The Model-Aware Node in Heterogeneous Networks

--- A supplementary document to a submitted conference paper

This is a supplementary document to our paper "***Carrier-Sense Multiple Access for Heterogeneous Wireless Networks Using Deep Reinforcement Learning***". This document derives the optimal sum throughputs that can be attained by a model-aware node when it co-exists with TDMA nodes (the nodes that adopt TDMA protocol) and ALOHA nodes (the nodes that ALOHA protocols).

We assume different MACs may have different slot granularities. The smallest is the basic slot used by the model-aware node to perform carrier sensing or to transmit packets. TDMA slot and ALOHA slots consists of multiple basic slots and are used by TDMA nodes and ALOHA nodes to transmit packets, respectively (i.e., a TDMA/ALOHA packet lasts a duration of a TDMA/ALOHA slot). We denote the ratio of TDMA slot and ALOHA slot to the basic slot by R_T and R_A.

Table 1 summarizes the MAC mechanism of TDMA and different variants of ALOHA. The model-aware node knows the MAC mechanisms of co-existing nodes as well as the number of nodes executing each MAC protocol. For example, for co-existence with TDMA and ALOHA, the model-aware node knows the time slots during which TDMA nodes transmit and the random access mechanism of the ALOHA nodes, as well as the number of TDMA nodes and the number of ALOHA nodes. The model-aware node executes an optimal MAC that maximizes the sum throughput based on this know

TABLE I: MAC mechanisms of different nodes.

Node Type	Description
TDMA	*A TDMA node transmits in X specific TDMA slots within a TDMA frame of Y TDMA slots in a repetitive manner from frame to frame.*
***q*-ALOHA**	*A q-ALOHA node transmits with a fixed probability q in each ALOHA slot in an i.i.d. manner from ALOHA slot to ALOHA slot.*
Fixed-window ALOHA	*A fixed-window ALOHA (FW-ALOHA) node generates a random counter value $w \in [0, W-1]$ after it transmits in an ALOHA slot. It then waits for w ALOHA slots before its next transmission. The parameter W is referred to as the window size.*
Exponential-backoff ALOHA	*Exponential backoff ALOHA (EB-ALOHA) is a variant of FW-ALOHA that uses a binary exponential backoff mechanism, in which the window size is doubled each time its transmission incurs a collision up to a maximum window size of $2^m W$, where m is the "maximum backoff stage". Upon a successful transmission, the window size reverts to the initial window size W.*

We now derive the optimal results used in our paper.

Co-existence with a *q*-ALOHA node

We first consider the co-existence of a model-aware node with one *q*-ALOHA node. The optimal MAC of the model-aware node depends on the value of the packet length R_A and the transmission probability q of the *q*-ALOHA node. Two possible strategies, one of which is the optimal strategy, are given as follows:

1) **Greedy strategy**: the model-aware node always transmits. The sum throughput is $1-q$.
2) **Polite strategy**: the model-aware node listens to the channel in the first slot of each ALOHA slot. If the channel is busy, the model-aware node will not transmit in the following R_A-1 slots; if the channel is idle, the model-aware node will transmit in the next R_A-1 slots. In this case, the sum throughput is $\left(1-q\right)\frac{R-1}{R}+q$.

By comparing $1-q$ and $(1-q)\dfrac{R-1}{R}+q$, we can conclude that when $q<1/(R+1)$, the greedy strategy is optimal; when $q=1/(R+1)$, the greedy strategy and the polite strategy achieve the same sum throughput; when $q>1/(R+1)$, the polite strategy is optimal.

It is straightforward to extend the number of q-ALOHA nodes to be arbitrary. Suppose that the number of q-ALOHA node is N and the sum throughput achieved by the greedy strategy and the polite strategy will be $(1-q)^N$ and $(1-q)^N\dfrac{R-1}{R}+Nq(1-q)^{N-1}$, respectively. From there, we can conclude that when $q<1/(NR+1)$, the greedy strategy is optimal; when $q=1/(NR+1)$, the greedy strategy and the polite strategy achieve the same sum throughput; when $q>1/(NR+1)$, the polite strategy is optimal.

