Some Practical Exercises for System Identification

Fall 2018

Introduction

The objective of the computer exercise sessions is to implement different algorithms learned during the lectures and make the student familiar with MATLAB and its system identification toolbox. Twelve computer exercise sessions are planned. The groups of two students should prepare two reports (one report per six sessions) and submit them in the *moodle* of the course in due time. The reports will be graded with 15 points each. The written exam, which will be held on 18th of December, includes some questions about the computer exercises (40 points in total). Only the students are capable to answer theses questions that worked on the computer exercises and understood them.

1 CE-1 : Nonparametric Methods

In order to solve the following exercises, you will require Matlab2014 or later. Make sure your version is up to date. During the first session of the exercises you will create a Simulink model of a fourth order transfer function, which is used for all simulations in CE-1.

1.1 Step response

1. Create a new Simulink model. Create a fourth order `Transfer Function` block with the following parameters :

$$G(s) = \frac{1}{s^4 + 0.4s^3 + 4.3s^2 + 0.85s + 1}$$

2. Define the sample time $T_e = 0.5$ s (Is this a reasonable choice ?).
3. Simulate measurement noise by creating a `Random Number` block with a variance of 0.01 and a sample time of T_e and adding it to the output of the transfer function block.

4. Create an input `Saturation` block with an upper limit of 0.5 and a lower limit of -0.5.
5. Finish the block diagram by creating a `From Workspace` source and a `To Workspace` sink to exchange data with the workspace. Set the sample time of the From Workspace and To Workspace blocks to T_e.

6. Apply a step with a magnitude equal to the upper saturation limit at the input and plot the response of the system. To do this, create an M-file. First we need to generate the input signal. Create a struct `simin` with the following fields :
   – `simin.signals.values` : A column vector representing the input signal (i.e. the step)
   – `simin.time` : The time vector corresponding to the input signal

   The time vector should have a length of 100 s and the step should occur after 1 s. Note that the simulation time should be at least 100 s (in the simulation parameters of Simulink).
7. Use the command `sim` to run the Simulink model with the above input signal and plot the output of the system.
8. In the same way compute the impulse response of the system.

1.2 Auto Correlation of a PRBS signal

1. Download the file `prbs.m` from the Moodle page of the course. The function `prbs(n,p)` generates a PRBS of `p` periods with an `n`-bit shift register.
2. Write a function for Matlab `[R,h] = intcor(u,y)` that computes $R_{uy}(h)$ for the periodic signals.
3. Check your function by computing the autocorrelation of a PRBS signal.

1.3 Impulse response by deconvolution method

The objective is to compute the impulse response of the system in simulink using the numerical deconvolution method. Note that the simulation time should be less than 250 s and the random signal should not exceed the saturation values. For this purpose,

1. Generate a random signal (see `help rand`) with a sampling time of $T_e = 0.5$s as the input to the model.
2. Use `toeplitz` command to construct the input matrix.
3. Generate a time vector for simulink : `t=0:Te:(N-1)*Te`, where N is the length of your input signal.
4. Compute the impulse response using the measured output and the matrix of the input signal.
5. Compare your results with the true impulse response of the discrete-time system obtained by `zoh` option for transformation (see `tf, c2d, impulse`).

1.4 Impulse response by correlation approach

1. Generate an input sequence `Uprbs` using the `prbs` command (with $p = 4$ and $n = 7$) and apply it to your system. Set the sample time to $T_e = 0.5$ s.
2. Using your `intcor` function that you have already developed, compute the impulse response of the discrete-time system $g(k)$ using the correlation approach.
3. Compute the impulse response using `xcorr` function of Matlab.
4. Compare your results (using `xcorr` and `intcor`) with the true impulse response of the discrete-time system.

1.5 Frequency domain Identification (Periodic signal)

Aim : Use the Fourier analysis method to identify the frequency response of a model excited by a PRBS signal.

1. Choose a PRBS signal with a length around $N = 2000$ and a sampling period of $T_e = 0.5$s. Apply the generated PRBS to the Simulink model in 1.1.
2. Compute the Fourier transform of the input and output signal (use `fft`) for each period and use the average.
3. Compute a frequency vector associated to the computed values (Slide 30, Chap. 2).
4. Generate a frequency-domain model in Matlab using the `frd` command.
5. Plot the Bode diagram of the identified model and compare it with the true one.

1.6 Frequency domain Identification (Random signal)

Aim : Use the spectral analysis method to identify the frequency response of a model excited by a random signal.

1. Apply a random signal of length $N = 2000$ to the system. Discuss the characteristics of the random signal (Gaussian or uniform, binary or multi-level) to obtain the highest energy of the excitation signal.
2. Compute the frequency-response of the model using the spectral analysis method.
3. Use a Hann or Hamming window to improve the results.
4. Cut the data to m groups and try to improve the results by averaging.
5. Plot the Bode diagrams of different methods and compare them.

