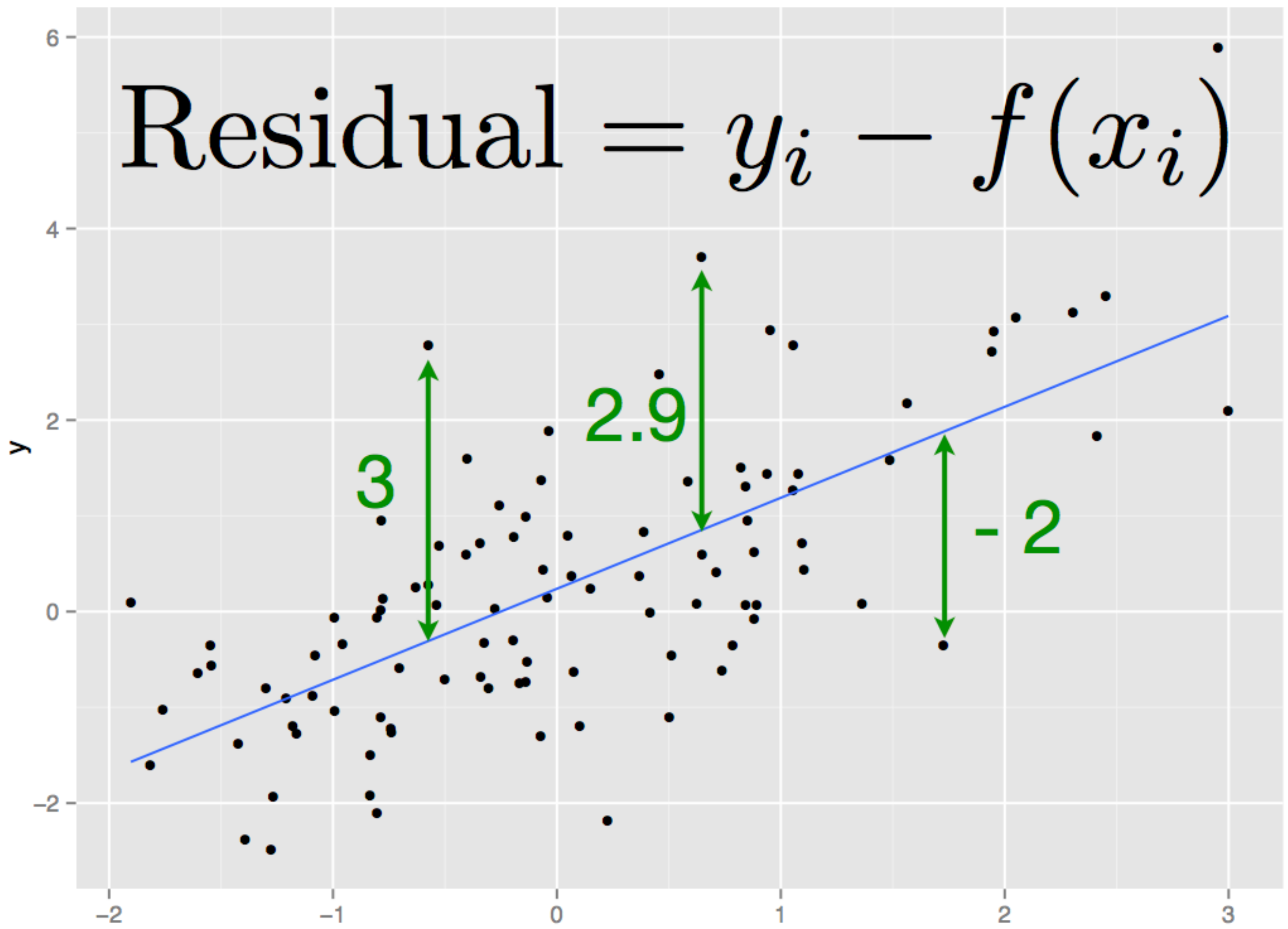


Another Linear Model



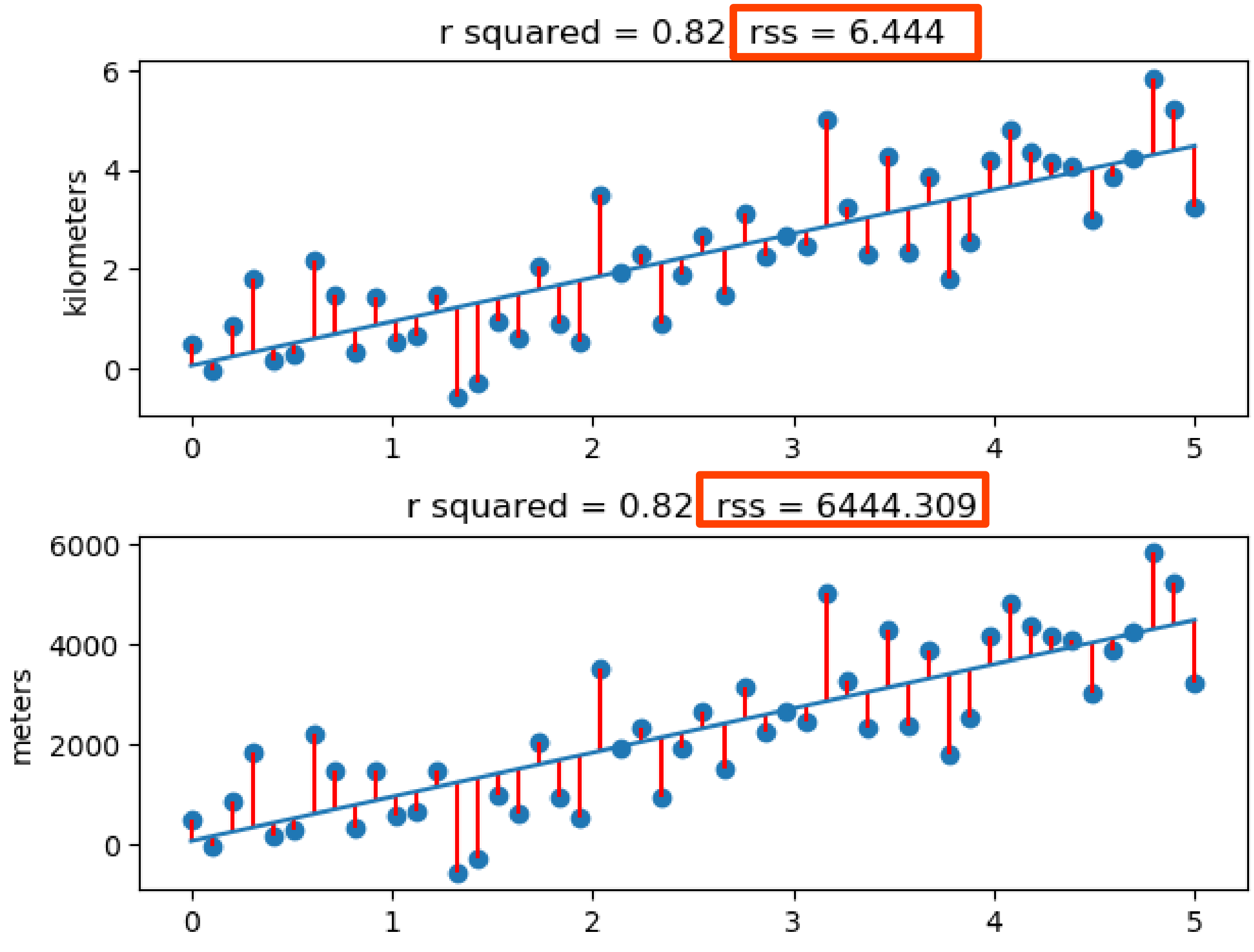
Residuals

- A residual is the difference between the actual value and the value predicted by the model (y-ŷ) for any given point.
- A least-squares regression model minimizes the sum of the squared residuals



A Difference Between the Expected and the Actual Data

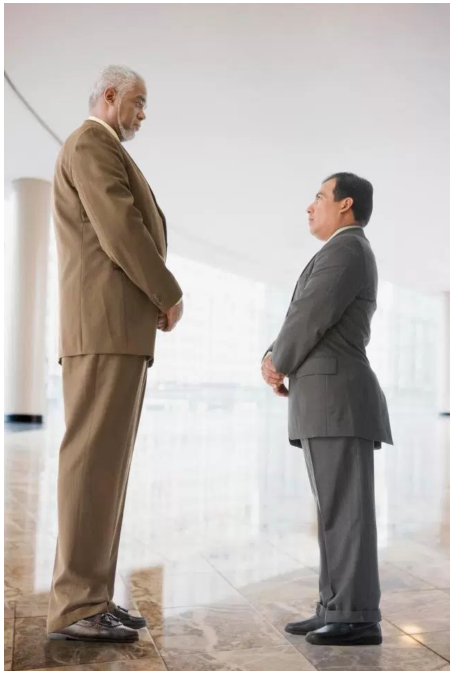
- The *Residual Sum of Squares* is a measure of the discrepancy between the data and an estimation model.
- A **small RSS** value indicates a **tight fit** of the model to the data; better prediction



Types of Questions to Address With Data

Q1: Is crime influenced by yearly temperature?

