Model-Aware Node in Heterogeneous Networks

--- A supplementary document to a submitted conference paper

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This document derives the optimal sum throughputs that can be attained by a model-aware node when it coexists with TDMA networks and ALOHA networks. This is a supplementary document to the work "DLMA with different packet lengths". The optimal sum throughputs derived here serve as benchmarks for the DRL model-free protocol.

In particular, we assume the model-aware node has the smallest packet length of one time slot. TDMA and ALOHA nodes have a larger packet length of R time slots ($R > 1$). For convenience, we use one TDMA-slot and one ALOHA-slot to represent a packet transmission time of TDMA node and ALOHA node, respectively.

Coexistence with a TDMA node

We first consider the coexistence of a model-aware node and a TDMA node. One TDMA-slot is R time slots and each TDMA frame lasts for K TDMA-slots. The TDMA node transmits in X specific TDMA-slots within each frame in a repetitive manner. The model-aware node has full knowledge of the X TDMA-slots used by the TDMA node and the value of R. To maximize the sum throughput, the model-aware node should transmit in all the slots not used by the TDMA node and should refrain from transmission in the slots occupied by the TDMA node. Thus, the optimal sum throughput is 1. Fig. 1 is an illustration of this setup when $R = 4, X = 2, K = 5$.

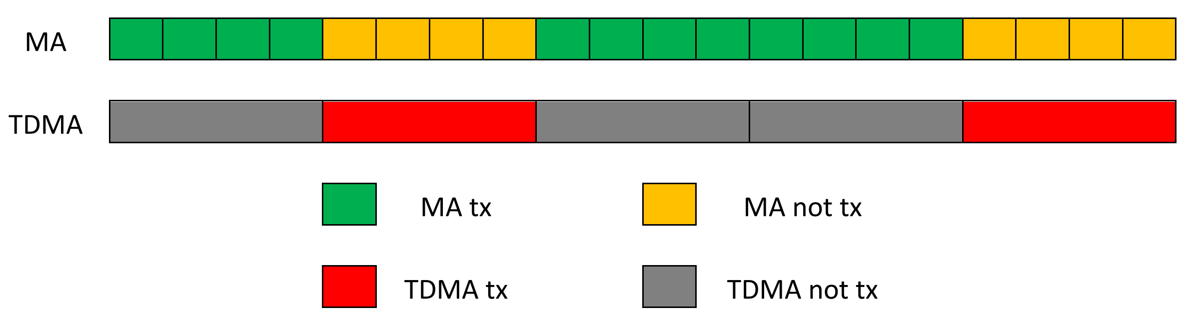

Fig. 1. An illustration of the optimal strategy of model-aware (MA) node when coexisting with a TDMA node.

Coexistence with a q-ALOHA node

We next consider the coexistence of a model-aware node and a q-ALOHA node. The q-ALOHA node transmits with a fixed probability q at the beginning of each ALOHA-slot and the ALOHA-slot is R time slots. The model-aware node knows the value of q and R. The optimal strategy of the model-aware node depends on q and R. Two possible strategies, one of which is the optimal strategy, are given as follows:

1) **Greedy strategy**: the model-aware node always transmits. The total throughput is $1-q$.

2) **Polite strategy**: the model-aware node listens to the channel in the first slot of each ALOHA-slot. If the channel is busy, the model-aware node will not transmit in the following $R-1$ slots; if the channel is idle, the model-aware node will transmit in the next $R-1$ slots. In this case, the total throughput is $(1-q)\frac{R-1}{R}+q$. Fig. 2 is an illustration of the polite strategy when $R=4$.

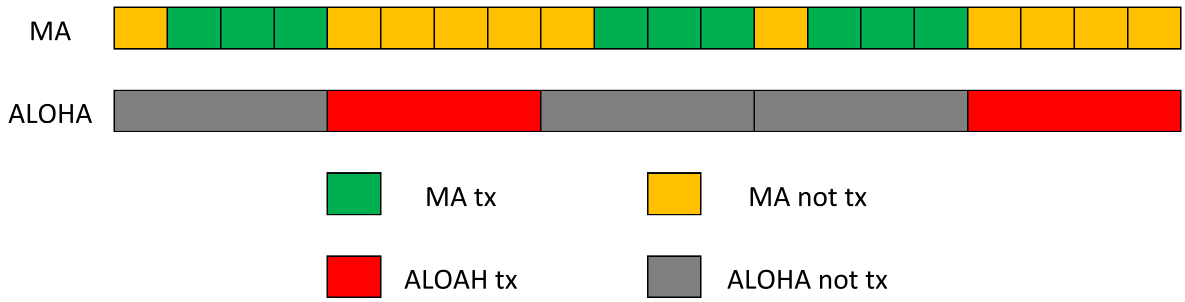

Fig. 2. An illustration of the polite strategy of model-aware (MA) node when coexisting with a q-ALOHA node.