2 CE-2 : Parametric Identification Methods

The objective of this exercise is to practice the parametric identification methods. The real data of two different systems are considered. In the first part a simple model for a laser beam is obtained with different methods. In the second part the complete identification (order estimation, parametric identification and model validation) of a mechatronic system is considered.

2.1 Identification of a laser beam stabilizing system

The Laser Beam Stabilization device consists of a low-power stationary laser beam source pointing at a single-axis, fast steering mirror. The reflected beam is picked up by a high-resolution position sensing detector (PSD). The PSD measures the relative displacement of the beam from the nominal position and the mirror mechanism is actuated using a high-bandwidth voice coil.

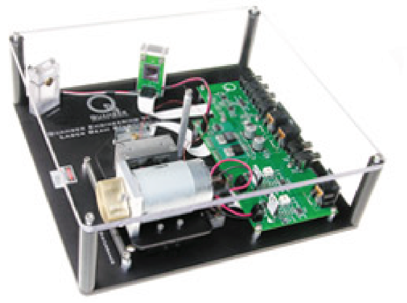


Figure 1. Laser beam stabilizing system

Since the open-loop system contains an integrator, a proportional controller is used to stabilize the system. The objective is to identify the whole closed-loop system. The excitation signal u is a PRBS and the beam position y is measured with a sampling frequency of 1 kHz. The input and output signal are saved in `laserbeamdataN.mat`. This file can be downloaded from the moodle of the course.

2.1.1 FIR model identification

Assume an FIR model for the data with $m = 50$ parameters :

$$\hat{y}(k,\theta) = b_1 u(k-1) + \cdots + b_m u(k-m)$$

1. Compute the vector of parameters $\hat{\theta}^T = [b_1 \ \dots \ b_m]$ using the least squares algorithm. Note that the matrix Φ is a Toeplitz matrix.
2. Compute the predicted output of the identified model, $\hat{y}(k,\hat{\theta})$. Plot the measured output $y(k)$ and the predicted output in the same figure and compute the loss function $J(\hat{\theta})$.
3. Compute the covariance of the parameter estimates assuming that the noise is white. Plot the finite impulse response of the system (the vector θ) together with $\pm 2\sigma$ confidence interval (use `errorbar`).

2.1.2 ARX model identification

Assume a second order ARX model for the data with the following predictor :

$$\hat{y}(k,\theta) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2)$$

1. Compute the vector of the parameters $\hat{\theta}^T = [a_1\ ,\ a_2\ ,\ b_1\ ,\ b_2]$ using the least squares algorithm.
2. Compute the predicted output of the identified model, $\hat{y}(k,\hat{\theta})$. Plot the measured output $y(k)$ and the predicted output in the same figure and compute the loss function $J(\hat{\theta})$.
3. Compute the output of the identified model, $y_m(k)$, for the input signal, $u(k)$, using `lsim`. Plot the measured output $y(k)$ and $y_m(k)$ in the same figure and compute the sum of squares of the error.
4. Try to improve the results using the Instrumental Variable method (note that $y_m(k)$ is a noiseless estimate of $y(k)$). Compare the result with that of ARX model.

2.1.3 State-space model identification

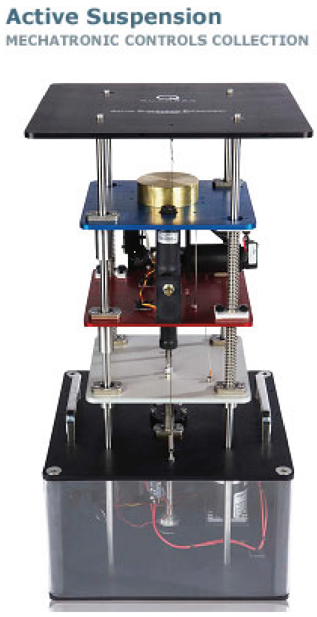
The objective of this part is to identify a sate-space model for the laser beam stabilizing system using the subspace method based on the extended observability matrix.

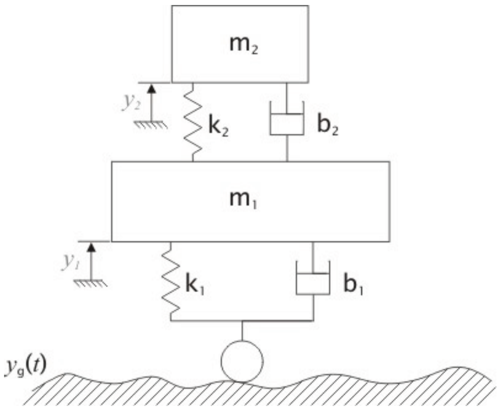
1. Construct the matrix $\boldsymbol{Y}$ and $\boldsymbol{U}$ based on the measured data (see Eq. 3.69 and 3.70 in the course-notes) and compute $Q = \boldsymbol{Y}\boldsymbol{U}^{\perp}$.
2. Compute the singular values of Q and conclude the number of states n (use `svd`). Compute the observability matrix O_r (the first n columns of Q).
3. Compute the matrix A and C from the observability matrix.
4. Assume that $D = 0$ and estimate B using the least squares algorithm.
5. Compute the output of the identified model, $y_m(k)$ for the input signal $u(k)$ using `lsim`. Plot the measured output $y(k)$ and $y_m(k)$ in the same figure and compute the the sum of squares of the error.

