Data Science
CS301
Basic Stats and Concepts

Week 5
Fall 2024
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Writing Functions
(You might need this later!)

```
functionName <- function(arg1, arg2, arg3=2, ...) {

  newVar <- sin(arg1) + sin(arg2) # do useful stuff

  newVar / arg3   # Return value }
```

```
functionName(2,3,1) # run function with inputs
```

- **functionName**: is the function's name

- **args**: arguments of the function, also called formals to import data into a function. No limit to the number for a function.

- **Return value**: The last line of the code is the value that will be returned by the function. It is not necessary that a function return anything

The getSin() Function
(for Trigonometry)

Name of function: A variable to refer to the function

Define function

Define the function's *mechanism. What does this function do when called?*

```
getSin <- function(myVal) {
  # return sin values in radians
  sin(myVal)
}

getSin(2)
```

The sumOfSquares() Function

```
#Return the sum of squares:

sumOfSquares <- function(x,y) {

  x^2 + y^2

 }

#run sumOfSquares () with x=2 and y=4

sumOfSquares(2,4) # returns 20
```

The redPlot() Function

```
# function to plot points on the canvas
redPlot <- function(x, y) {
        plot(x, y, col="red")
        }
# run the function
redPlot (2,4) # plot a red point
redPlot (c(2:10), c(2:10)) # a series of points
```

The colorPlot() Function

```
# determine points and color

colorPlot <- function(x, y, c) {

        plot(x, y, col=c)

        }

# run the function

colorPlot(x, y, "red") # plot a red point

colorPlot (c(2:10), c(2:10), "blue")
```

Yet, Another Example: Using An `If-Else` Statement

```
GimmeAtLeastFive <- function(inNum){

 if(inNum >= 5){

     print("That is at least five")

  }

 else{

     print("not enough")

 }

}
```

Basic Stats

- We will spend some time looking at different types of statistical tests so that they can be implemented in code.

Medians

Median

First, arrange the observations in an ascending order.

If the number of observations (n) is **odd**:
the median is the value at position

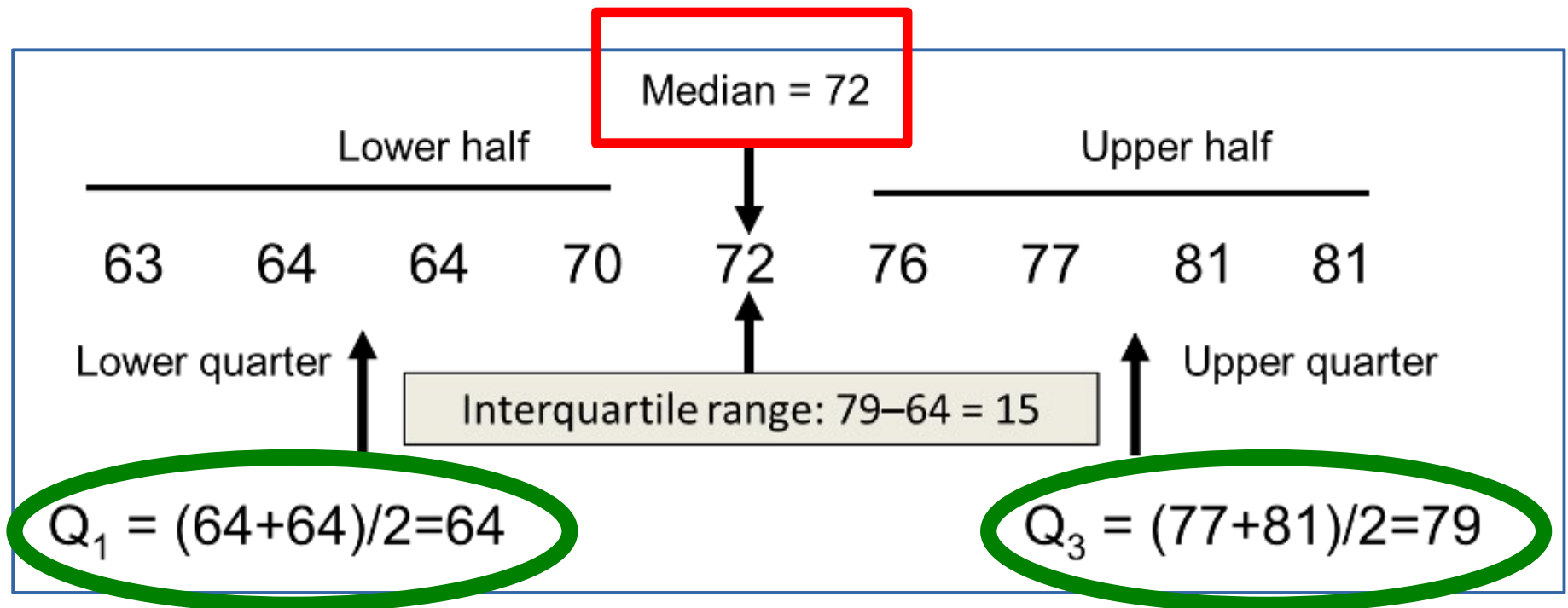$$\left(\frac{n+1}{2}\right)$$

If the number of observations (n) is **even**:

1. Find the value at position $\left(\dfrac{n}{2}\right)$

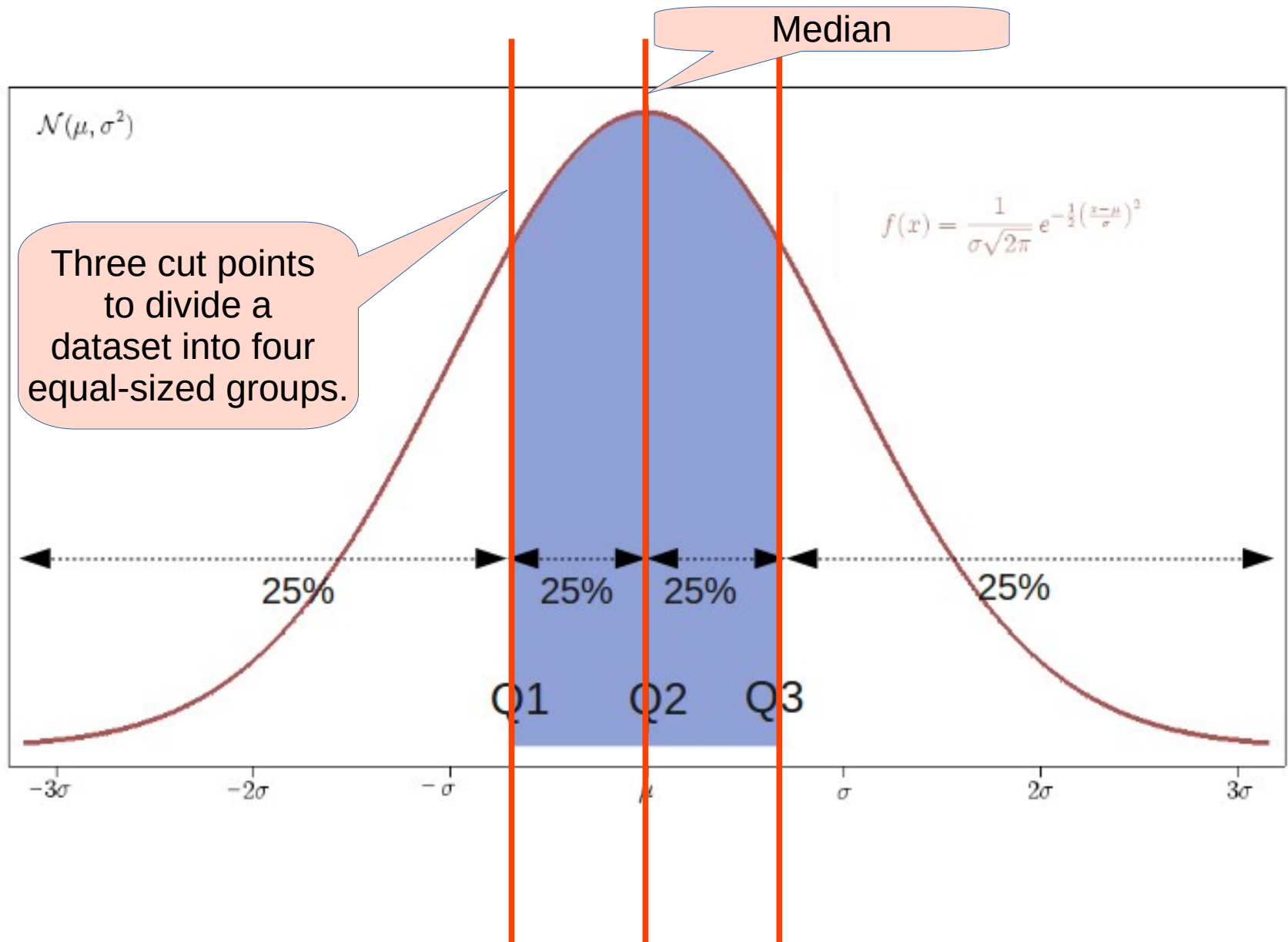2. Find the value at position $\left(\dfrac{n+1}{2}\right)$