Co-existence with a FW-ALOHA node

We next consider the co-existence of a model-aware node and a FW-ALOHA node. The model-aware node knows the transmission scheme of the FW-ALOHA node, i.e., it knows the FW-ALOHA node will generate a random backoff counter after each transmission. Also, the model-aware node knows the value of W and whether the FW-ALOHA node transmitted in the past slots by listening to the channel. A subtlety here is that the model-aware node does not know the exact value c of the FW-ALOAH node.

We can derive the optimal strategy of the model-aware node as follows. During two adjacent transmissions of FW-ALOHA node, the model-aware node adopts the greedy strategy in the first x ALOHA slots (after a transmission by the ALOHA node) and adopts the polite strategy after x ALOHA-slots, where x is a parameter to be optimized on to achieve the maximum sum throughput of the system. For generality, we let $x\in[0,W]$ (when $x=W$, the model-aware not is fully greedy; when $x=0$, the model-aware node is fully polite). Given an x, we can use a renewal process to analyze the strategy.

We define one round to be the number of ALOHA-slots between the end of a transmission by the ALOHA node and the end of the next transmission by the ALOHA node (i.e., there is an ALOHA transmission per round). We can calculate the expected total number of ALOHA slots, the successful ALOHA slots of the model-aware node, and the successful ALOHA slots of the FW-ALOHA node, per round as follows:

$$E[\text{number of total ALOHA slots in one round}]=\frac{1}{W}\left[1+2+\cdots+(W-1)+W\right](*)=\frac{W+1}{2}(1)$$

$$\begin{aligned}&E[\text{number of model-aware successful ALOHA slots in one round}]\\&=\frac{1}{W}\left[0+1+2+\cdots+(x-1)+x+\left(x+\frac{R-1}{R}\right)+\left(x+2\bullet\frac{R-1}{R}\right)+\cdots+\left(x+(W-(x+1))\frac{R-1}{R}\right)\right](**)\\&=\frac{1}{W}\left[\frac{x(x-1)}{2}+(W-x)x+\frac{(W-x)(W-x-1)}{2}\bullet\frac{R-1}{R}\right](2)\end{aligned}$$

$$E[\text{number of FW-ALOHA successful ALOHA slots in one round}]=\frac{1}{W}\underbrace{[0+0+\ldots+0+1+1+\cdots 1]}_{\text{number of 0 is x, number of 1 is W-x}}(***)=\frac{W-x}{W}(3)$$

Note that in $(*)(**)(***)$, the addends correspond to the number of model-aware node's successful ALOHA slots, the number of FW-ALOHA's successful ALOHA slots and the number of total ALOHA slots for $c = 0,1,2,\ldots,W-1$, respectively. Also, we point that when $c < x$ the number of model-aware node's successful ALOHA slots is c and the number of FW-ALOHA's successful ALOHS slots is 0; when $c \ge x$, the number of model-aware node's successful ALOHA slots is $x + (c-x)\dfrac{R-1}{R}$ c and the number of FW-ALOHA's successful ALOHS slots is 1.

Therefore, the throughput of the model-aware node is $(2)/(1)$, the throughput of the FW-ALOHA node is $(3)/(1)$, and the sum throughput is $(2)/(1)+(3)/(1)$.

