Model-aware node coexisting with different legacy networks

This document derives the optimal total network throughputs that can be attained by a model-aware node when it coexists with TDMA networks, ALOHA networks and a mix of TDMA and ALOHA networks. This is a supplementary document to the paper we submitted to IEEE ICC 2018 of a model-free wireless MAC protocol based on deep reinforcement learning (DRL). The throughputs achievable by the model-aware protocol serve as benchmarks for the DRL model-free protocol.

■ Co-existence with TDMA networks

A TDMA node transmits in X specific slots within each frame of K slots in a repetitive manner. In this case, the model-aware node is a TDMA-aware node that has full knowledge of the X slots used by the TDMA node. To maximize the total network throughput, the TDMA-aware node will transmit in all the slots not used by the TDMA node. Thus, the optimal total throughput is 1.

■ Co-existence with q-ALOHA networks

A *q*-ALOHA node transmits with a fixed transmission probability q in each time slots. The transmission probabilities of different time slots are i.i.d. We consider the co-existence of $(N-1)$ *q*-ALOHA nodes and one model-aware node. The model-aware node is referred to as a *q*-aware node. It knows the value *q* as well as *N*.

Given the *q*-ALOHA nodes transmit in time slots in an i.i.d. manner, the transmission strategy of the *q*-aware node should also be the identical and independent in all the time slots. Consider a particular time slot, let p be the probability of transmission for the *q*-aware node and $f(p)$ be the associated total network throughput. It is given by

$$f(p) = (N-1)q(1-q)^{N-2}(1-p) + p(1-q)^{N-1}.$$

Take the derivative of $f(p)$ with respect to p :

$$f'(p) = -(N-1)q(1-q)^{N-2} + (1-q)^{N-1}$$

From the derivative $f'(p)$, we can see when $q > 1/N$, $f'(p) < 0$; when $q < 1/N$, $f'(p) > 0$. Therefore, to maximize $f(p)$, the optimal transmission probability for the *q*-aware node is

$$p^* = \begin{cases} 0 & \text{if } q > 1/N \\ 1 & \text{if } q \le 1/N \end{cases} \qquad (1)$$

or

$$p^* = \begin{cases} 0 & \text{if } q \geq 1/N \\ 1 & \text{if } q < 1/N \end{cases} \quad (2)$$

The corresponding optimal total throughput $f(p^*)$ is

$$f\left(p^*\right) = \begin{cases} (N-1)q(1-q)^{N-2} & \text{if } q \geq 1/N \\ (1-q)^{N-1} & \text{if } q < 1/N \end{cases}$$

■ Co-existence with FW-ALOHA networks

A fixed-window ALOHA (*FW*-ALOHA) node generates a random counter value c in the range of $[0, W-1]$ after it transmits in a time slot. It then waits for c slots before its next transmission.

A FW-aware node knows the transmission scheme of the FW-ALOHA node and aims to maximize the total throughput. For simplicity, we consider the case of the co-existence of one FW-ALOHA node and one FW-aware node. The FW-aware node has two optimal strategies:

❖ Optimal strategy 1

After a transmission of the FW-ALOHA node, the FW-aware node transmits in all the subsequent time slots except after observing consecutive $(W-1)$ idle slots of the FW-ALOHA node, in which case the FW-aware node refrains from transmission in the next time slot. The reason is as follows. Immediately after a transmission by the FW-ALOHA node, the probability of FW-ALOHA node transmitting in the next time slot is $1/W$ (i.e., it will transmit only when the counter value c is 0). This is equivalent to the single *q*-ALOHA case where $q = 1/W$ (with N=2 in (1)). Thus, if $W \geq 2$, the FW-aware node should transmit (see (1)). If the FW-ALOHA node does not transmit, the probability it will transmit in the time slot after next is $1/(W-1)$. Again, according to (1), the FW-aware node will transmit unless $1/(W-1) = 1$. Reasoning it this way, we see that, the FW-aware node will only transmit after it observes $(W-1)$ idle time slots of the FW-ALOHA node.

Following this strategy, the achieved throughputs are given by:

FW-ALOHA node: $\dfrac{2}{W(W+1)}$ FW-Aware node: $\dfrac{W-1}{W+1}$ Total Throughput: $\dfrac{W^2-W+2}{W(W+1)}$

These throughputs are derived using a renewal process. We present the derivation as follows.