3. Find the average of the two values to get the median.

Medians

Median = 72

Lower half   Upper half

63   64   64   70   72   76   77   81   81

Lower quarter   Upper quarter

Interquartile range: 79–64 = 15

$Q_1 = (64+64)/2=64$ $Q_3 = (77+81)/2=79$

- What does Q1 and Q3 indicate?
  - Quantiles: allow us to determine placements in the set of numbers

Quantiles:
Quarters of Data

Finding Quantiles

- Finding 1st and 3rd quantiles is to determine the positions at the ¼ and ¾ marks, respectively.

Quartile	Calculation	Result
Zeroth quartile	Although not universally accepted, one can also speak of the zeroth quartile. This is the minimum value of the set, so the zeroth quartile in this example would be 3.	3
First Quantile	The rank of the first quartile is 10×(1/4) = 2.5, which rounds up to 3, meaning that 3 is the rank in the population (from least to greatest values) at which approximately 1/4 of the values are less than the value of the first quartile. The third value in the population is 7.	7
Second Quantile	The rank of the second quartile (same as the median) is 10×(2/4) = 5, which is an integer, while the number of values (10) is an even number, so the average of both the fifth and sixth values is taken—that is (8+10)/2 = 9, though any value from 8 through to 10 could be taken to be the median.	9
Third Quantile	The rank of the third quartile is 10×(3/4) = 7.5, which rounds up to 8. The eighth value in the population is 15.	15
Fourth quartile	Although not universally accepted, one can also speak of the fourth quartile. This is the maximum value of the set, so the fourth quartile in this example would be 20. Under the Nearest Rank definition of quantile, the rank of the fourth quartile is the rank of the biggest number, so the rank of the fourth quartile would be 10.	20

1st 2nd 3rd

Original Data: 3, 6, **7**, 8, **8, 10**, 13, **15**, 16, 20

Rounded

Quantiles:
Quarters of Data

```
qnums <- c(3, 6, 7, 8, 8, 10, 13, 15, 16, 20)

summary(qnums) # quick preparation of stats

quantile(qnums)
```

```
> qnums   <- c(3, 6, 7, 8, 8, 10, 13, 15, 16, 20)
> summary(qnums) # quick preparation of stats
   Min.  1st Qu.  Median     Mean  3rd Qu.    Max.
   3.00     7.25    9.00    10.60    14.50   20.00
> quantile(qnums)
     0%     25%     50%     75%    100%
   3.00    7.25    9.00   14.50   20.00
```

Quantile Divisions, Code 1

```r
library(tidyverse)
library(tibble)

qnums  <- c(3, 6, 7, 8, 8, 10, 13, 15, 16, 20)
summary(qnums) # quick preparation of stats
quantile(qnums) # just report the quantiles

# lets make a plot to see where these quantile divisions.
mySummary <- summary(qnums) # quick preparation of stats

#define the quantile variables for 1st, 2nd and 3rd values
firstQuantile <- as.numeric(mySummary[2])
secondQuantile <- as.numeric(mySummary[3])
thirdQuantile <- as.numeric(mySummary[5])
cat(" 25%:", firstQuantile," mean:", secondQuantile, "
75%:",thirdQuantile)


# output: 25%: 7.25  mean: 9  75%: 14.5
```

File: quantiles.R

Quantile Divisions, Code 2

```r
#View(qnums) # OH NO! This is not a spreadsheet-like form to
create a plot with ggplot().


# We will need to make a data frame using tibble().
qnumsDat <- tibble(num = 1:length(qnums),
                   vals = c(qnums) )


ggplot(data = qnumsDat,
  mapping = aes(x = 1:length(qnums), y = qnums )) +
  geom_point() +
  geom_hline(yintercept = firstQuantile, color = "red") +
  geom_hline(yintercept = secondQuantile, color = "blue") +
  geom_hline(yintercept = thirdQuantile, color = "purple") +
  geom_smooth() +
  labs(x = "Nums") +
  labs(y = "Qnums and Quantile Divisions")
```

File: quantiles.R

Quantile Divisions

3rd quantile

2nd quantile

1st quantile
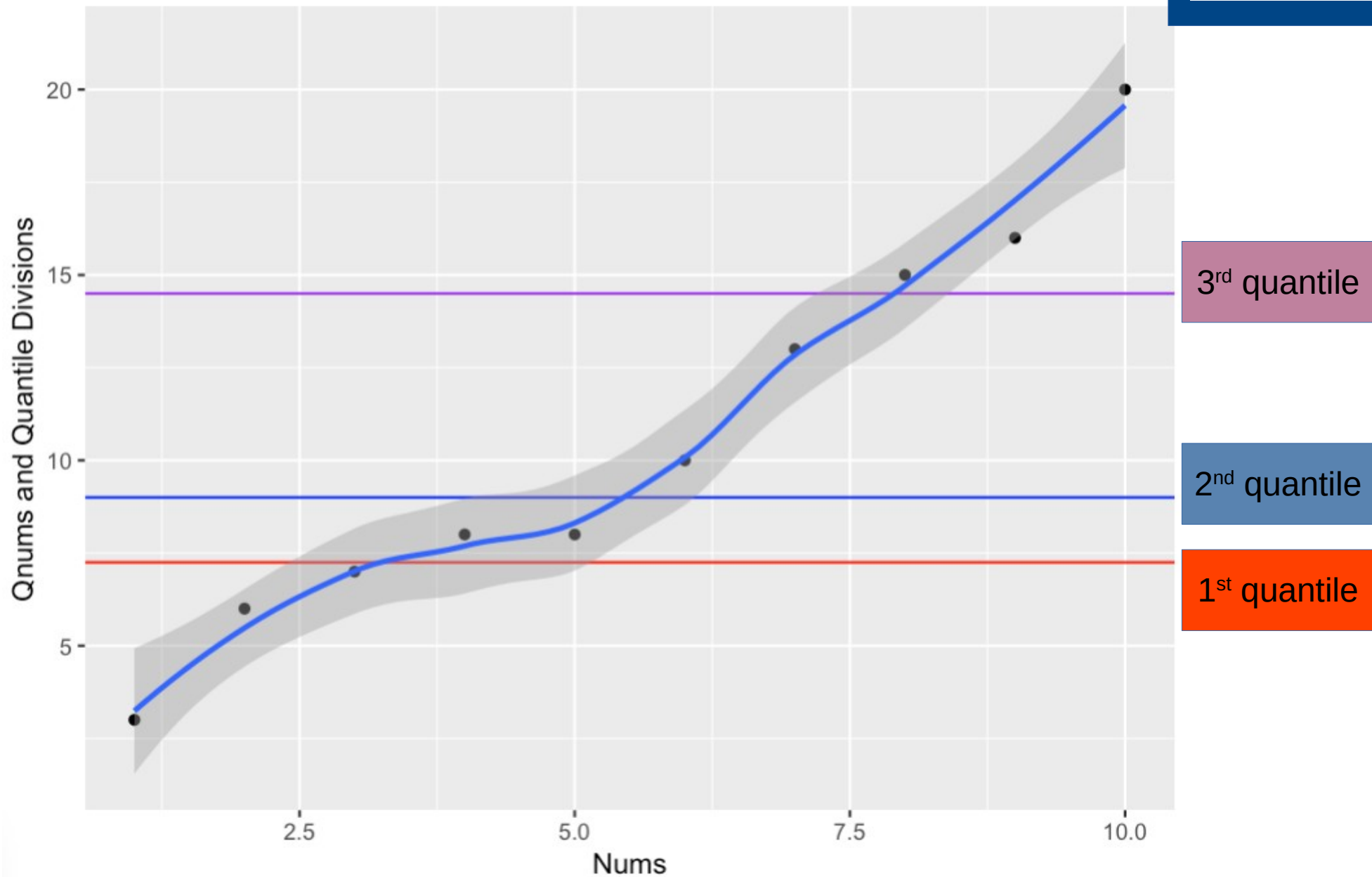
Consider this … `summary()`

Choose "AirPassengers" having only one column.