File: crime.csv



Q2: What influence is there on earning potential and personal height?

File: wages.csv

Crime Data Set

- Is there a relationship between *crime* and *temperature*? State statistics from 2009.

```
rm(list = ls()) # remove old vars in memory
library(tidyverse)
# open the crime dataset from the data.
c <- file.choose() # set the filename
crime <- read.csv(c) # load and read the data.
```

Crime Data Set

```
View(crime) #or
tibble::as_tibble(crime) # dataframe
```

```
          state  abbr    low  murder  tc2009
          <chr> <chr>  <int>   <dbl>   <dbl>
1       Alabama    AL    -27     7.1  4337.5
2        Alaska    AK    -80     3.2  3567.1
3       Arizona    AZ    -40     5.5  3725.2
4      Arkansas    AR    -29     6.3  4415.4
5    California    CA    -45     5.4  3201.6
6      Colorado    CO    -61     3.2  3024.5
7   Connecticut    CT    -32     3.0  2646.3
8      Delaware    DE    -17     4.6  3996.8
9       Florida    FL     -2     5.5  4453.7
10      Georgia    GA    -17     6.0  4180.6
...
```

Yearly low temp

Murder rate

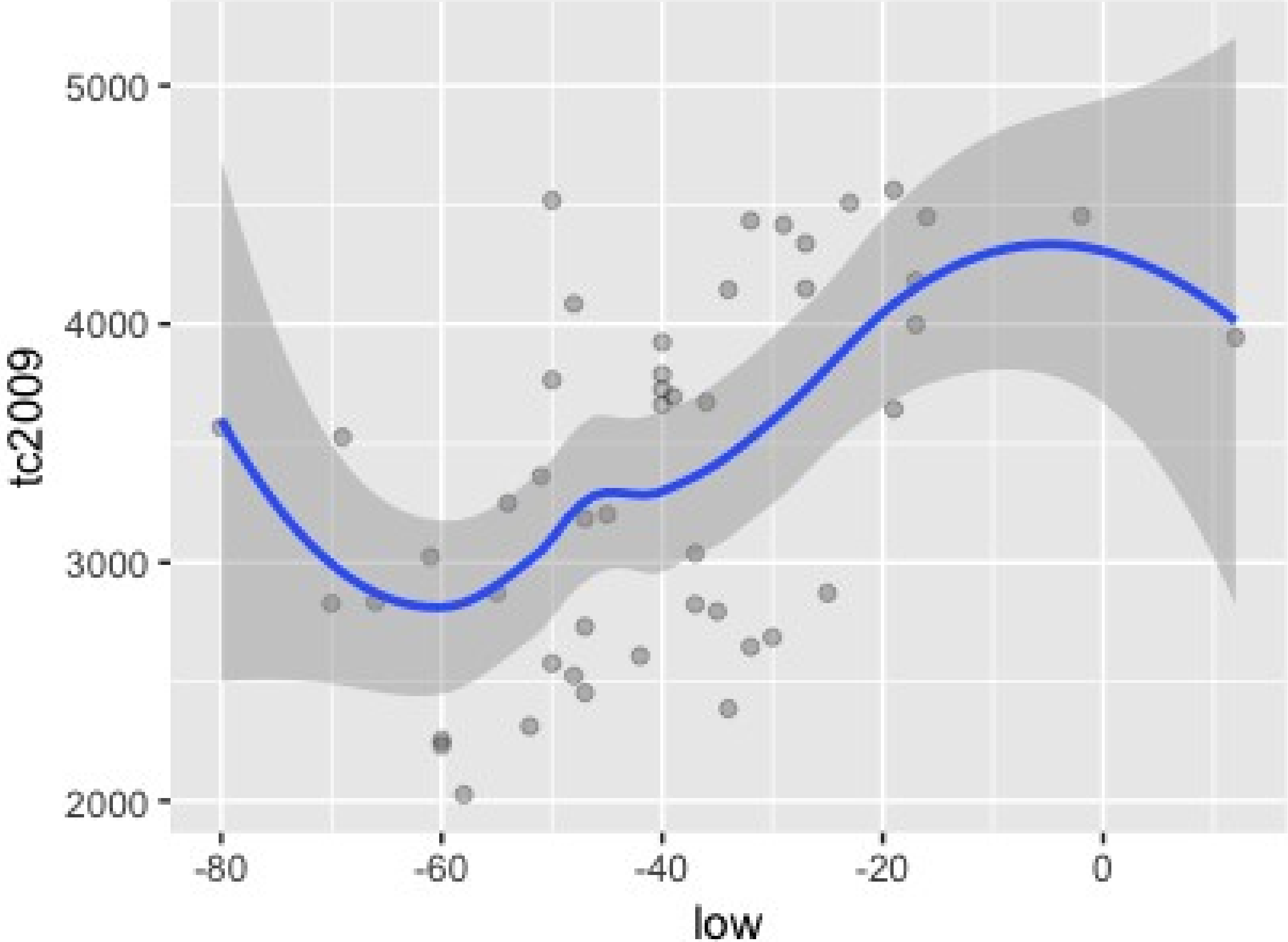
Training data

Exploratory Plots

```
#plot with general trend line

crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth()


#plot with linear model line

crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) +
geom_smooth(method = lm)
```

No Model: Just General Trends

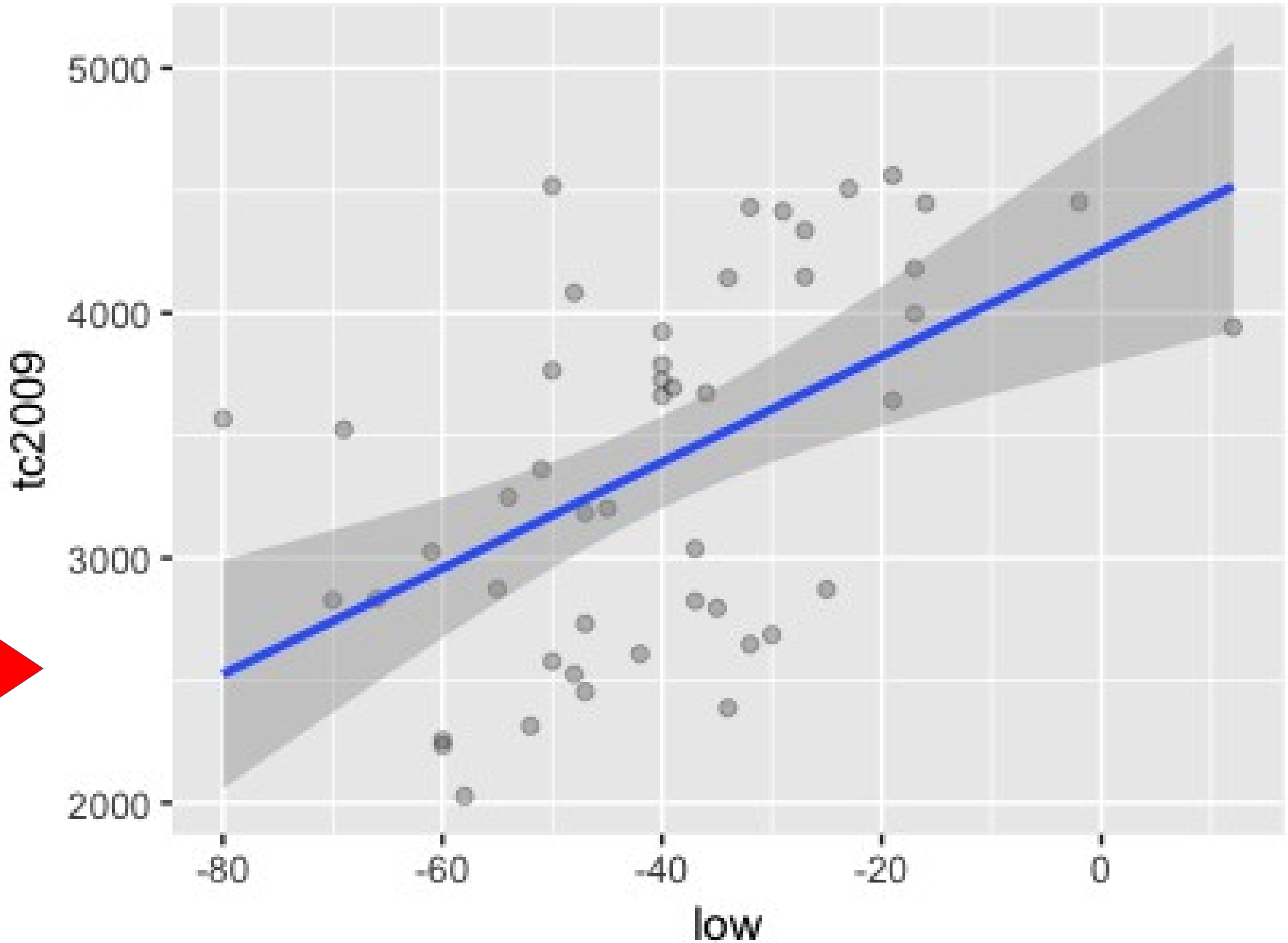
```
crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth()
```