Now we derive the value x that maximizes the sum throughput.

$$
\begin{aligned}
&(2)/(1)+(3)/(1)\\
&=\left\{\frac{1}{W}\left[\frac{x(x-1)}{2}+(W-x)x+\frac{(W-x)(W-x-1)}{2}\bullet\frac{R-1}{R}\right]+\frac{W-x}{W}\right\}\bigg/\left(\frac{W+1}{2}\right)\\
&=\frac{1}{W(W+1)}\left[-\frac{1}{R}x^2+\frac{2W-(2R+1)}{R}x+\frac{R-1}{R}\left(W^2-W\right)+2W\right](4)
\end{aligned}
$$

Because $-\dfrac{1}{R}<0$, (4) is maximized when $x=-\dfrac{\frac{2W-(2R+1)}{R}}{2\left(-\frac{1}{R}\right)}=W-R-\dfrac{1}{2}$. Since $x\in\{0,1,\ldots,W\}$, we can conclude that when $W\le R$, $x=0$ maximizes (4); when $W\ge R+1$, $x=W-R$ or $x=W-R+1$ maximizes (4).

As an example, when $R=4$, for different W we have the following results:

W	2	3	4	5	6	7
x	0	0	0	0 or 1	1 or 2	2 or3

Co-existence with an EB-ALOHA node

We now consider the co-existence of a model-aware node and an EB-ALOHA node.

We define the stage of the EB-ALOHA node to be the number of previous collisions experienced by the current packet, i.e., stage 0, stage 1, …, stage m. The model-aware node knows the transmission scheme of EB-ALOHA node. In particular, it knows the initial window size W and the maximum backoff stage m of the EB-ALOHA node. The model-aware node knows whether the EB-ALOHA node transmitted in past slots by listening to the channel (when the EB-ALOHA node did not transmit) and the lack of ACK from its receiver (when the model-aware node transmitted and there was a collision). Thus, the model-aware node knows the stage of the EB-ALOHA node by counting the number of collisions the current EB-ALOHA packet has encountered. Similar to the FW-aware node, the EB-aware node has no knowledge of the backoff counter value c of the EB-ALOHA node at each stage, although the EB-aware node knows the stage value of the EB-ALOHA node.

To analyze the optimal strategy of the model-aware node when $m = 2$ (we only consider $m = 2$ in this document), we construct a 3-state (state 0, state 1 and state 2) "coarse" Markov chain where the state is the stage of the EB-ALOHA node. By "coarse", we mean we do not care how many ALOHA-slots has transpired in each stage, but only focus on the stage the EB-ALOHA is in. For each stage, the model-aware node adopts a similar strategy as it does when co-existing with the FW-ALOHA node, i.e., between two transmissions of the EB-ALOHA node, the model-aware node first adopts the greedy strategy in $x_i \left(i = 0,1,2\right)$ ALOHA-slots and then adopts the polite strategy after that, where i here refers to the state (stage) the EB-ALOHA node is in. We can draw the coarse Markov chain, as illustrated in Fig. 1.

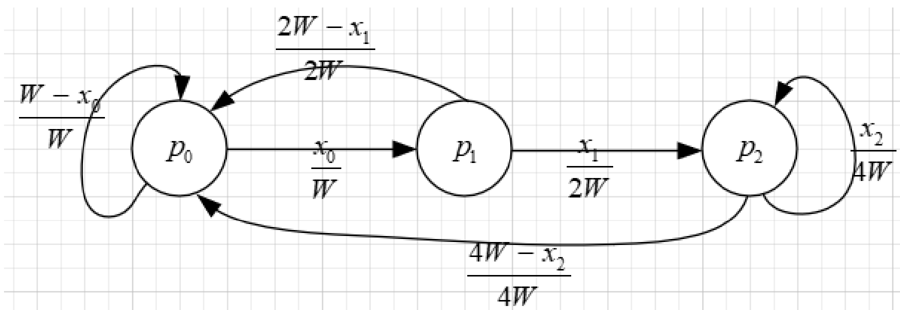

Fig. 1. A coarse Markov chain for the strategy of the model-aware node when co-existing with an EB-ALOHA node.