View(AirPassengers)

general meta data

summary(AirPassengers)

	AirPassengers
1	112
2	118
3	132
4	129
5	121
6	135
7	148
8	148

```
> summary(AirPassengers)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  104.0   180.0   265.5   280.3   360.5   622.0
```

Consider this... `summary()`

- Min: Minimum value (lower bound)

- Max: Maximum value (upper bound)

- Mean: Average value across the set

- Median:
  - The middle number (if num of observations is odd)
  - The average of the middle pair (if num of observations is even)
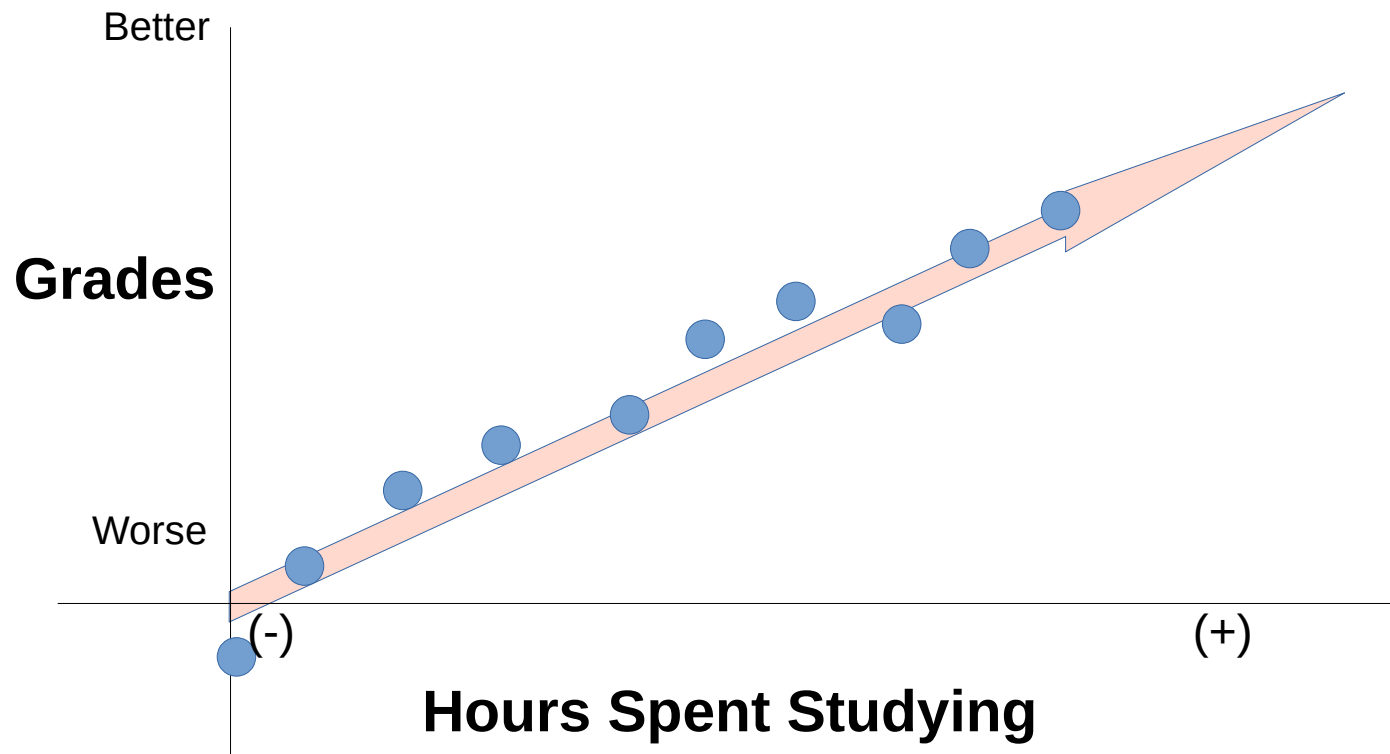
```
> summary(AirPassengers)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  104.0   180.0   265.5   280.3   360.5   622.0
```

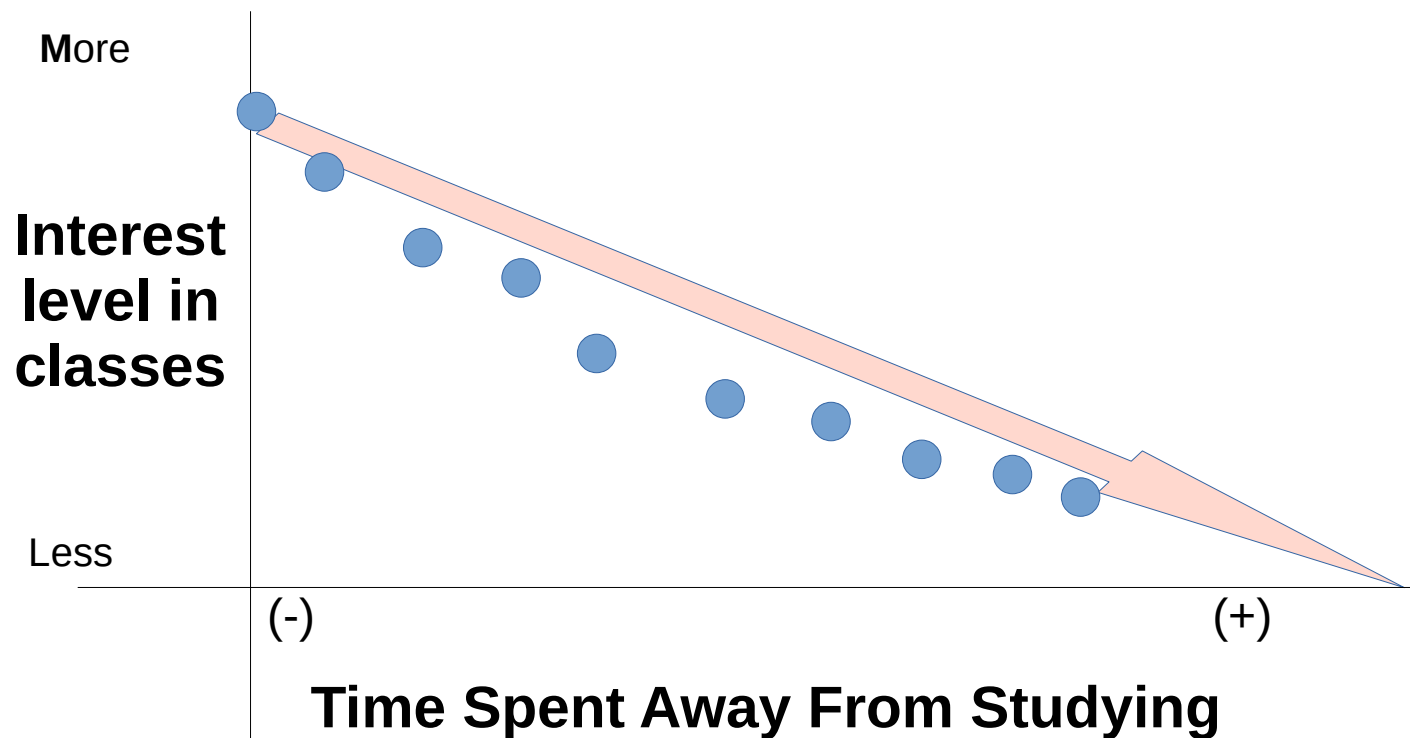# Correlation:
Relatedness Between Sets

- **Positive correlation** exists when two variables move in the same direction.