This is the data's general pattern

Linear Model: Predictions

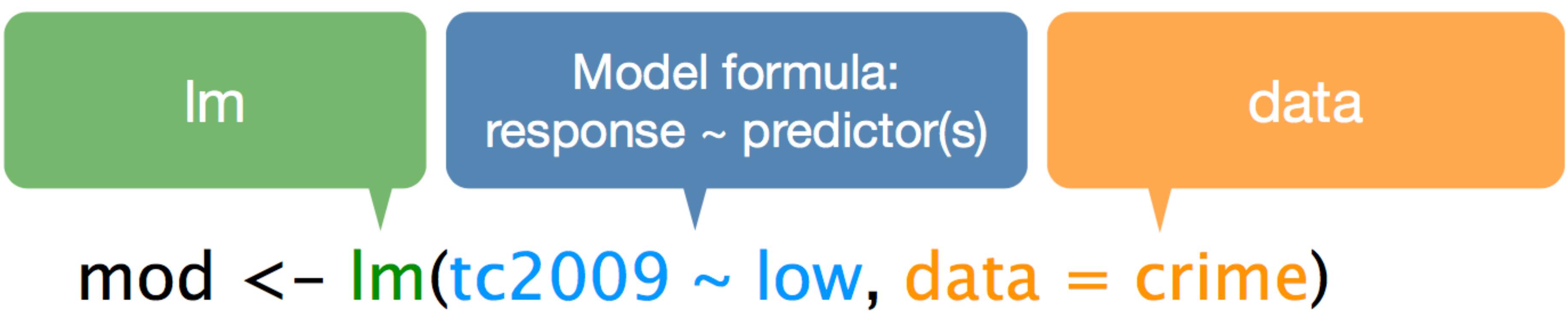
```
crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth(method = lm)
```



The linear Model: used for predictions

The Linear Model

- How much does ***low*** *(indep Var)* influence ***tc2009*** *(dep Var)*
- Linear model syntax



Syntax

- Formulas only need to include the response and predictor variables

$$y = f(x) = \alpha + \beta x + \epsilon$$

Syntax to Build the linear model:

$$y \sim x$$

Models Use Formulas

- R formulas are expressions built with "~" (*tilda*)

tc2009 ~ low

Note: ***tc2009*** is **dependent** variable and ***low*** is **independent** variable

class(tc2009 ~ low)

gives: [1] "formula" meaning that is a line equation

- $tc2009 = f(low) = \boldsymbol{\beta} * low + \epsilon$

Different Names, Same Meanings

```
response ~ explanatory

dependent ~ independent

outcome ~ predictors
```

Build Your Model!

```
mod <- lm(tc2009 ~ low, data = crime)
```

Dependent ~ independent

```
Call:
lm(formula = tc2009 ~ low, data = crime)

Coefficients:
(Intercept)          low
    4256.86        21.65
```

Intercept and Coefficient

```
mod
```

```
> mod

Call:
lm(formula = tc2009 ~ low, data = crime)

Coefficients:
(Intercept)          low
    4256.86        21.65
```

Coef

- Shows the model’s coefficients (I.e., intercept, slopes)

slope

```
coef(mod)
coefficients(mod)
# (Intercept)          low
#  4256.86158     21.64725
```

α β

Interpreting Models

Linear models are very easy to interpret

$$y = \alpha + \beta x + \epsilon$$

α is the expected value of y when x is 0.

β is the expected increase in y associated with a one unit increase in x

Coefficients

```
coef(mod)
coefficients(mod)
# (Intercept)          low
#  4256.86158     21.64725
```

The best estimate of
tc2009 for a state with low = -10 is
4256.86 + 21.6 * (-10) = 4040.86

(x,y) ← (-10, 4040.86)

Coefficient Calculator

This function is now my data!!

Based on our training using data, if x = -10, then y = 4040.86

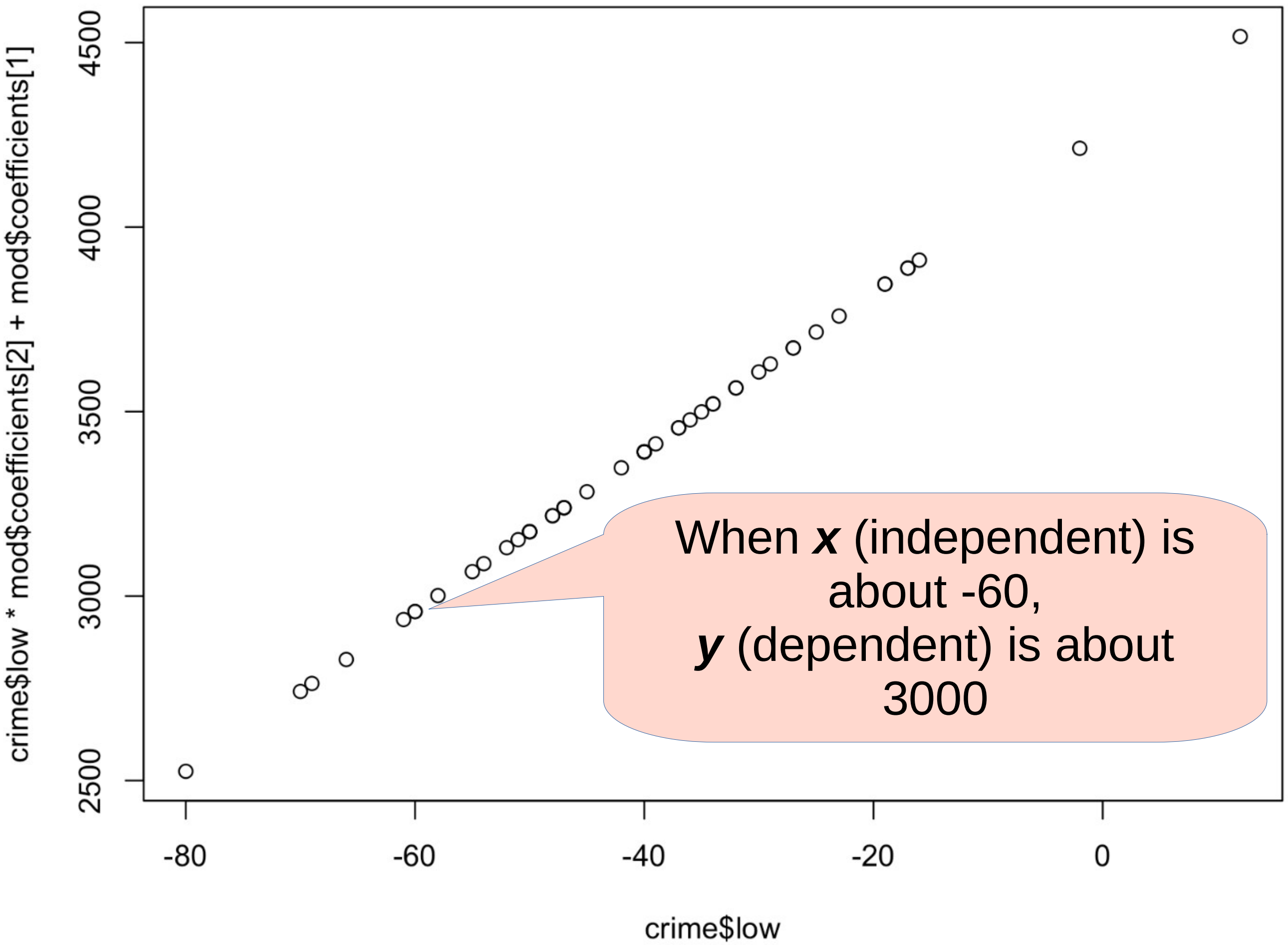
The best estimate of tc2009 for a state with low = -10 is 4256.86 + 21.6 * (-10) = 4040.86

I can predict *y*, based on values of x!