We define one round as the time slots between two successful transmissions of the *FW*-ALOHA node. We now have

$$E\left[\text{number of slots in one round}\right]=\frac{1}{W}\left[1+2+\ldots+(W-1)+W\right]=\frac{W+1}{2} \tag{1}$$

$$E\left[\text{number of total successuful slots in one round}\right]=\frac{1}{W}\left[0+1+2+\ldots+(W-2)+W\right]=\frac{W^2-W+2}{2W} \tag{2}$$

$$E\left[\text{number of FW-ALOHA successful slots in one round}\right]=\frac{1}{W}\bullet 1=\frac{1}{W} \tag{3}$$

$$E\left[\text{number of FW-aware successful slots in one round}\right]=\frac{1}{W}\left[0+1+2+\ldots(W-2)+(W-1)\right]=\frac{W-1}{2} \tag{4}$$

The throughputs are therefore as follows:

$$\text{FW-ALOHA throughput} = \frac{(3)}{(1)}=\frac{2}{W(W+1)}$$

$$\text{FW-aware throughput} = \frac{(4)}{(1)}=\frac{W-1}{W+1}$$

$$\text{Total throughput} = \frac{(2)}{(1)}=\frac{W^2-W+2}{W(W+1)}$$

❖ Optimal strategy 2

After observing a transmission of the FW-ALOHA node, the FW-aware node transmits in all subsequent time slots except after observing consecutive $(W-2)$ idle slots of the FW-ALOHA node, in which case FW-aware node refrains from transmission in the next 2 time slots. The difference between this strategy and the previous strategy is that the previous strategy follows (1) while this strategy follows (2).

Following this strategy, the achieved throughputs are given by

FW-ALOHA node: $\frac{4}{W(W+1)}$ FW-Aware node: $\frac{W-2}{W}$ Total Throughput: $\frac{W^2-W+2}{W(W+1)}$

Similar to optimal strategy 1, these throughputs of optimal strategy 2 are also derived using a renewal process.

$$E\left[\text{number of slots in one round}\right]=\frac{1}{W}\left[1+2+\ldots+(W-1)+W\right]=\frac{W+1}{2} \tag{5}$$

$$E\left[\text{number of total successful slots in one round}\right]=\frac{1}{W}\left[0+1+2+\ldots+(W-3)+(W-1)+(W-1)\right]=\frac{W^2-W+2}{2W} \quad (6)$$

$$E\left[\text{number of FW-ALOHA successful slots in one round}\right]=\frac{1}{W}\bullet 2=\frac{2}{W} \quad (7)$$

$$E\left[\text{number of FW-aware successful slots in one round}\right]=\frac{1}{W}\left[0+1+2+\ldots(W-2)+(W-2)\right]=\frac{W^2-W-2}{2W} \quad (8)$$

$$\text{FW-ALOHA throughput}=\frac{(7)}{(5)}=\frac{4}{W(W+1)}$$

$$\text{FW-aware throughput}=\frac{(8)}{(5)}=\frac{W-2}{W}$$

$$\text{Total throughput}=\frac{(6)}{(5)}=\frac{W^2-W+2}{W(W+1)}$$

We note that although the two optimal strategies of the FW-aware node are different, they achieve the same total throughputs.

■ Co-existence with EB-ALOHA networks

An exponential backoff ALOHA (EB-ALOHA) node is a variant of a FW-ALOHA node. Instead of using a fixed window size W, the EB-ALOHA node doubles its window size each time when its transmission encounters a collision until a maximum window size is met. We denote the maximum window size as $W_{\max}=2^m W$, where m is the "maximum backoff stage". We adopt the notation $W_i=2^i W$, where $i\in(0,m)$ is called "backoff stage". Upon a successful transmission, the window size is reset to the initial value W.

A EB-aware node knows the transmission scheme of the EB-ALOHA node, i.e., it knows that the EB-ALOHA node will double its window size when EB-ALOHA node encountering a collision. Also, the initial window size W and the maximum backoff stage m are known to the EB-aware node.

Here, we consider the co-existence of one EB-ALOHA node and one EB-aware node. The maximum backoff stage m of the EB-ALOHA node is set to 2. We can see that when $m=2$, the EB-ALOHA node could be in three backoff stages: stage 0, stage 1 and stage 2.