Better

Grades

Worse

(-) (+)

Hours Spent Studying

Points lie close to a straight line that has a **positive** gradient.

Correlation

- **Negative correlation** exists when two variables move in the opposite directions.

More

Interest level in classes

Less

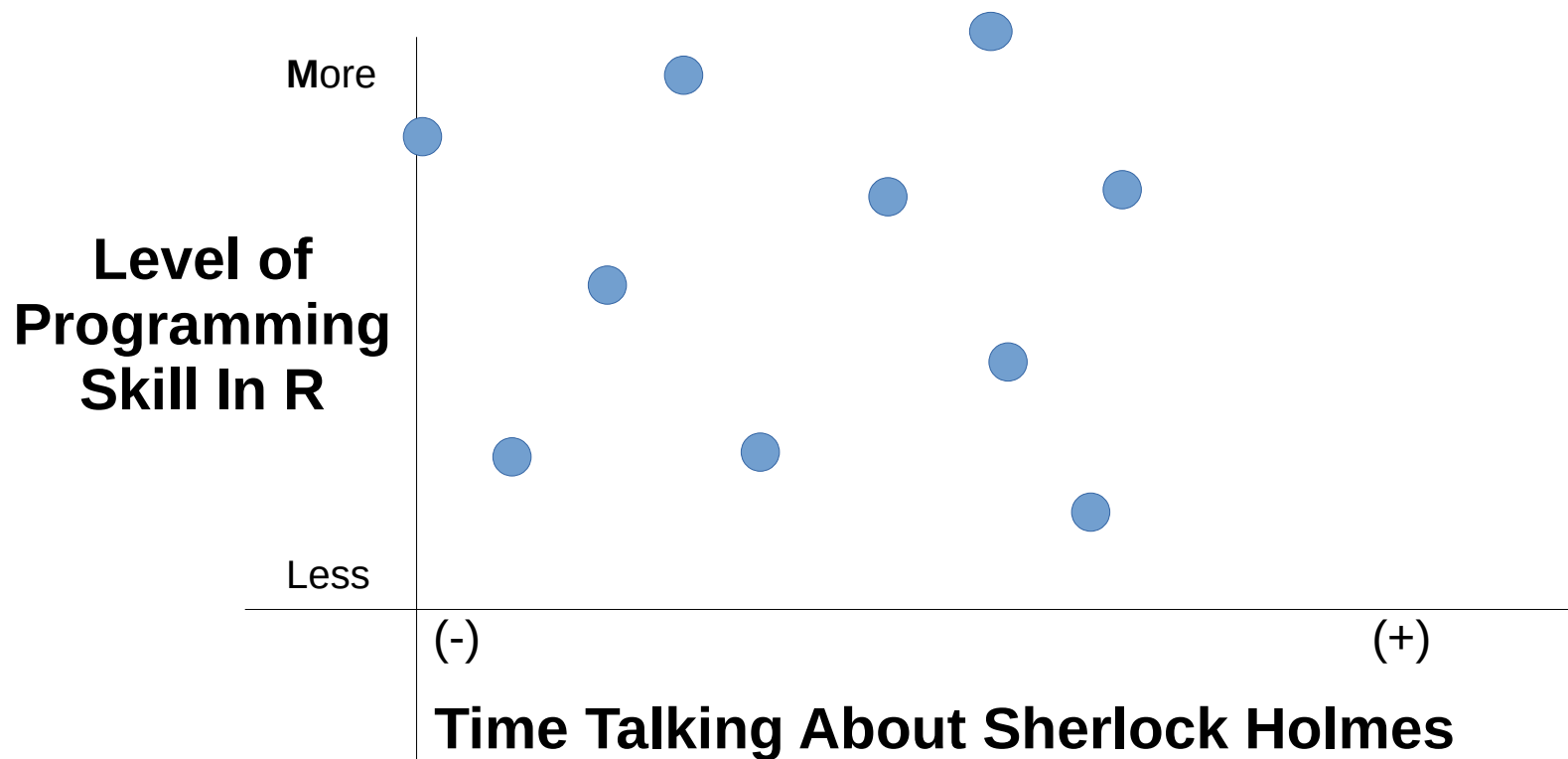(-)   (+)

Time Spent Away From Studying

Points lie close to a straight line that has a **negative** gradient.

Correlation

- **No correlation** exists when two variables are independent of each other.

More

Level of Programming Skill In R

Less

(-)   (+)

Time Talking About Sherlock Holmes

No pattern exists in the layout of points. :-(
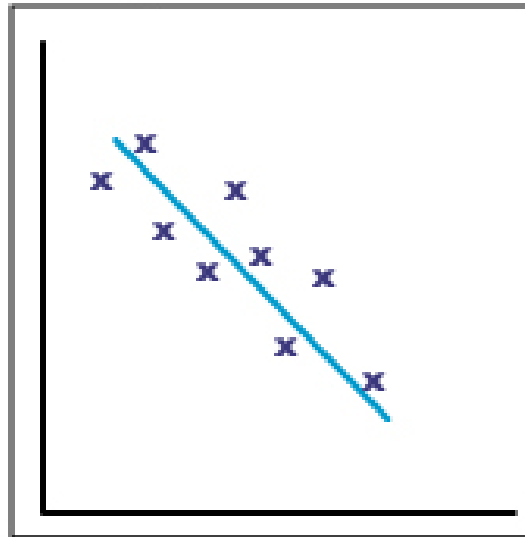
Correlation

Positive correlation
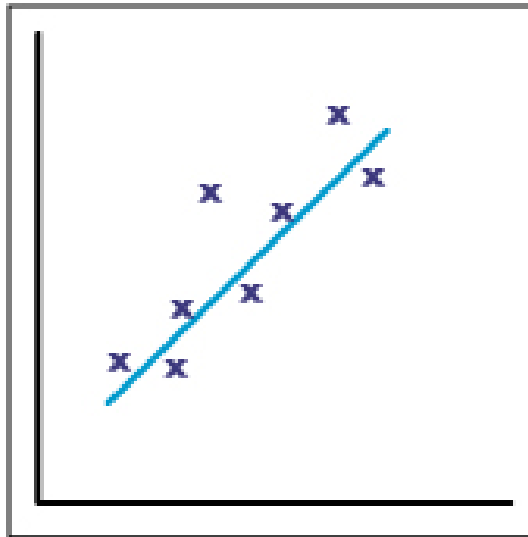
The points lie close to a straight line, which has a positive gradient.

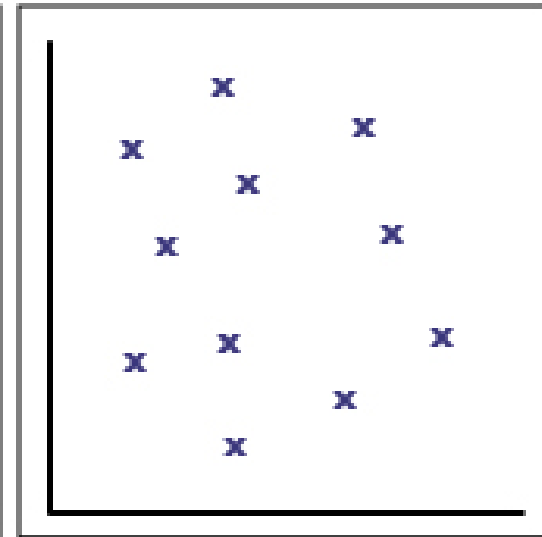This shows that as one variable **increases** the other **increases**.

Negative correlation

The points lie close to a straight line, which has a negative gradient.

This shows that as one variable **increases**, the other **decreases**.

No correlation

There is no pattern to the points.

This shows that there is **no connection** between the two variables.

Measurements of Correlation (By the Numbers)

- A correlation of 1 indicates a perfect positive correlation.

- A correlation of -1 indicates a perfect negative correlation.

- A correlation of 0 indicates that there is no relationship between the different variables.

- Values between -1 and 1 denote the strength of the correlation, as shown in the example below.