We can calculate the stationary probability of each state using the following equations:

$$\begin{cases} p_0 = \dfrac{W - x_0}{W} p_0 + \dfrac{2W - x_1}{2W} p_1 + \dfrac{4W - x_2}{4W} p_2 \\ p_1 = \dfrac{x_0}{W} p_0 \\ p_2 = \dfrac{x_1}{2W} p_1 + \dfrac{x_2}{4W} p_2 \\ p_0 + p_1 + p_2 = 1 \end{cases}$$

There are four possibilities in calculating the sum throughput, given as follows:

(i) $x_0 = 0$ (no matter what x_1, x_2 are)

(ii) $x_0 \neq 0, x_1 = 0$ (no matter what x_2 is)

(iii) $x_0 \neq 0, x_1 \neq 0, x_2 = 0$

(iv) $x_0 \neq 0, x_1 \neq 0, x_2 \neq 0$

For (i)(ii)(iii), we can get $p_0 = \dfrac{1}{1+\dfrac{x_0}{W}+\dfrac{2x_0x_1}{W\left(4W-x_3\right)}}$, $p_1 = \dfrac{x_0}{W}p_0$, $p_2 = \dfrac{2x_0x_1}{W\left(4W-x_2\right)}p_0$. Therefore, we can use the method in FW-ALOHA case to calculate throughputs.

$$E\left[\text{number of total ALOHA-slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (5)$$

$$\begin{aligned} &E\left[\text{number of model-aware successful ALOHA-slots in one round}\right] \\ &= p_0 \bullet \frac{1}{W}\left[\frac{x_0\left(x_0-1\right)}{2}+\left(W-x_0\right)x_0+\frac{\left(W-x_0\right)\left(W-x_0-1\right)}{2}\bullet\frac{R-1}{R}\right] \\ &+p_1 \bullet \frac{1}{2W}\left[\frac{x_1\left(x_1-1\right)}{2}+\left(2W-x_1\right)x_1+\frac{\left(2W-x_1\right)\left(2W-x_1-1\right)}{2}\bullet\frac{R-1}{R}\right] \\ &+p_2 \bullet \frac{1}{4W}\left[\frac{x_2\left(x_2-1\right)}{2}+\left(4W-x_2\right)x_2+\frac{\left(4W-x_2\right)\left(4W-x_2-1\right)}{2}\bullet\frac{R-1}{R}\right] \quad (6) \end{aligned}$$

$$E\left[\text{number of EB-ALOHA successful ALOHA-slots in one round}\right] = p_0 \bullet \frac{W-x_0}{W} + p_1 \bullet \frac{2W-x_1}{2W} + p_2 \bullet \frac{4W-x_2}{4W} \quad (7)$$

Therefore, the throughput of the model-aware node is $(6)/(5)$, the throughput of the FW-ALOHA node is $(7)/(5)$, and the sum throughput is $(6)/(5)+(7)/(5)$.

For 4), $p_0 = 0, p_1 = 0, p_2 = 1$. This corresponds to a greedy strategy in FW-ALOHA case with a fixed window size of $4W$. The throughput of the model-aware node is $\dfrac{4W-1}{4W+1}$, the throughput of the EB-ALOHA node is 0, and the sum throughput is $\dfrac{4W-1}{4W+1}$.

When R and W is fixed, we can do an exhaustive searching of every combination of x_0, x_1, x_2 and find the optimal sum throughput. For example, when $R=4$, we can conclude that when $W=2,3$, $x_0=0\left(x_1=0, x_2=0\right)$, the polite strategy is optimal; when $W>3$, $x_0=W, x_1=2W, x_2=4W$, the greedy strategy is optimal.

Co-existence with a TDMA node and a *q*-ALOHA node

The optimal strategy of a model-aware node when co-existing with a TDMA node and a *q*-ALOHA node depends on the following factors: the packet lengths of TDMA and *q*-ALOHA, the TDMA transmission pattern, and the transmission probability of *q*-ALOHA nodes. There is no general optimal strategy for the model-aware node. In this document, we only focus on the two particular cases used in our paper.