Due to error, there may be a slight difference between expected and actual values

Forecasting **Trends** With a Simple Line

```
plot(crime$low, crime$low*mod$coefficients[2] + mod$coefficients[1] )
```



Coefficient Calculator Function for *mod*

```
# Function to compute estimated y value for an entered x
value

tellMeY <- function(x_int){

  cat("  intercept :",mod$coefficients[1] )
  cat("\n  slope     :",mod$coefficients[2] )
  y = mod$coefficients[1] + x_int * mod$coefficients[2]
  cat("\n Model predicts y = ",y, "from x = ",x_int)
}

# what if x = -10?
tellMeY(-10) # note: x = -10
```

```
> # what if x = -10?
> tellMeY(-10) # note: x = -10
  intercept : 4256.862
  slope     : 21.64725
 Model predicts y =  4040.389 from x =  -10
>
```

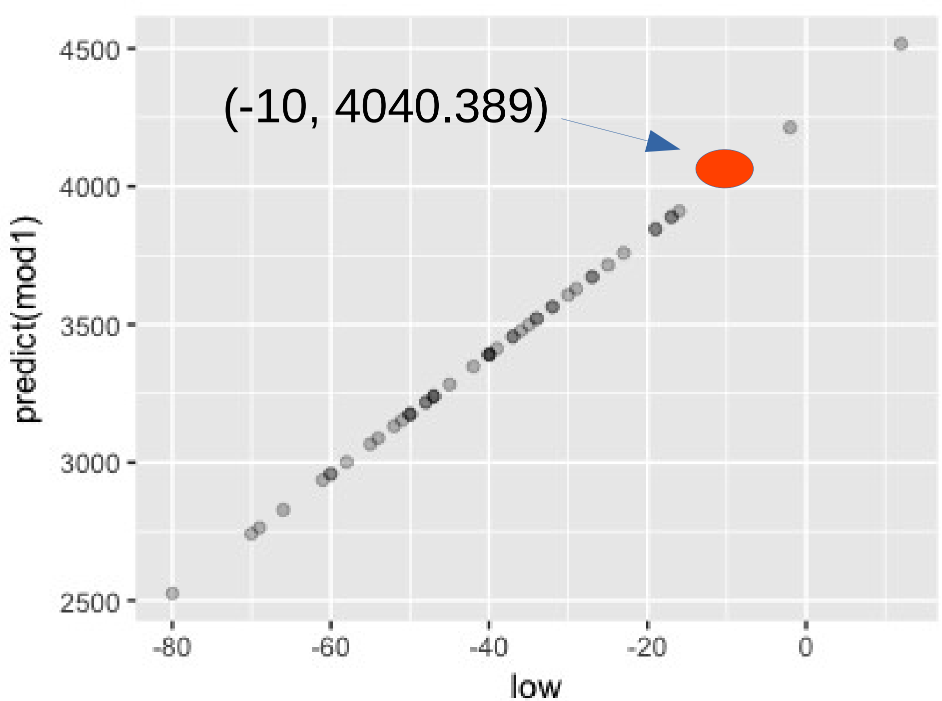
y = *intercept* + *slope* * *x*
= alpha + beta**x*
= *b* + *mx*

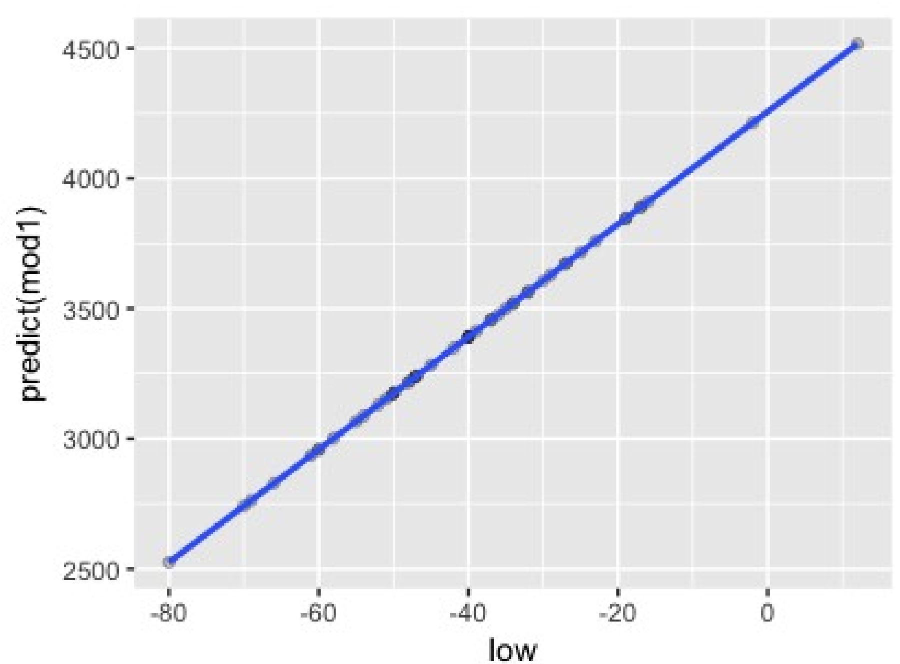
Forecasting with `predict()`

?predict

crime %>% ggplot(aes(x = low, y = **predict(mod)**)) +
geom_point(alpha = I(1/4))

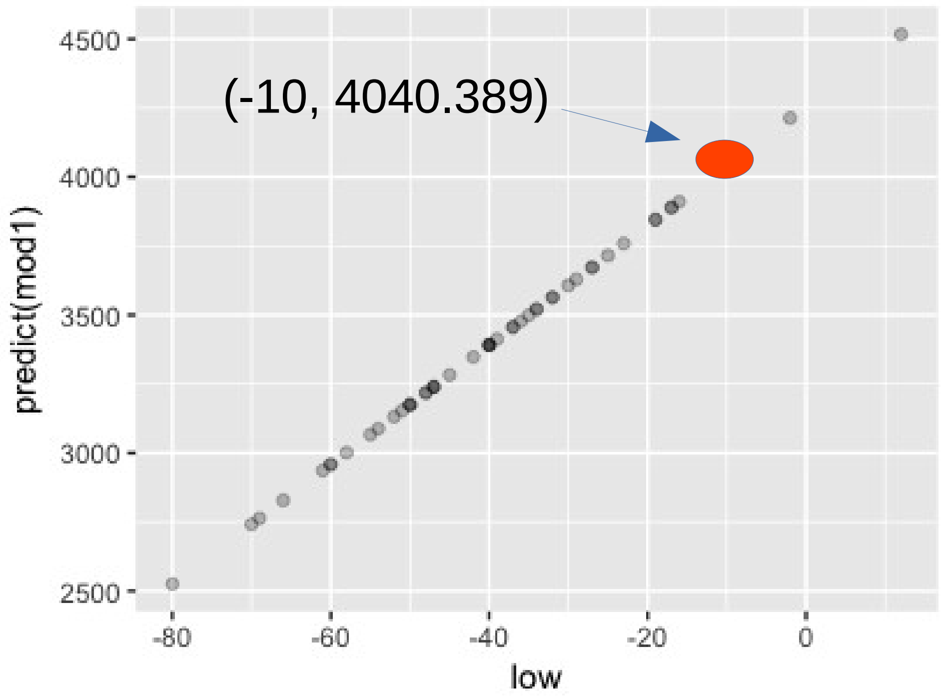
crime %>% ggplot(aes(x = low, y = **predict(mod)**)) +
geom_point(alpha = I(1/4)) + **geom_smooth()**

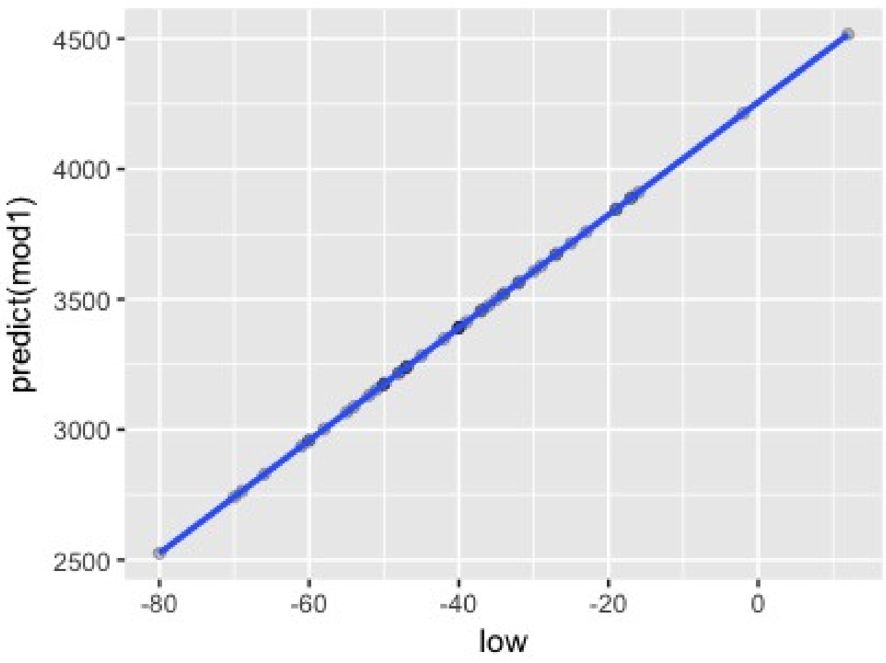




What Conclusions to Draw?