Measurements of Correlation

$r = 0.34; p = 0.332$
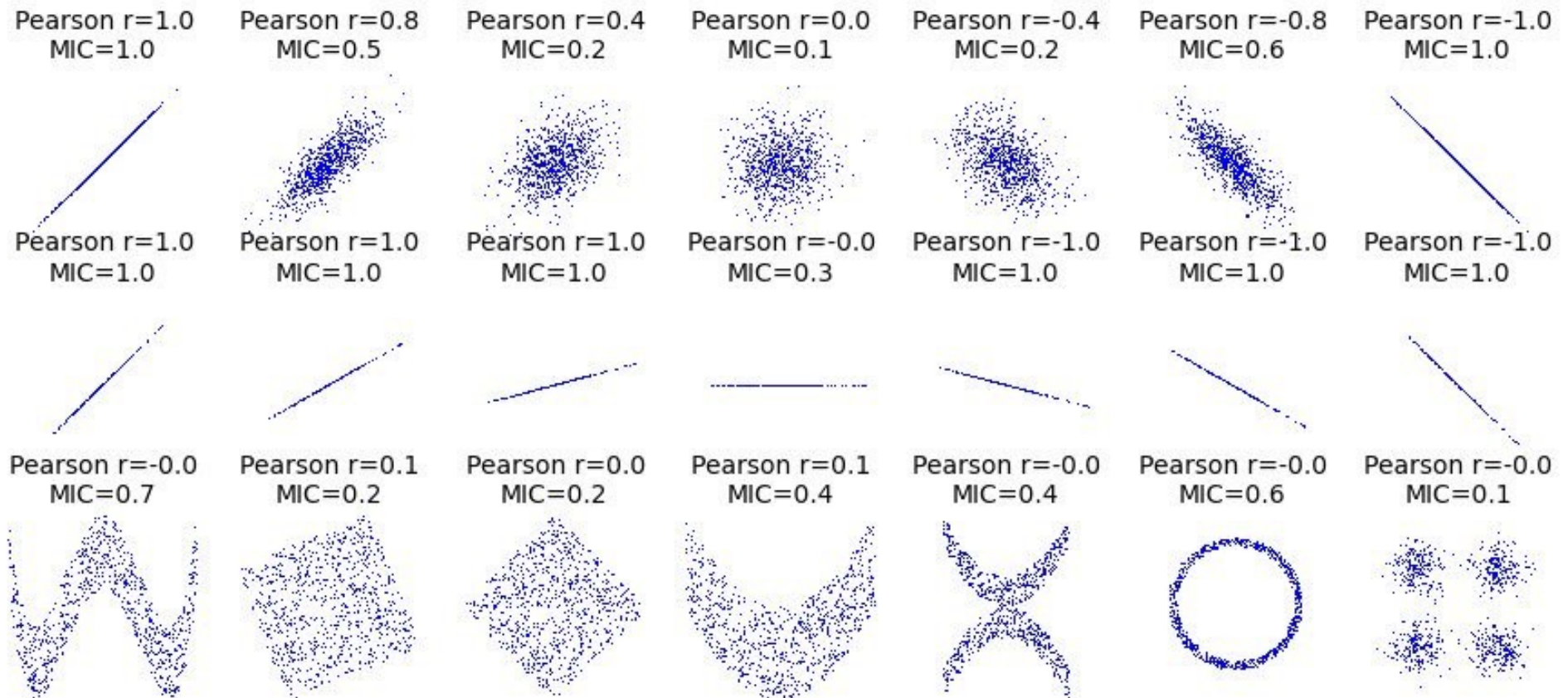
$r = 0.96; p < 0.0001$

$r = 0.72; p = 0.018$

$r = -0.99; p < 0.0001$

Measurements of Correlation

Examples Taken From Online Help

```r
? cor # online help
## Two simple vectors
cor(1:10, 2:11) # == 1
cor(1:10, -2:-11) # == -1

## matrix
cor(longley)

## Correlation Matrix of Multivariate sample:
(Cl <- cor(longley))
## Graphical Correlation Matrix:
symnum(Cl) # highly correlated

## Pearson's r
symnum(clP <- cor(longley, method = "pearson")) #default

## Spearman's rho,  Kendall's tau and
# Data is of non-bivariate normal distribution
symnum(clS <- cor(longley, method = "spearman"))
symnum(clK <- cor(longley, method = "kendall"))

## How much do they differ?
i <- lower.tri(Cl)
cor(cbind(P = Cl[i], S = clS[i], K = clK[i]))
```
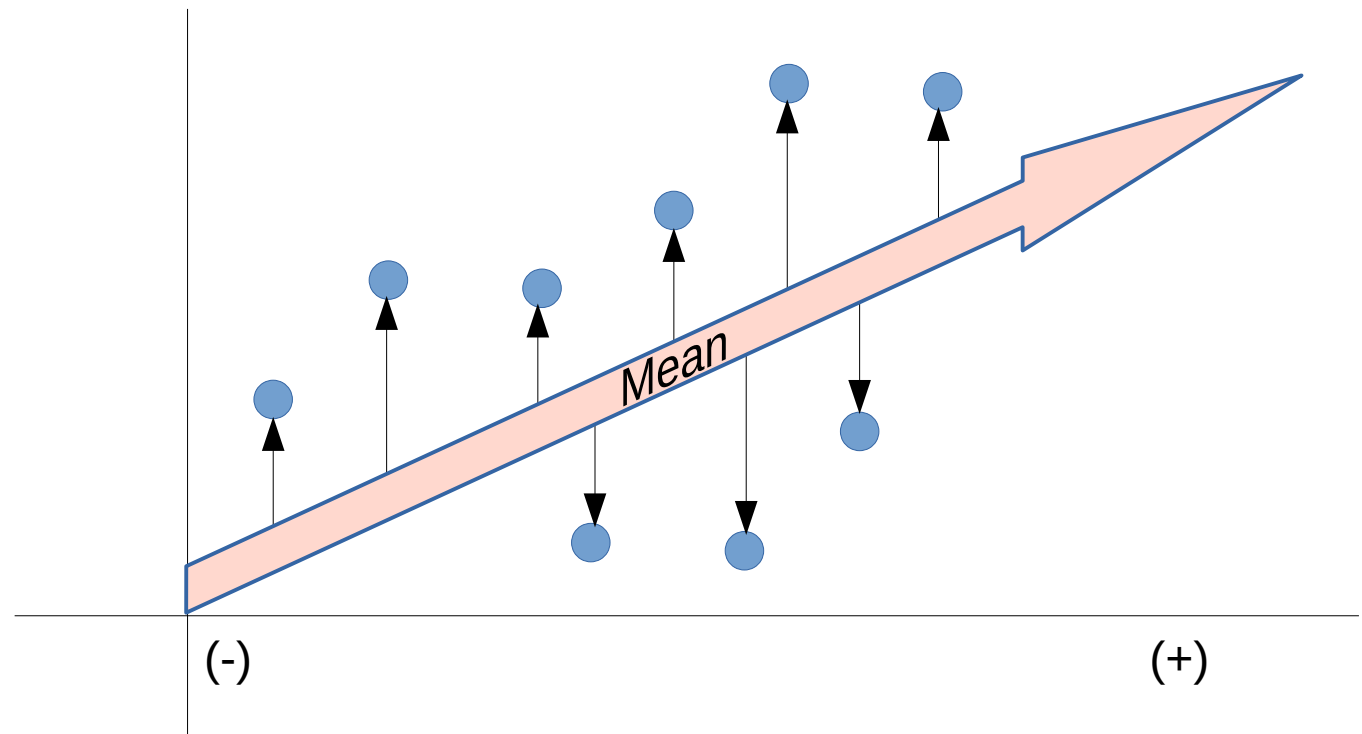
Variance