For the optimal strategy, the EB-aware node should take sub-strategies for different stages. Borrowing the idea of FW-aware node, the EB-aware node may transmit in successive time slots until it hears

$2^i W - 1$ idle time slots at stage i ($i = 0,1,2$) and we denote this strategy as Strategy 000. However, the reasoning turns out to be not correct. For example, when m is large, a better strategy could be to cause lots of collisions to the EB-ALOHA node so that it seldom transmits. Then, the success probability of the EB-aware node is close to 1 and the optimal total throughput is close to 1, the upper bound for throughput. There are 8 strategies for the EB-aware node according to the decision made after hearing $2^i W - 1$ idle time slots at stage i.

To derive the optimal strategy for the EB-aware node, we use the following analysis.

Upon each transmission of the EB-ALOHA node, the EB-aware node transmits in all subsequent time slots except after observing $W-1$, $2W-1$ and $4W-1$ consecutive idles slots of EB-ALOHA node, in which we can conclude two strategies for the EB-aware node.

批注 [s1]: This is not correct.
Actually, we did simulations to check which strategies work the best.

You should first explain what is meant by model-aware here. It knows m and W and the EB procedure.
It also knows when it collides with the EB-ALOHA node (thus, the "stage" of the EB-ALOHA protocol). Need to define state.

The tricky thing is even if it knows the state, it cannot break down the strategy to be the same as in the previous section for the FW-ALOHA case. For example, one might surmise that the optimal strategy is that after I collisions, the EB-aware node should transmit in successive time slots until it hears 2^i*W-1 idle time slots. This reasoning turns out to be note correct. For example, when m is large, a better strategy could be to cause lots of collisions to the EW-ALOHA node so that it seldom transmits. Then, the success probability of the EB-aware node is close to 1 and the optimal total throughput is close to 1, the upper bound for throughput.

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❖ Strategy xx1

The first strategy is referred to as Strategy xx1. The EB-aware node will always transmit when observing $4W-1$ consecutive idle slots of EB-ALOHA node, no matter what the decision made after observing $W-1$ idle slots and 2W-1 idle slots. Note that Strategy xx1 is actually a greedy strategy, i.e., the EB-ALOHA node with Strategy xx1 will always transmit (after moving to state 2).

批注 [TW2]: This sentence is not clear to me

批注 [YY3R2]: Here I am the EB-aware node may choose to transmit or not to transmit when observing W-1 or 2W-1 idle slots.

It is easy to analyze Strategy xx1. Strategy xx1 of EB-ALOHA actually is equivalent to the optimal strategy of FW-ALOHA with a fixed window size of $4W$. We can derive the throughput using the renewal process. Define one round as the number of slots it takes when EB-ALOHA counter goes to 0. We then have:

$$E\left[\text{number of slots in one round}\right] = \frac{1}{4W}\left(1+2+\ldots+\left(4W-1\right)+4W\right) = \frac{4W+1}{2} \tag{9}$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = \frac{1}{4W}\left(0+1+2+\ldots\left(4W-2\right)+\left(4W-1\right)\right) = \frac{4W-1}{2} \tag{10}$$

$$\text{EB-ALOHA throughput} = 0$$

$$\text{EB-aware throughput} = \frac{(10)}{(9)} = \frac{4W-1}{4W+1}$$

$$\text{Total throughput} = \frac{4W-1}{4W+1}$$

❖ Strategy 000

The second strategy is referred to as Strategy 000. Now, the EB-aware node will not transmit when observing $W-1$, $2W-1$ or $4W-1$ consecutive idle slots of the EB-ALOHA node.

To analyze Strategy 000, we construct a Markov chain with 3 states: sates 0, 1 and 2. For the Markov chain, we focus on the "coarse Markov chain" in which we do not care how many time slots are spent in each state.

For example, if it is in state 0, and the counter value is c; then it takes $c+1$ slots before going to the next round. Here, we do not care about the $c+1$ time slots. We define the following probabilities:

P(going from stage 0 to stage 0 at the end of the current round | at stage 0) = $1/W$ (by assuming 000, there is no collision only if the counter value was initiated at $W-1$ at the beginning of the current round)

P(going from stage 1 to stage 1 at the end of the current round | at stage 0) = $1-1/W$.

The "coarse Markov chain" is illustrated in Fig. 1. We can calculate the above probabilities accordingly.