After you have completed the analysis, what does the data indicate?





So, back to the Question… ?

Q1: Is crime influenced by yearly temperature?

File: crime.csv

A: *The data and its trained model suggest that there is a positive correlation between crime and temperature in the US*

Aside: intercept terms

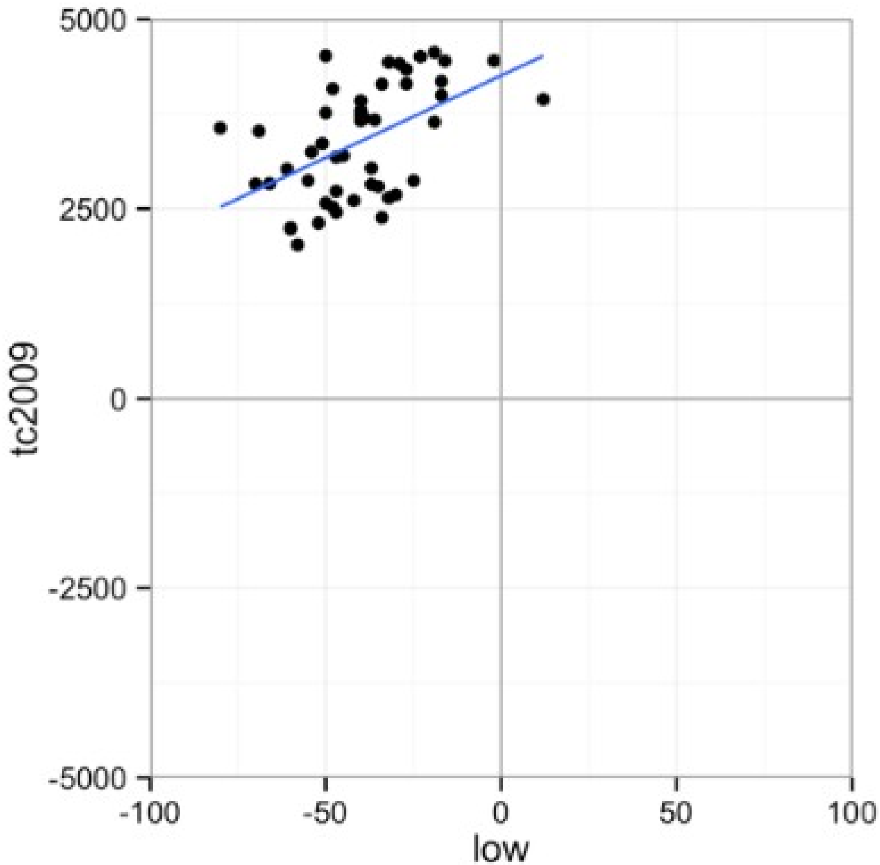
R includes an intercept term in each model by default

$$y = \alpha + \beta x + \epsilon$$

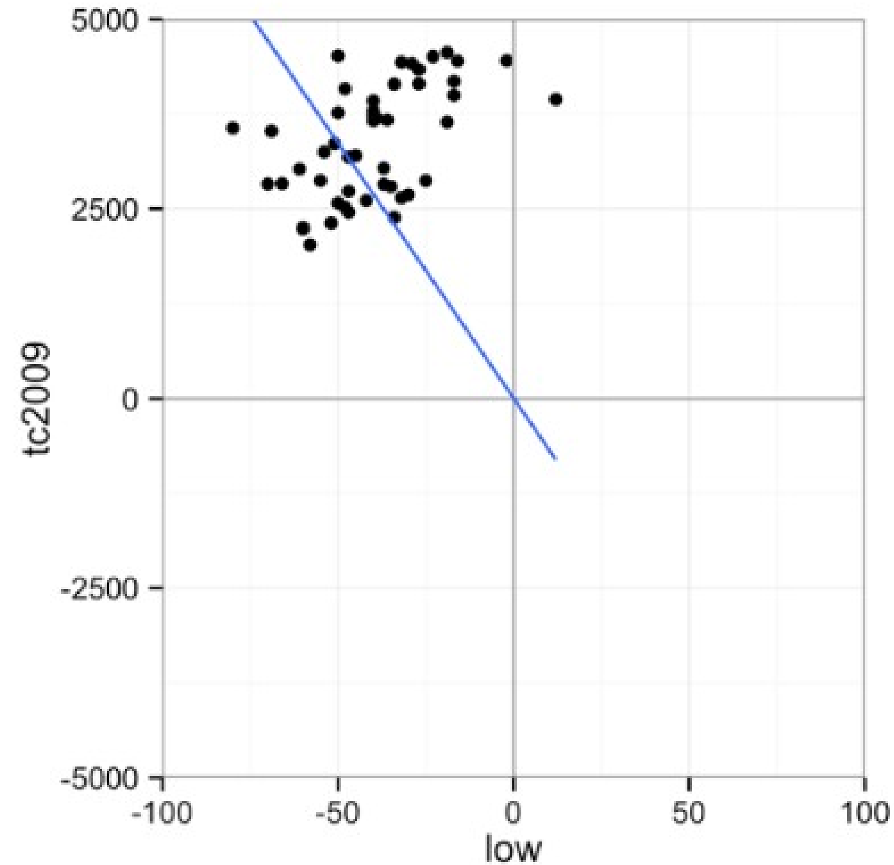
```
y ~ x
```

Study at x = 0?

(Does x = 0 make sense here?)



With a



Without a

Every linear model has a y intercept. Including a lets this term vary. Not including a forces the intercept to (0, 0).

Study at $x = 0$?
(Does $x = 0$ make sense here?)

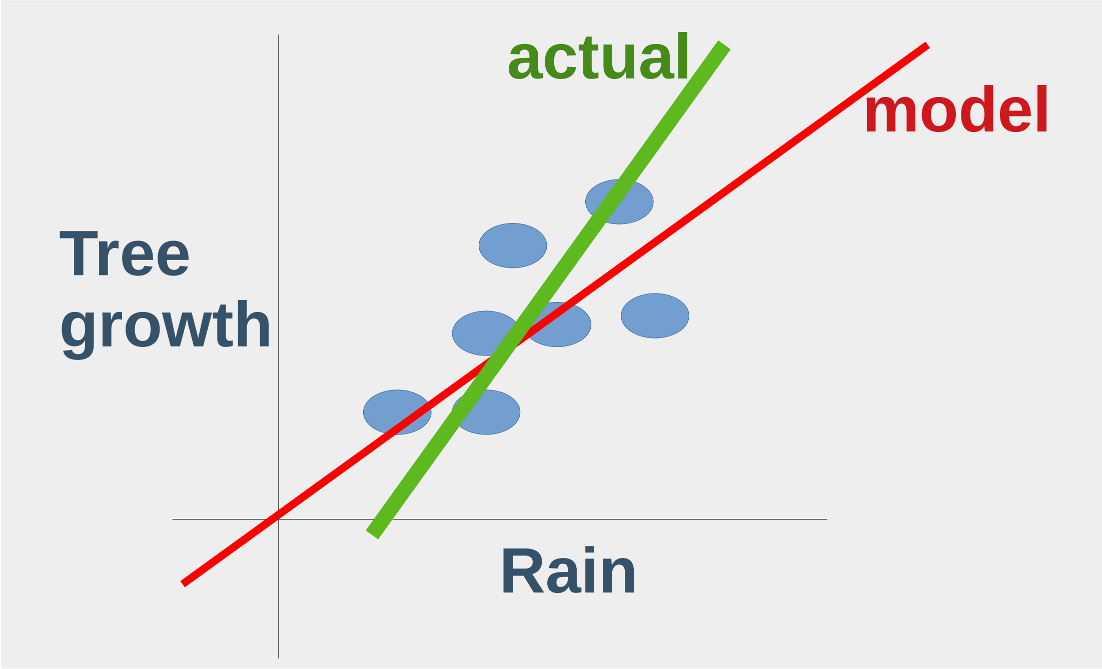
- The y-intercept is the place where the regression line crosses the y-axis (where $x = 0$), and is denoted by b from $y = mx + b$
- **Meaningful interpretation**: Sometimes the y-intercept is relevant (and sometimes it is not)
- No meaning for the y-intercept when data is not present near the point where $x = 0$ (and the model suggests that data is present at this point)

Study at $x = 0$?
(Does $x = 0$ make sense here?)