- The difference between our results and the expectation

- An idea of the *spread* of data from the mean


```
myData <- c( 9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4)
var(myData) # Variance is 9.368421

# Standard Deviation
sqrt(var(myData)) # Standard deviation is 3.060788
```
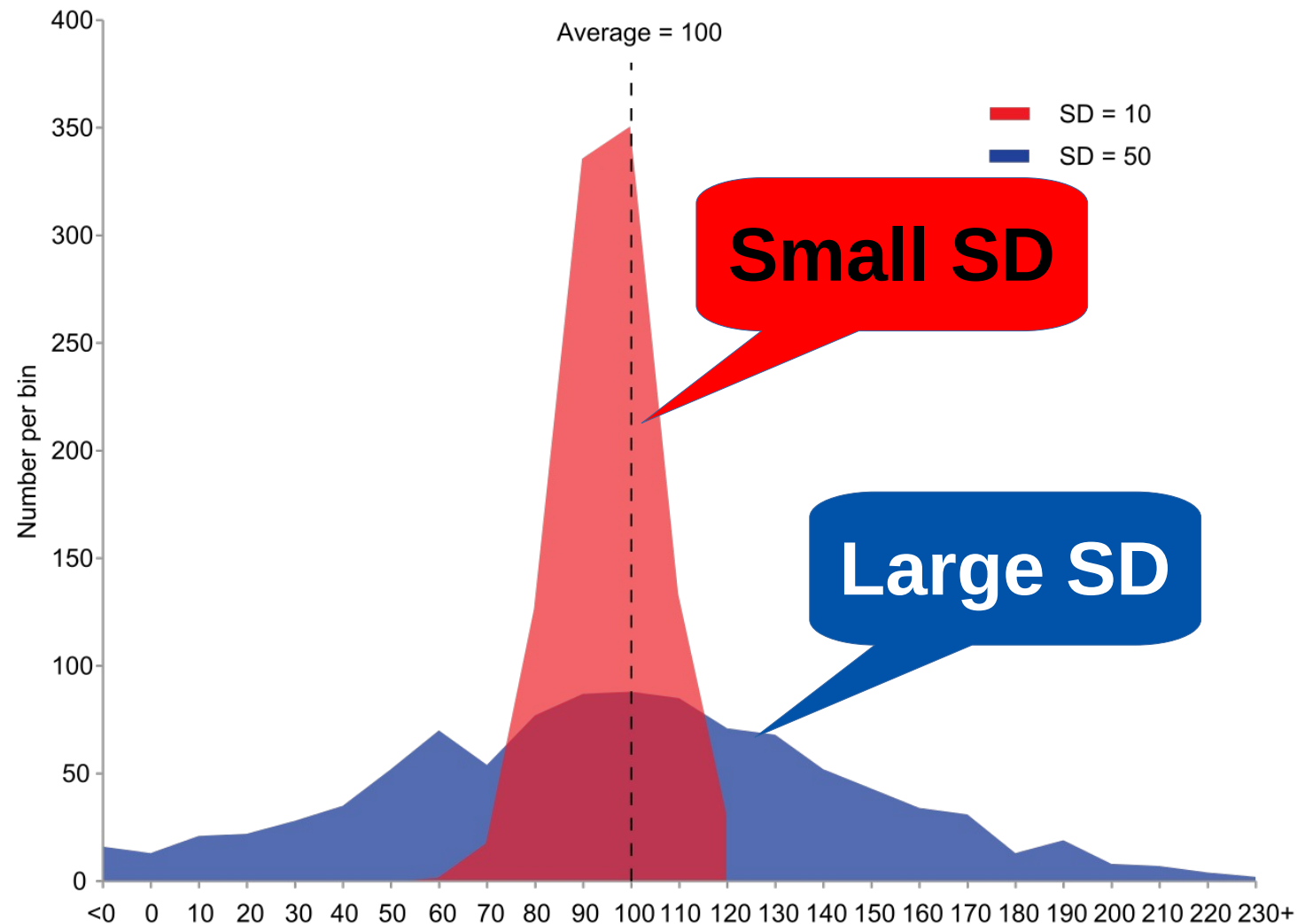
Standard Deviation

- A quantity calculated to indicate the extent of deviation for a group as a whole.

- The general average distance between each point and the mean

Standard Deviation

- Equal to the square root of variance(X)
  - $\mathrm{sqrt}(\mathrm{var}(X))$, $\mathrm{sd}(X)$

- A measure that is used to quantify the amount of variation or dispersion of a set of data values.

- A **low standard deviation** indicates that the data points tend to be **close to the mean** (also called the expected value) of the set

- A **high standard deviation** indicates that the data points are **spread out** over a wider range of values.
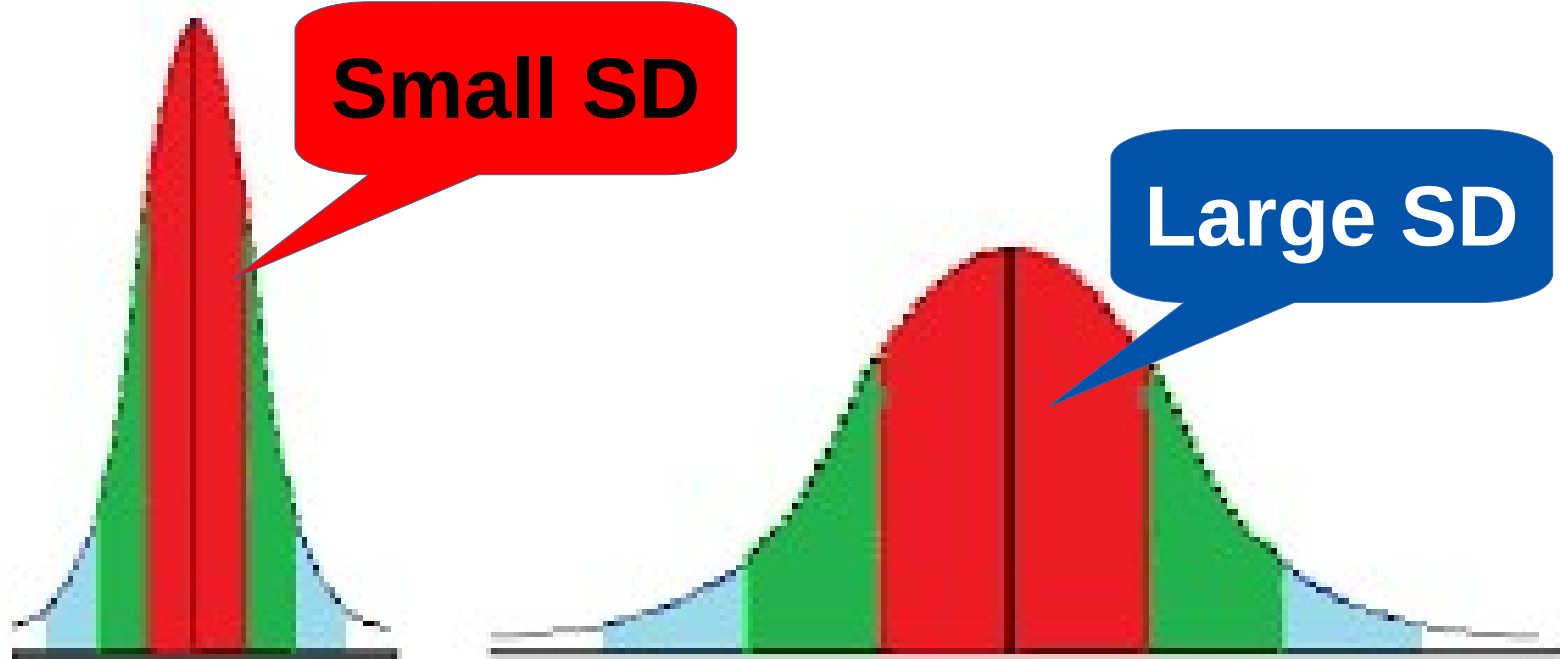
Histograms for IQ Test Components
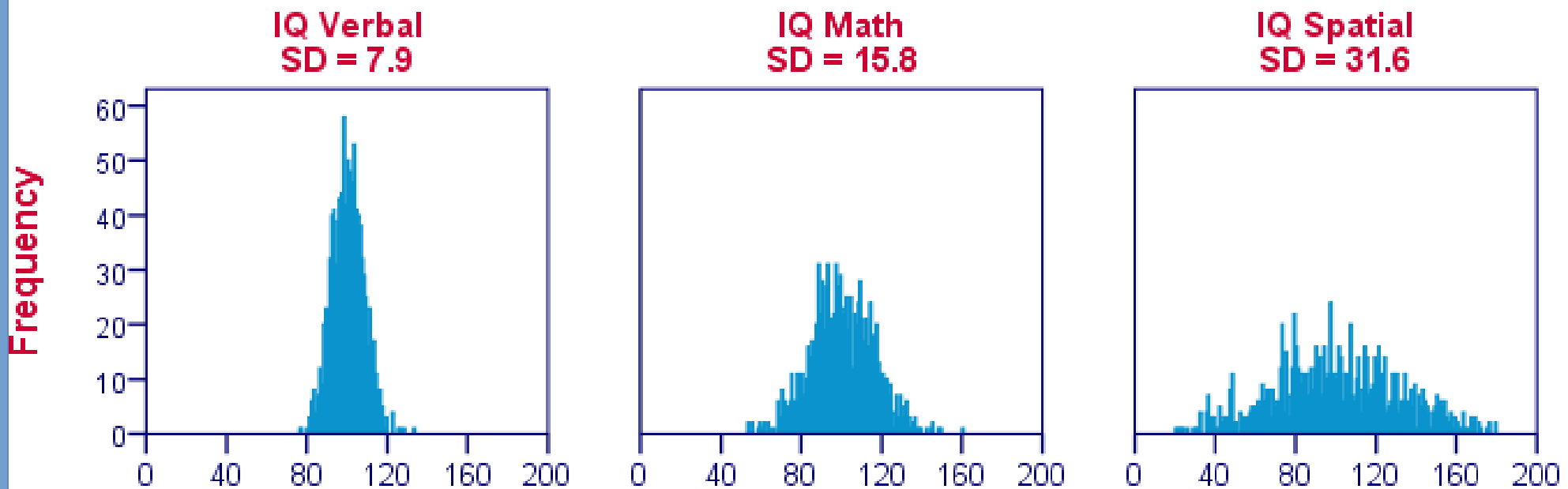
Putting Things Together:
Find Some Basic Stats

```r
library(tidyverse)

library(tibble)

library(dplyr) # and load tidyverse too!

data_people <- tibble::tribble(
  ~EyeColour, ~Height, ~Weight, ~Age,
  "Blue",          1.8, 110L, 18L,
  "Brown",         1.9, 150L, 34L,
  "Blue",          1.7, 207L, 28L,
  "Brown",         1.9, 170L, 21L,
  "Blue",          1.9, 164L, 29L,
  "Brown",         1.9, 183L, 31L,
  "Brown",         1.9, 175L, 20L,
  "Blue",          1.9, 202L, 27L
)
```