2.2 Parametric Identification of an Active Suspension System

The objective of this part is to identify and validate a parametric black-box model for an active suspension system. In the first part the order of the system is estimated using the data and then different parametric models are identified and validated.

The active suspension system consists of two masses, each supported by a spring as shown in Figure 2. The upper mass (blue, sprung mass) represents the vehicle body supported above the suspension while the second mass (red, unsprung mass) corresponds to one of the vehicle's wheels. The upper mass is controlled through a DC motor. The lower plate (silver) simulates the road surface by moving vertically. Ride Comfort is related to vehicle body motion sensed by the passengers. A measure for the Ride Comfort can be the acceleration of the sprung mass in the quarter car model.





(a) (b)

Figure 2 : (a) Photo of active suspension system. (b) Active suspension system modeled as a double mass-spring-damper

We try to identify a parametric model between the current reference for the DC motor (which is proportional to its mechanical torque) and the acceleration of the sprung mass. In this experiment the silver plate is fixed (no road disturbance).

Download the file `CE2.mat` that contains the experimental data from an active suspension system containing the input signal `u` and the sampled output signal `y` with a sampling period ($T_e = 0.03$ s). Use the command `iddata` to generate a data object that can be used for the System Identification Toolbox. Remove the mean value of the data using the command `detrend`.

Remark : Matlab uses the following notation :

$$A(q^{-1})y(t) = \frac{B(q^{-1})}{F(q^{-1})}u(t-n_k) + \frac{C(q^{-1})}{D(q^{-1})}e(t)$$

where :

$$\begin{aligned} A(q^{-1}) &= 1 + a_1q^{-1} + \cdots + a_{n_a}q^{-n_a} & B(q^{-1}) &= b_1 + b_2q^{-1} + \cdots + b_{n_b}q^{-n_b+1} \\ C(q^{-1}) &= 1 + c_1q^{-1} + \cdots + c_{n_c}q^{-n_c} & D(q^{-1}) &= 1 + d_1q^{-1} + \cdots + d_{n_d}q^{-n_d} \\ F(q^{-1}) &= 1 + f_1q^{-1} + \cdots + f_{n_f}q^{-n_f} \end{aligned}$$

For ARX structure we have $n_c = n_d = n_f = 0$, for ARMAX $n_d = n_f = 0$, for OE : $n_a = n_c = n_d = 0$ and for BJ : $n_a = 0$.

2.2.1 Order estimation

1. Using the `arx` command, identify 10 models of orders from 1 to 10 ($n_k = 1, n_a = n_b = n, n = 1, \ldots, 10$) and plot the "loss function" of the models versus the model order (loss function is available for the model `M` in the following variable : `M.EstimationInfo.LossFcn`). Estimate the order of the system from this curve.
2. Validate the chosen order by checking the zero/pole cancellation approach using several models identified by the ARMAX structure ($n_k = 1, n_a = n_b = n_c = n, n = n_{\min}, \ldots, n_{\max}$). Use `h=iopzplot(M)` to plot the zeros and poles of M and `showConfidence(h,2)` to plot their confidence intervals ($\pm 2\sigma$).
3. Estimate the delay n_k by inspecting the coefficients of $B(q^{-1})$ and their standard deviations (use `M.b` and `M.db`. Compute the number of parameters in the numerator ($1 \leq n_b \leq n - n_k + 1$).
4. Compare your results with those proposed by `struc, arxstruc, selstruc`.

2.2.2 Parametric identification

1. Divide the data into two parts and use one part for identification and the other part for validation.
2. Using the estimated values of n_a, n_b and n_k, identify different parametric models by the following methods : `arx, iv4, armax, oe, bj, n4sid`. For the structures with noise model take $n_c = n_d = n_a$. For the state-space model choose number of states equal to the global order n.

2.2.3 Model validation

1. Compare the output of the identified models with the measured output using the command `compare`. What is the best model ?
2. Compare the frequency response of the models with a nonparametric frequency-domain identified model. What is the best model ?
3. Validate the identified models by the whiteness test of the residuals as well as the cross-correlation of the residuals and past inputs using the command `resid`. Which models are validated ?