Problem Set 4

Due date: 16 October

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Please upload your completed assignment to the ELMs course site (under the assignments menu). Remember to include an annotated script file for all work with R and show your math for all other problems (if applicable, or necessary). Please also upload your completed assignment to the Github repository that you have shared with us. *We should be able to run your script with no errors.*

Total points: 30

Question 1

Points: 5

Suppose I am playing poker with a regular deck of 52 cards. I have three cards in my hand, one of which is a king. What is the probability that the next two cards I draw will both be kings?

Ans 1:

Probability that the next two cards are kings

Probability that the next card drawn is a king = 3/49

Probability that the card after that is also a king = 2/48.

Hence the joint probability is 3/49 * 2/48 = 0.002 or 0.2% chance

Question 2

Points: 5

A survey of 1,500 randomly selected individuals reveals that 46% of them have a college degree and that 31% of them have a gun in their house. If you were to assume that college graduation and gun ownership are independent of one another, what is the probability that an individual is both a college graduate and does not have a gun in his or her house?

Ans:2

College Degree = 1,500 *46/100 = 690 individuals

Has a gun = 1,500 * 31/100 = 465 individuals

Probability that I have a college degree and do not have a gun will be:

P(Has college Degree) * P(Does not have a gun)

= 46/100 * 69/100

= 0.3174 or 31.74% given that they are independent.

Question 3

Points: 5

I roll a fair six-sided dice and I get a number greater than three. What is the probability that the next two dice rolls will also generate numbers greater than three?

Ans 3:

Each roll will have a 0.5 chance of becoming greater than three, or the first roll will be greater than three and the second will be greater than three.

Hence total probability for two rolls will be 0.5*0.5 = 0.25 or 25% chance

Question 4

Points: 10

Assume that in the United States, 15% of people are smokers and, of smokers, 60% smoke at least one pack a day. Among those who smoke at least a pack a day, 60% will develop lung cancer at some point in their lives while, in the rest of the population (which includes smokers who smoke less than a pack a day and non-smokers), only 10% will. What is the probability that a person who develops lung cancer was a smoker who smoked at least a pack a day?

Ans 4:

Defining some things:

A = Person that smokes more than one pack a day

B = Person has cancer

Hence we need to find P(A|B) or the probability of smoking a pack a day given that the person has developed cancer.

We can define

P(A) = 0.15 * 0.6 = 0.09 [Probability of a smoker and smoking more than a pack]

P(B|Ac) = 0.1 [Total Probability of getting cancer when the person is not a smoker nor a heavy smoker]

P(B|A) = 0.6 [Probability that you get cancer if you are a heavy smoker]

P(Ac) = Probability the person is not a heavy smoker

= Probability the person is not a smoker or the person smokes less than a pack a day.

= (0.15 *0.4) + 0.85

= 0.91

We need to find P(A|B);

P(A|B) = P(B|A) * P(A)/ P(B) [Bayes theorm]

= P(B|A) * P(A)/ (P(B|A) * P(A) + P(B|Ac) * P(Ac)) [Law of multiplication]

= 0.6 * 0.09 / 0.6 * 0.09 + 0.1 * 0.91

= 0.054/0.145

= 0.372

Or there is a 37.2% chance that a person who develops lung cancer was a smoker who smoked more than one pack.

Question 5

Points: 5

Assume that in the world, 60% of countries are democracies and 80% of countries have ratified the Rome Statute for the International Criminal Court. Assume further that 90% of democracies have ratified the Rome Accords. What is the probability that a country is either a democracy or has ratified the Rome Accords?

Ans 5:

P(D) -> 0.60

P(RS) -> 0.80

P(D ( ) RS)/P(D) = 0.9

=> Probability that it is either a democracy or has ratified the accords = P(D) + P(RS) - P(D ( ) RS)

= 0.6 + 0.8 - (0.9*0.6)

= 0.86 or 86% chance that a country is a democracy or has ratified the Rome accords.