File: bmi.r

Calculate the Body Mass Index (BMI)

```
# Find the average BMI of people with blue eyes using piping

# Note: BMI = (height / (weight * weight))

data_people %>% select(EyeColour, Height, Weight) %>%
filter(EyeColour=="Blue") %>% mutate(BMI = Weight / Height^2)
%>% summary(averageBMI == mean(BMI))
```

```
  EyeColour           Height          Weight            BMI
Length:4          Min.   :1.700   Min.    :110.0   Min.    :33.95
Class :character  1st Qu.:1.775   1st Qu.:150.5    1st Qu.:42.56
Mode  :character  Median :1.850   Median :183.0    Median :50.69
                  Mean   :1.825   Mean    :170.8   Mean    :51.74
                  3rd Qu.:1.900   3rd Qu.:203.2    3rd Qu.:59.87
                  Max.   :1.900   Max.    :207.0   Max.    :71.63
```
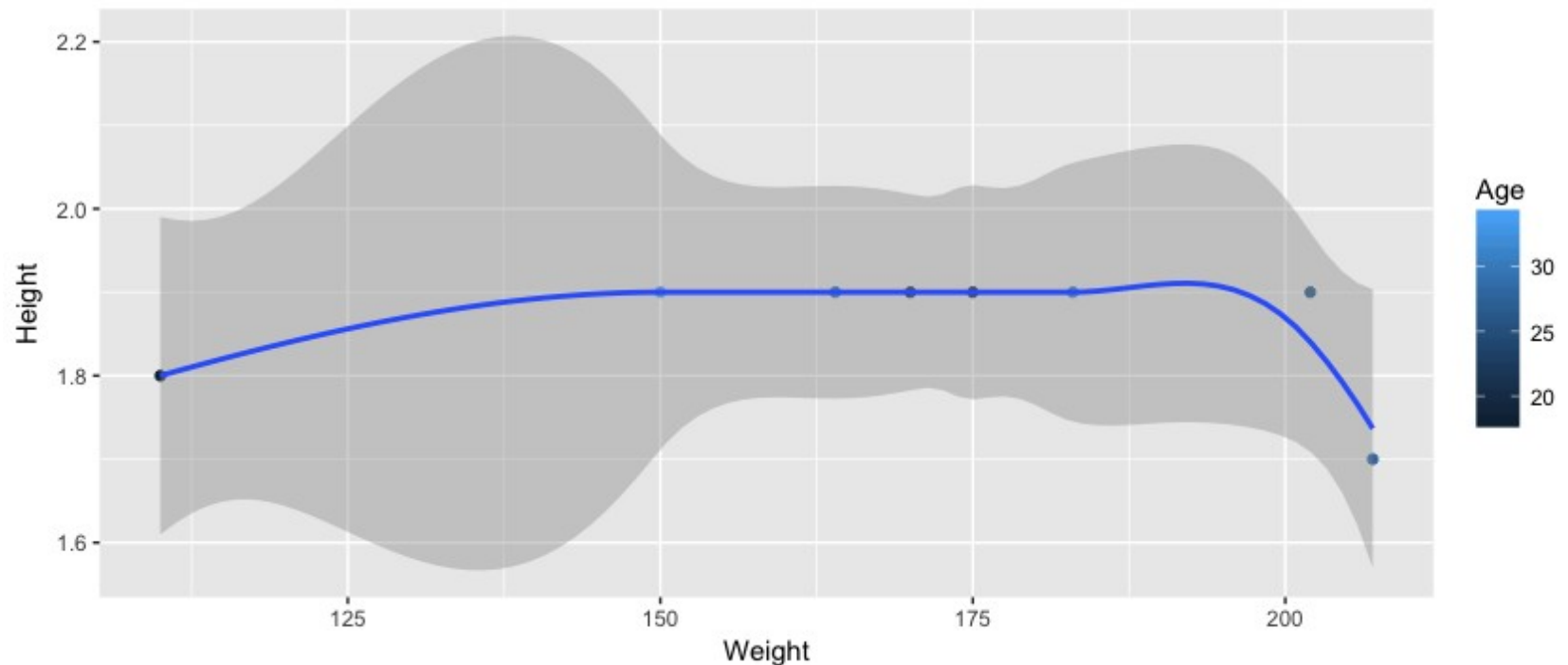
The actual data

data_people %>% select(EyeColour, Height, Weight) %>%
filter(EyeColour=="Blue") %>% mutate(BMI = Weight / Height^2)

ggplot()

```
ggplot(data = data_people) +
geom_point(mapping = aes(y = Height, x = Weight, color = Age )) +
geom_smooth(mapping = aes(y = Height, x = Weight))

# Try playing with the settings!!
```

