

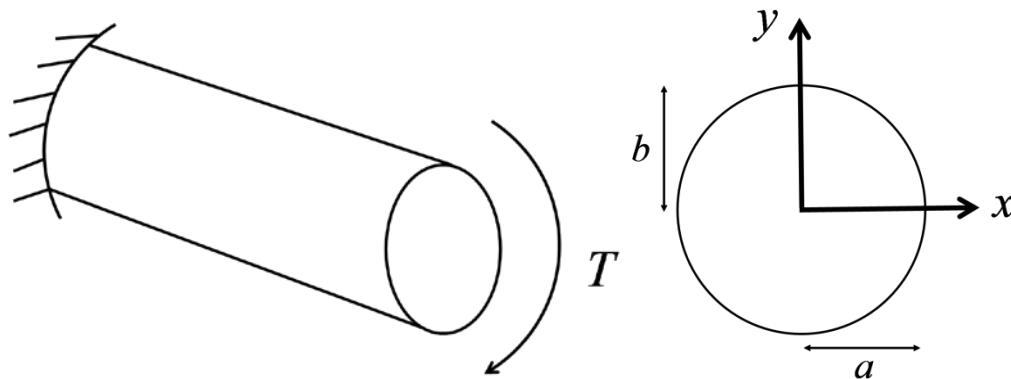
## 113-2 Finite Element Method 期末報告(個人)

The Prandtl theory of a cylindrical member leads to

$$\begin{aligned} \text{GE:} \quad & -\nabla^2 u = 2G\theta \quad \text{in } \Omega \\ \text{BCs:} \quad & u = 0 \quad \text{on } \Gamma \end{aligned}$$

where  $\Omega$  is the cross-section of the cylindrical member being twisted,  $\Gamma$  is the boundary of  $\Omega$ .  $G$  is the shear modulus of material of the member,  $\theta$  is the angle of twist, and  $u$  is the stress function. Solve the equation for the case in which  $\Omega$  is a circular section using the mesh of linear triangular elements. Compare the finite element solution with the exact solution given by

$$u(x, y) = G\theta \left( \frac{a^2 b^2}{a^2 + b^2} \right) \left( 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right), \text{ in which } a=1, b=1 \text{ and } G\theta=5.$$



Hint:

Suggest you to develop a small program to estimate the element stiffness matrix and forcing vector, and then following the standard FEM process to obtain the FEM solution.

網格方式:

1. 依照課堂中的方法，四個三角形線性元素。
2. 自行選擇切割方法，四個三角形線性元素。
3. 超過五個元素。(可用電腦輔助計算)
4. 超過十個元素，越多越好。(可用電腦輔助計算)(期末加分題)