Ex: A model where rain (x) is used to predict tree growth (y)

If *rain* = 0, then does *tree_growth* = 0? Always? Trees may still grow when sunny out

Intercept not relevant: The regression line may cross y-axis at some other point (other than zero)



An Intercept Term: To Use or Not?

FYI: You can explicitly ask for an intercept by including the number one, 1, as a formula term. You can remove the intercept by including a zero or negative 1.

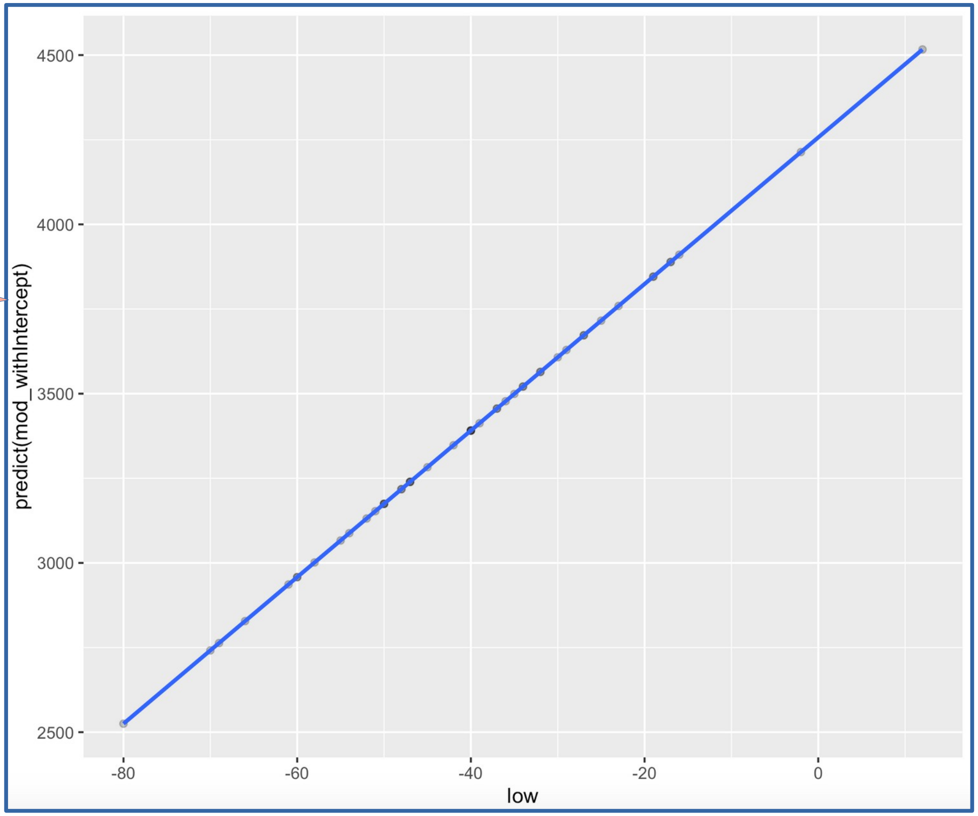
```
# equivalent - includes intercept
lm(tc2009 ~ 1 + low, data = crime)
lm(tc2009 ~ low, data = crime)

# equivalent - removes intercept
lm(tc2009 ~ low - 1, data = crime)
lm(tc2009 ~ 0 + low, data = crime)
```

Let’s Test An Intercept

Add the intercept

```
# equivalent - includes intercept

mod_withIntercept <- lm(tc2009 ~ 1 + low, data = crime)

crime %>% ggplot(aes(x = low, y =
predict(mod_withIntercept))) + geom_point(alpha = I(1/4))
+ geom_smooth()
```

Does this represent your data?

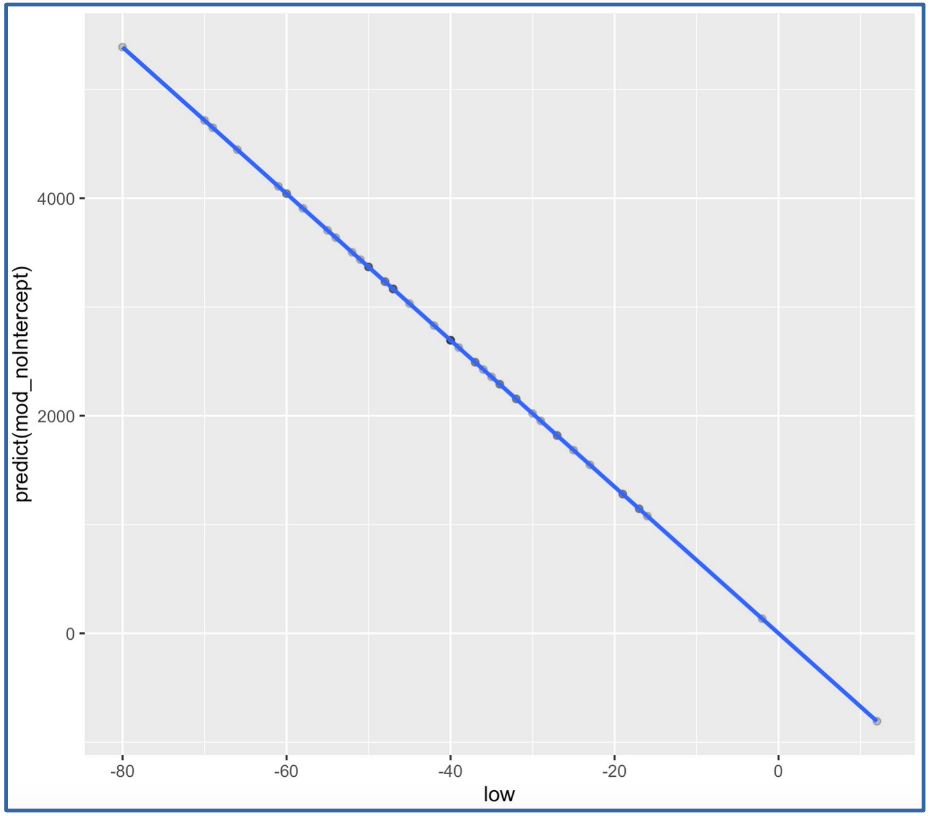


Let’s Test An Intercept

Remove the intercept

```
# equivalent - removes intercept

mod_noIntercept <- lm(tc2009 ~ low - 1, data = crime)

crime %>% ggplot(aes(x = low, y =
predict(mod_noIntercept))) + geom_point(alpha = I(1/4)) +
geom_smooth()
```

Does this represent your data?



Results: `summary(mod)`

```
> summary(mod)

Call:
lm(formula = tc2009 ~ low, data = crime)

Residuals:
     Min       1Q   Median       3Q      Max 
-1134.36  -647.13    98.03   533.62  1344.30 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4256.86     233.44  18.236  < 2e-16 ***
low            21.65       5.33   4.061 0.000188 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 649.9 on 46 degrees of freedom
Multiple R-squared:  0.2639,	Adjusted R-squared:  0.2479 
F-statistic: 16.49 on 1 and 46 DF,  p-value: 0.000188
```

R-squared Value

- Goodness of fit of a model: The R2 coefficient of determination describes how well the regression predictions approximate the real data points.
- ***How well do the indep vars explain the dep var?***

R2 = 1, indep variable(s) predict dep variables

R2 = 0, no prediction

```
Residual standard error: 649.9 on 46 degrees of freedom
Multiple R-squared:  0.2639,    Adjusted R-squared:  0.2479
F-statistic: 16.49 on 1 and 46 DF,  p-value: 0.000188
```

P-Value and Model Significance

- ***How do I know if this model is any good?***
- If,
  - *p*-value is between 0 and 0.01 (or)
  - *p*-value is between 0 and 0.05
- Then the model is significant
- The closer to zero, the better the model.

```
Residual standard error: 649.9 on 46 degrees of freedom
Multiple R-squared:  0.2639,    Adjusted R-squared:  0.2479
F-statistic: 16.49 on 1 and 46 DF,  p-value: 0.000188
```

Study the p-Value

```
#Create a simple model
myX <- 0:100
myY <- myX + 1
mod <- lm(myY ~ myX)
summary(mod)
```