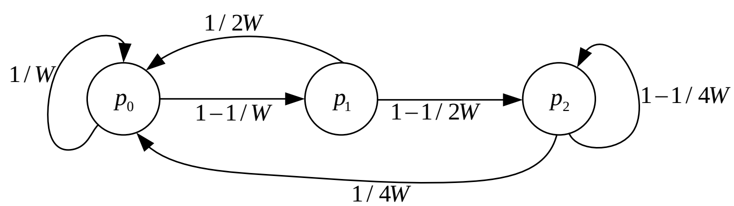

Fig. 1 Coarse Markov Chain

$$\begin{cases} \frac{1}{W}p_0 + \frac{1}{2W}p_1 + \frac{1}{4W}p_2 = p_0 \\ \left(1-\frac{1}{W}\right)p_0 = p_1 \\ \left(1-\frac{1}{2W}\right)p_1 + \left(1-\frac{1}{4W}\right)p_2 = p_2 \\ p_0 + p_1 + p_2 = 1 \end{cases} \Rightarrow p_0 = \frac{W}{(2W-1)^2},\ p_1 = \frac{W-1}{(2W-1)^2},\ p_2 = \frac{2(W-1)}{2W-1}$$

Also, the throughputs can be derived using a renewal process.

$$E[\text{number of slots in one round}] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \tag{11}$$

$$E[\text{number of total successful slots in one round}] = p_0 \bullet \frac{W^2-W+2}{2W} + p_1 \bullet \frac{(2W)^2-(2W)+2}{2(2W)} + p_2 \bullet \frac{(4W)^2-(4W)+2}{2(4W)} \tag{12}$$

$$E\left[\text{number EB-ALOHA successful slots in one round}\right] = p_0 \bullet \frac{1}{W} + p_1 \bullet \frac{1}{2W} + p_2 \bullet \frac{1}{4W} \tag{13}$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{4W-1}{2} \tag{14}$$

$$\text{EB-ALOHA throughput} = \frac{(13)}{(11)}$$

$$\text{EB-aware throughput} = \frac{(14)}{(11)}$$

$$\text{Total throughput} = \frac{(12)}{(11)}$$

We can calculate the total throughputs of Strategy xx1 and Strategy 000 numerically. The optimal strategy can be concluded as follows:

When $W = 2$, the optimal strategy is Strategy 000;

When $W = 3$, the optimal strategies are Strategy 000 and Strategy xx1.

When $W \geq 4$, the optimal strategy is Strategy xx1.

	strategy xx1			strategy 000			
W	EB-aware	EB-ALOHA	Total	EB-aware	ALOHA	Total	optimal strategy
2	0.7777777778	0	0.7777777778	0.7230769231	0.0615384615	0.7846153846	strategy 000
3	0.8461538462	0	0.8461538462	0.8251748252	0.0209790210	0.8461538462	strategy 000 and xx1
4	0.8823529412	0	0.8823529412	0.8712220762	0.0105124836	0.8817345598	strategy xx1
5	0.9047619048	0	0.9047619048	0.8978562421	0.0063051702	0.9041614123	strategy xx1
6	0.9200000000	0	0.9200000000	0.9152957648	0.0042002100	0.9194959748	strategy xx1
7	0.9310344828	0	0.9310344828	0.9276231263	0.0029978587	0.9306209850	strategy xx1
8	0.9393939394	0	0.9393939394	0.9368066283	0.0022468754	0.9390535037	strategy xx1
9	0.9459459459	0	0.9459459459	0.9439161653	0.0017465554	0.9456627207	strategy xx1

■ Co-existence with a mix of TDMA and q-ALOHA networks

We consider the coexistence of one model-aware node with one TDMA node and $(N-1)$ *q*-ALOHA node. The optimal strategy for the model-aware node is given as the following. For the slots used by the

TDMA node, the model-aware node refrains from transmission. For the slots not used by the TDMA node, it will transmit according to the value of N and q, as per (1) and (2). Suppose that TDMA is assigned X slots of 10 slots, the optimal throughputs are given in the following table:

Throughput	TDMA	q-ALOHA	Mode-aware	Total throughput
$q \geq 1/N$	$\frac{X}{10}(1-q)^{N-1}$	$\left(1-\frac{X}{10}\right)q(1-q)^{N-2}$	0	$\frac{X}{10}(1-q)^{N-1}+\left(1-\frac{X}{10}\right)q(1-q)^{N-2}$
$q < 1/N$	$\frac{X}{10}(1-q)^{N-1}$	0	$\left(1-\frac{X}{10}\right)(1-q)^{N-1}$	$(1-q)^{N-1}$