The first case we consider is that the packet length of both TDMA and *q*-ALOHA is 4 basic slots, i.e., $R_T = R_A = 4$. The TDMA frame lasts 5 five TDMA slots and the TDMA node transmits in the 2$^{\text{nd}}$ and 5$^{\text{th}}$ TDMA slot in each frame. The transmission probability of the *q*-ALOHA node is $q = 0.4$. The optimal strategy of the model-aware node is for the slots occupied by the TDMA node, the model-aware do nothing; for the slots not occupied by the TDMA node, for each ALOHA slot, the model-aware node adopts the polite strategy. Fig. 2 illustrates the optimal strategy of the model-aware node.

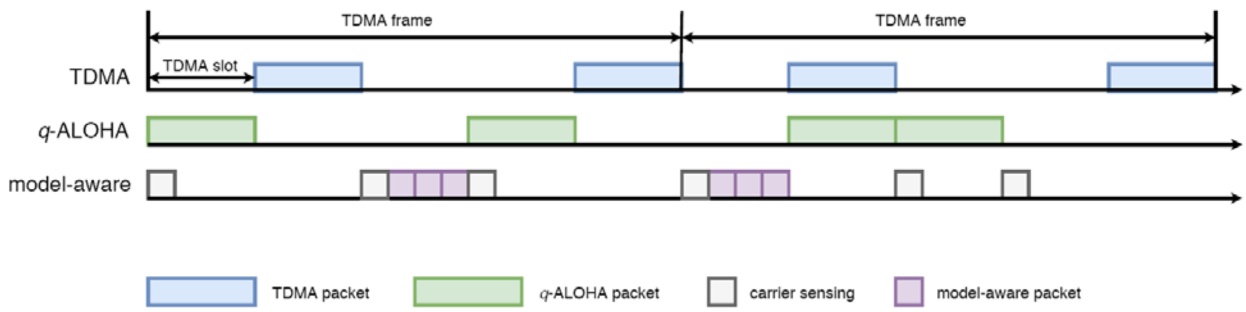

Fig. 2 Optimal strategy of the model-aware node when co-existing with one TDMA node and one q-ALOHA node (case 1).

The second case we consider is the same as the first case except the packet length of the *q*-ALOHA node is $R_A = 2$. The optimal strategy of the model-aware node is the same as the above setting. Fig. 2 illustrates the optimal strategy of the model-aware node in the second case.

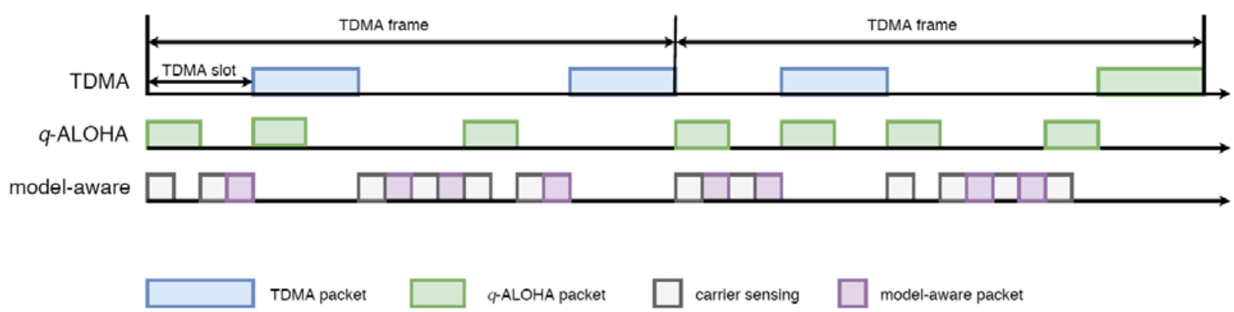

Fig. 3 Optimal strategy of the model-aware node when co-existing with one TDMA node and one q-ALOHA node (case 2).