Study the p-Value

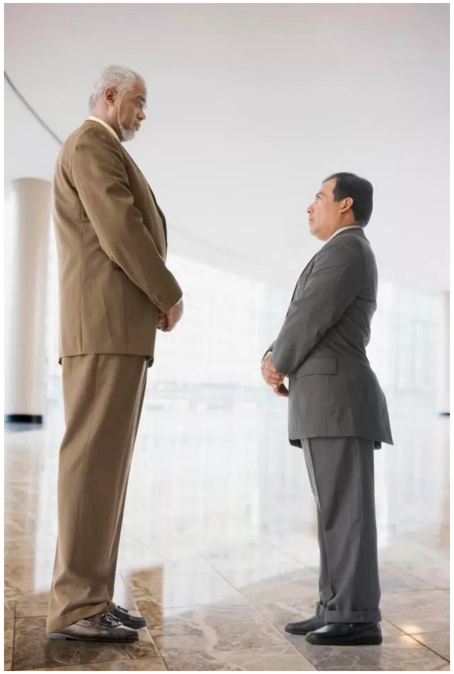
```
#Create a simple model
myX <- 0:100
myY <- myX + 1
mod <- lm(myY ~ myX)
summary(mod)
```

```
Call:
lm(formula = myY ~ myX)

Residuals:
       Min         1Q     Median         3Q        Max
-1.000e-13 -3.420e-16  1.157e-15  2.304e-15  1.242e-14

Coefficients:
             Estimate Std. Error   t value Pr(>|t|)
(Intercept) 1.000e+00  2.057e-15 4.862e+14   <2e-16 ***
myX         1.000e+00  3.553e-17 2.814e+16   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.041e-14 on 99 degrees of freedom
Multiple R-squared:      1,     Adjusted R-squared:      1
F-statistic: 7.921e+32 on 1 and 99 DF,  p-value: < 2.2e-16

Warning message:
In summary.lm(mod) : essentially perfect fit: summary may be unreliable
```

Types of Questions to Address With Data

Q1: Is crime influenced by yearly temperature?

File: crime.csv



Q2: What influence is there on earning potential and personal height?

File: wages.csv

Consider This!

Let's try making a model with the other data set. Is there a connection between height on earning potential.

Load the Wages Data

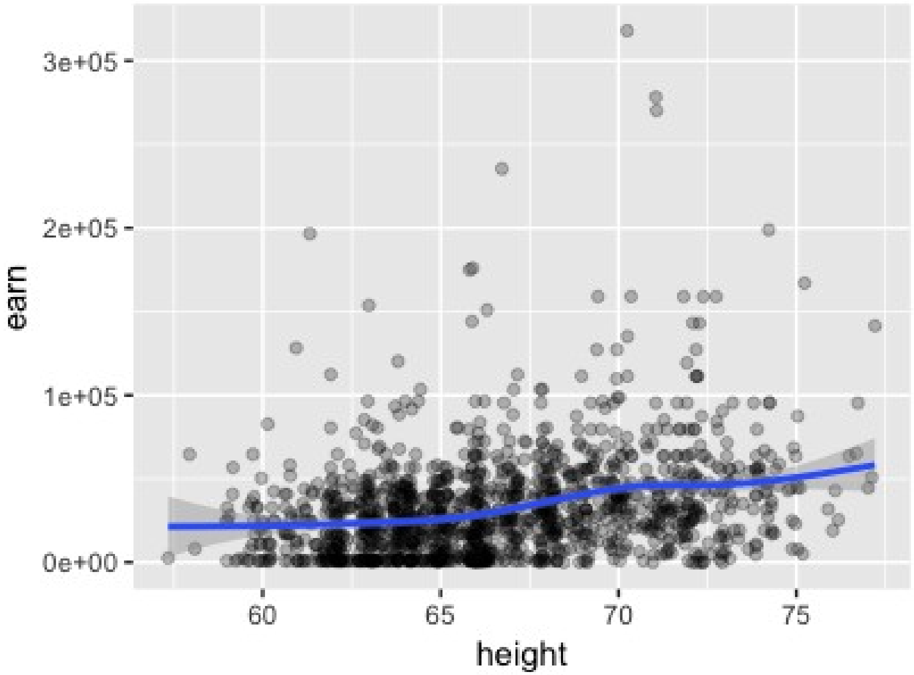
Fit a linear model to the wages data set that predicts ***earn*** with ***height***.

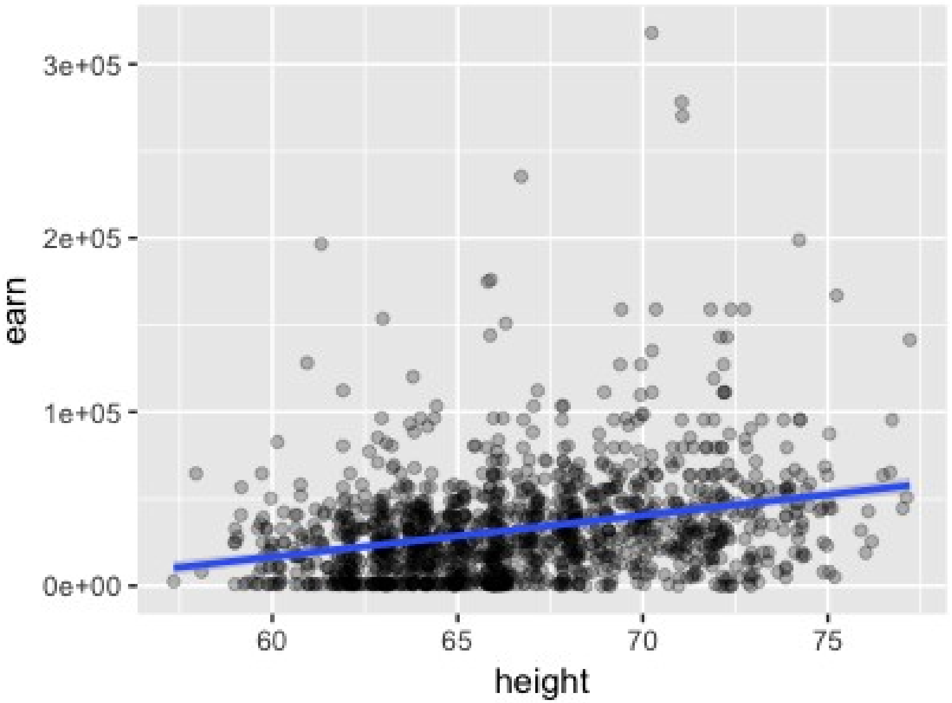
```
rm(list = ls()) # remove old vars

# open the wages.csv dataset from the data.

w <- file.choose() # set the filename

wages <- read.csv(w) # load and read the data.
```

Do *Tall* People *Earn* More?

```
wages %>% ggplot(aes(x = height, y = earn)) + geom_point(alpha = I(1/4)) + geom_smooth() # add a line

wages %>% ggplot(aes(x = height, y = earn)) + geom_point(alpha = I(1/4)) + geom_smooth(method = lm) # linear model line
```

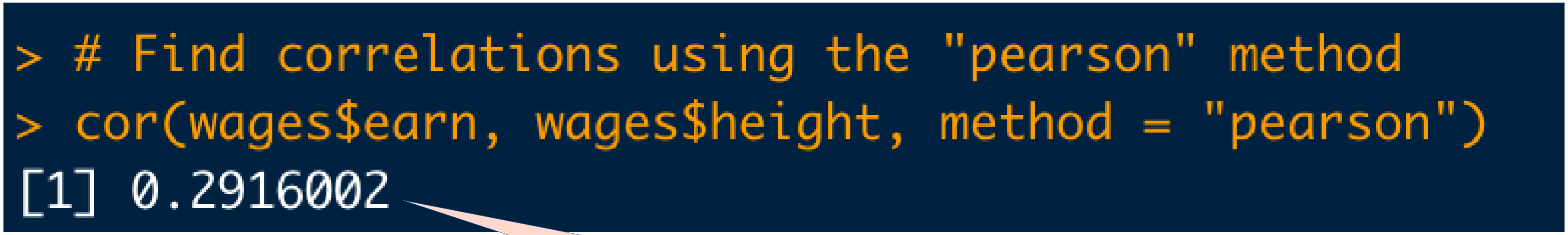
 # Try switching the x's and y's for another view.





Correlations:
Earn and Height

```
# Find correlations using the "pearson" method

cor(wages$earn, wages$height, method = "pearson")
```



Positive correlation

Make a Model

```
hmod <- lm(earn ~ height, data = wages)
summary(hmod)
```

Where **dependent** var is ***earn***

And **independent** var is ***height***

$$y = \alpha + \beta x + \epsilon$$

Summary of Model

summary(hmod)

```
> summary(hmod)

Call:
lm(formula = wages$earn ~ wages$height)

Residuals:
   Min      1Q Median     3Q    Max
-47903 -19744  -5184  11642 276796

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   -126523      14076  -8.989   <2e-16 ***
wages$height     2387        211  11.312   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 29910 on 1377 degrees of freedom
Multiple R-squared:  0.08503,	Adjusted R-squared:  0.08437
F-statistic:   128 on 1 and 1377 DF,  p-value: < 2.2e-16
```

Build your model’s line equation from these coefficients!