More With ggplot()!

```
ggplot(data = data_people) + geom_point(mapping = aes(y =
Height, x = Weight, color = Age )) + geom_smooth(mapping =
aes(y = Height, x = Weight )) + facet_wrap(~EyeColour)
```
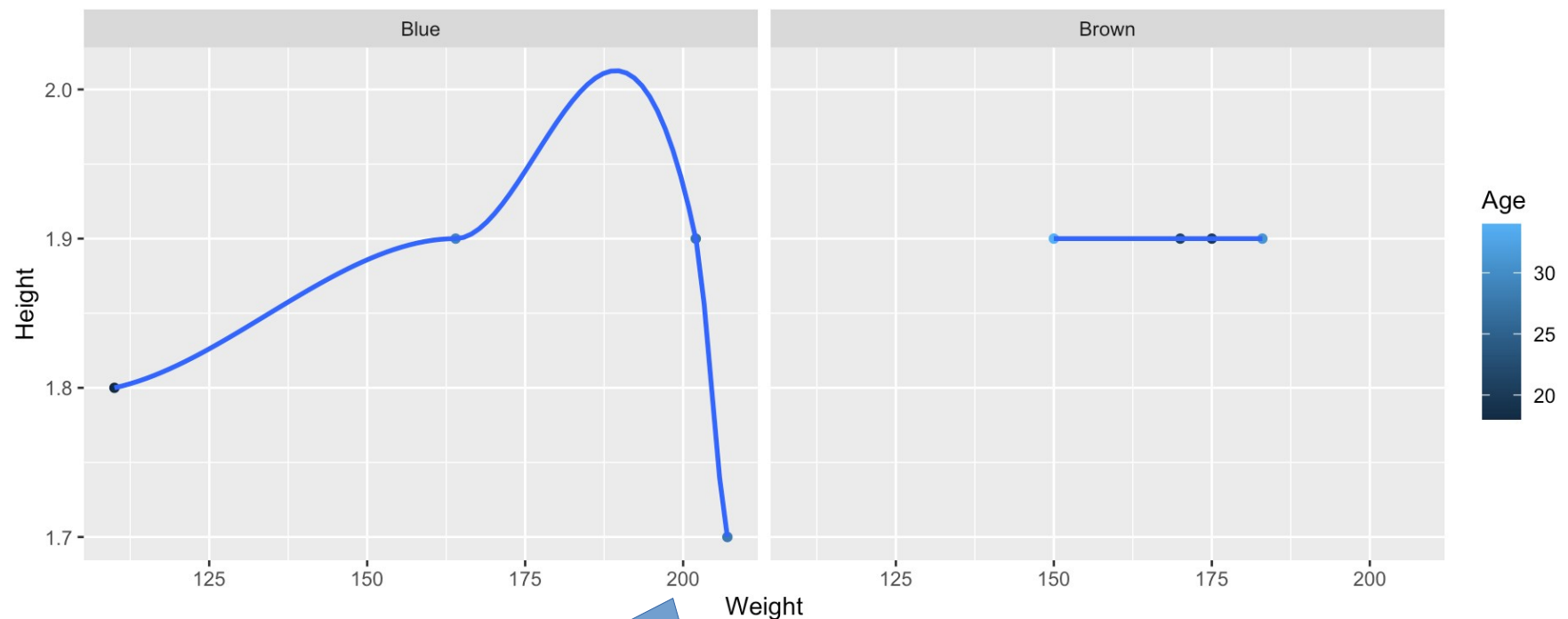

Why does the data indicate this drop?

Basic Stats: Working With *p*-values

- Suppose: We are the producers of two kinds of drinks: green and purple. Each drink comes in a bottle and we would like to know whether the green and the purple drinks are filled to the same levels.

- We randomly select 9 bottles from our entire set of 100000 bottles

Comparing Populations

- By inspection,
  - **Purple bottles seem a little *under-filled***
  - **Green bottles seem a little *over-filled***

- **Q: Can we use a statistical test to conclude whether the whole batch is under- or over-filled?**

Hypothesis Testing

- **Is there a statistically significant *difference* between the two groups in terms of the average extent to which the bottles are filled?**
  - **Null hypothesis (Ho)**: The bottles are filled at the same levels. (*Nothing unusual is happening.)*
  - **Alternative hypothesis (Ha)**: There is a difference between the levels of drink in the bottles.
- Remember: we have a sample of ***only nine bottles from the super set of 100000 bottles***.
- Statistics is used to extrapolate our findings from the small set to the larger set of bottles.
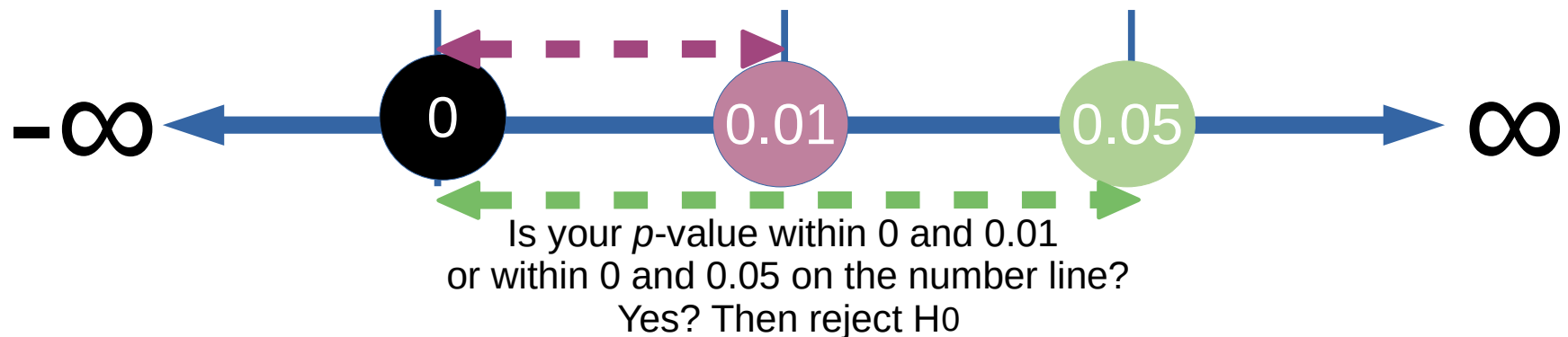
Is Our Sample Telling the Truth?

- We admit that our sample-selection may not necessarily represent our larger stock of bottles

- The selected sample may still show that the green and purple bottles have been filled differently (*by accident*) and not represent the larger set of all bottles.

Accidentally selected

Use *p*-Values

- The p-Value says that we are sure that our randomly selected sample size is a good representation of our larger super set.

- We use a 95 per cent confidence interval range: Our selected bottles fit within (i.e., *represent*) 95 % of the entire set. This means that our selected sample is still a good representation of the entire set of 100000 bottles.

- **Reject the Null Hypothesis (H_0) when p < 0.05** (when *p* is close to zero)

- Rejecting H_0 means that something *non-random* is happening.

$$-\infty \qquad 0 \qquad 0.01 \qquad 0.05 \qquad \infty$$

Is your *p*-value within 0 and 0.01
or within 0 and 0.05 on the number line?
Yes? Then reject H_0

Data for the T-Test

```
data_drinks <- tibble::tribble(
  ~Observation, ~Colour, ~percentFull,
  1,"Green", 70,
  2,"Purple",30,
  3,"Green",50,
  4,"Purple",20,
  5,"Purple",15,
  6,"Green",90,
  7,"Purple",40,
  8,"Green",60,
  9,"Purple",15)
```

Code for the T-Test

```
data_drinks <- data_drinks %>%
    select(Colour, percentFull) #lose obs. num
# Run the t-test: a comparison of means.
t.test(data = data_drinks, percentFull ~ Colour)
```

 # Check the p-value:

- **# If p-val <= alpha = 0.05: reject H$_0$.**
- **# If p-val > alpha = 0.05: do not reject H$_0$.**

- **What do we conclude about our *data_drinks*?**

Automate Your Analysis
With a Function

```
myOut <- t.test(data = data_drinks, percentFull ~ Colour)

myOut$p.value

rejectOrWhat <- function(pValue){

  if(pValue >= 0.05){

    print("Accept Null Hypothesis: nothing happening")

  }

  else{

    print("Reject Null Hypotheis: something is going
on...")

  }}

rejectOrWhat(myOut$p.value)
```

```
#If p-val <= alpha = 0.05: reject H0.

#If p-val > alpha = 0.05: do not reject H0.
```

R's Built-In Data
Our Built in Data

R studio (R statistics) has plenty of included data-sets for practicing t-tests work.

```
# find sets

data()
```

```
Data sets in package 'datasets':

AirPassengers                    Monthly Airline Passenger Numbers 1949-1960
BJsales                          Sales Data with Leading Indicator
BJsales.lead (BJsales)           Sales Data with Leading Indicator
BOD                              Biochemical Oxygen Demand
CO2                              Carbon Dioxide Uptake in Grass Plants
ChickWeight                      Weight versus age of chicks on different diets
DNase                            Elisa assay of DNase
EuStockMarkets                   Daily Closing Prices of Major European Stock Indices,
                                 1991-1998
Formaldehyde                     Determination of Formaldehyde
HairEyeColor                     Hair and Eye Color of Statistics Students
Harman23.cor                     Harman Example 2.3
```