Earn Regressed Over *height*

```
hmod <- lm(earn ~ height, data = wages)
coef(hmod)
## (Intercept)      height
## -126523.359    2387.196
```

$$earn = \alpha + \beta \times height + \epsilon$$

$$\boxed{earn = -126523.36 + 2387.20 \times height + \epsilon}$$

An Estimation

The best estimate of earn for someone 68 inches tall is

$$earn = -126523.36 + \boxed{2387.20 \times 68} + \epsilon$$

$$earn = 35806.24$$

Conclusions

Earn Regressed Over *height*

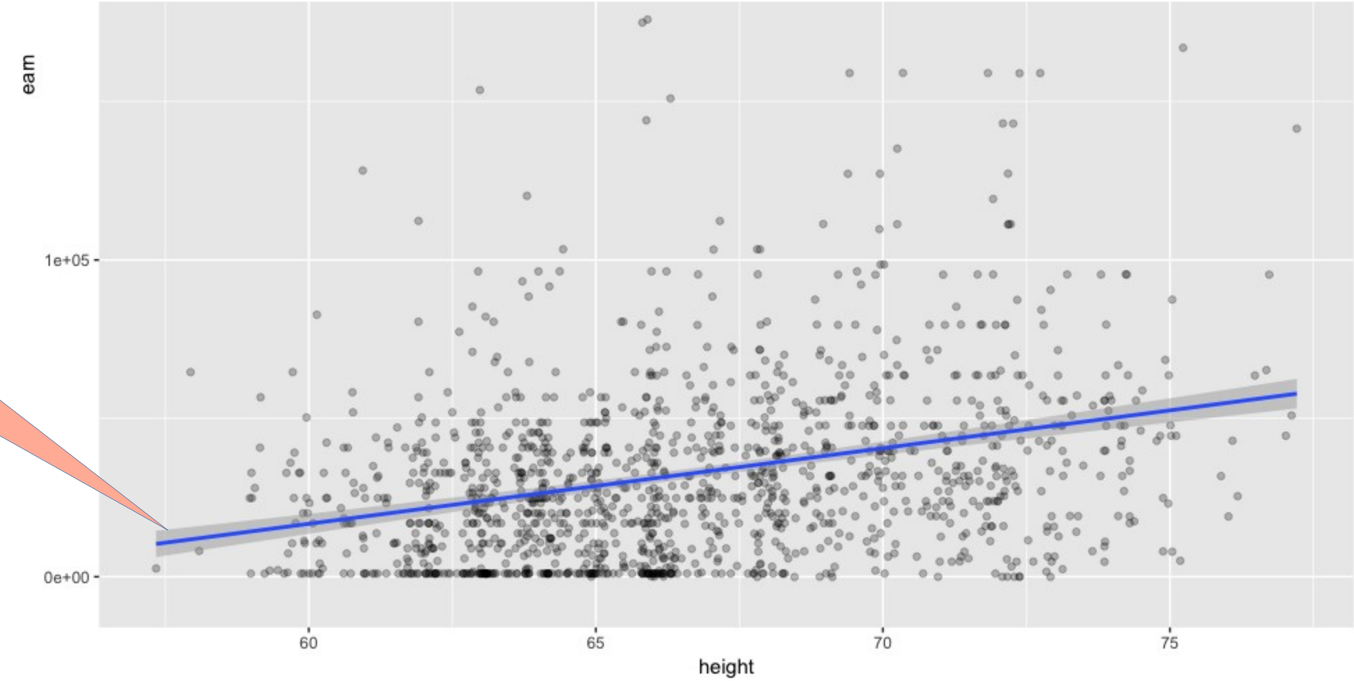
```
cor(wages$earn, wages$height, method = "pearson")
```

– Slight, but positive

```
hmod <- lm(earn ~ height,  data = wages)
```

– Significant

Our model with the data points



Build a Model

- Fit a linear model to the wages data set
- How do we interpret the results?

Q: What happens when we regress *earn* over *race*?

Or, How does ***race*** influence ***earn***?

Summary

```
rmod <- lm(earn ~ race, data = wages)
coef(rmod) # get the model's y-intercepts and slopes
```

```
coef(rmod)
# (Intercept) racehispanic   raceother   racewhite
#    28372.09     -2886.79     3905.32     4993.33
```

summary(rmod)

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)     28372       2781  10.204   <2e-16 ***
racehispanic    -2887       4515  -0.639   0.5227
raceother        3905       6428   0.608   0.5436
racewhite        4993       2929   1.705   0.0885 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

Estimates From Coefficients

```
coef(rmod)
# (Intercept) racehispanic   raceother   racewhite
#    28372.09     -2886.79     3905.32     4993.33
```

The estimate for a white person is
28372.09 + 4993.33 = 33365.42

The estimate for a other person is
28372.09 + 3905.32 = 32277.41

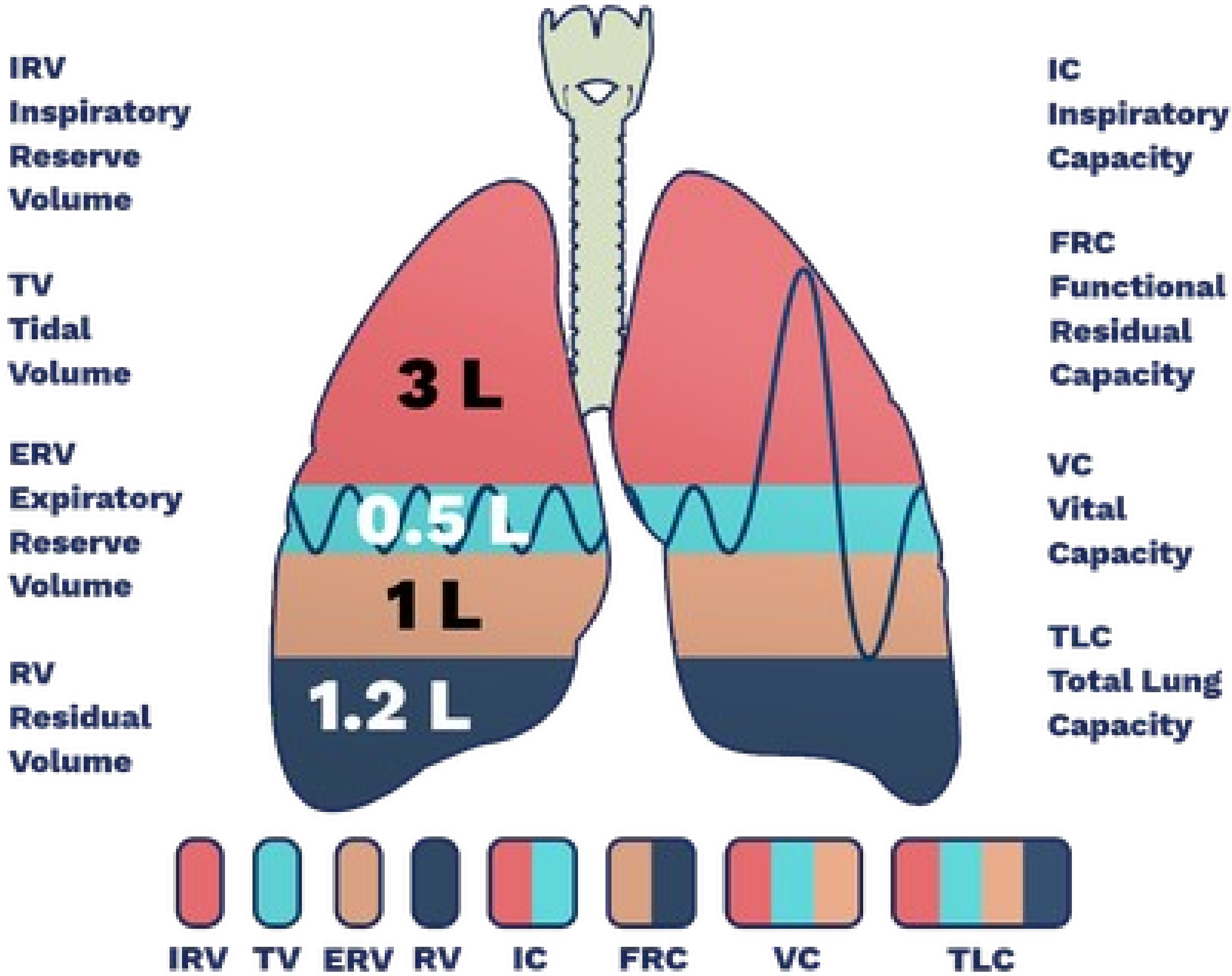
The estimate for a hispanic person is
28372.09 + -2886.79 = 25485.30

The estimate for a black person is
28372.09 = 28372.09

What Do Plots Also Indicate??


```
ggplot(data = wages) +
  geom_point(mapping = aes(y = earn, x = height, color = race )) +
  geom_smooth(mapping = aes(y = earn, x = height )) + facet_wrap(~race)
```

Lungs Capacity



File: lungCapDemo.R