By comparing $1-q$ and $(1-q)\frac{R-1}{R}+q$, we can conclude that when $q<1/(R+1)$, the greedy strategy is optimal; when $q=1/(R+1)$, the greedy strategy and the polite strategy achieve the same sum throughput; when $q>1/(R+1)$, the polite strategy is optimal.

It is straightforward to extend the number of q-ALOHA nodes to be arbitrary. Suppose that the number of q-ALOHA node is N and the sum throughput achieved by the greedy strategy and the polite strategy will be $(1-q)^N$ and $(1-q)^N\frac{R-1}{R}+Nq(1-q)^{N-1}$, respectively. From there, we can conclude that when $q<1/(NR+1)$, the greedy strategy is optimal; when $q=1/(NR+1)$, the greedy strategy and the polite strategy achieve the same sum throughput; when $q>1/(NR+1)$, the polite strategy is optimal.

Coexistence with a fixed-window ALOHA node

Next, we consider the coexistence of a model-aware node and a fixed-window ALOHA (FW-ALOHA) node. The ALOHA-slot is R time slots. The FW-ALOHA node generates a uniformly random backoff counter value $c\in[0,W-1]$ after it transmits in an ALOHA-slot. It then waits for c ALOHA-slots before its next transmission. The value of c is decremented at the end of each ALOHA-slot. When c reaches 0, the FW-ALOHA node transmits in the next ALOHA-slot. The model-aware node knows the transmission scheme of the FW-ALOHA node, i.e., it knows the FW-ALOHA node will generate a random backoff counter after each transmission. Also, the model-aware node knows the value of W and whether the FW-ALOHA node transmitted in the past slots by listening to the channel. A subtlety here is that the model-aware node does not know the exact value c of the FW-ALOAH node.

We can derive the optimal strategy of the model-aware node as follows. During two adjacent transmissions of FW-ALOHA node, the model-aware node adopts the greedy strategy in the first x ALOHA-slots (after a transmission by the ALOHA node) and adopts the polite strategy after x ALOHA-slots, where x is a parameter to be optimized on to achieve the maximum sum throughput of the system. For generality, we let $x \in [0, W]$ (when $x = W$, the model-aware not is fully greedy; when $x = 0$, the model-aware node is fully polite). Given an x, we can use a renewal process to analyze the strategy. We define one round to be the number of ALOHA-slots between the end of a transmission by the ALOHA node and the end of the next transmission by the ALOHA node (i.e., there is an ALOHA transmission per round). We can calculate the expected total number of ALOHA-slots, the successful ALOHA-slots of the model-aware node, and the successful ALOHA-slots of the FW-ALOHA node, per round as follows:

$$E\left[\text{number of total ALOHA-slots in one round}\right] = \frac{1}{W}\left[1 + 2 + \cdots + (W-1) + W\right](*) = \frac{W+1}{2} (1)$$

$$E\left[\text{number of model-aware successful ALOHA-slots in one round}\right]$$
$$= \frac{1}{W}\left[0 + 1 + 2 + \cdots + (x-1) + x + \left(x + \frac{R-1}{R}\right) + \left(x + 2\bullet\frac{R-1}{R}\right) + \cdots + \left(x + \left(W - (x+1)\right)\frac{R-1}{R}\right)\right](**)$$
$$= \frac{1}{W}\left[\frac{x(x-1)}{2} + (W-x)x + \frac{(W-x)(W-x-1)}{2}\bullet\frac{R-1}{R}\right](2)$$

$$E\left[\text{number of FW-ALOHA successful ALOHA-slots in one round}\right] = \frac{1}{W}\underbrace{\left[0 + 0 + \ldots + 0 + 1 + 1 + \cdots 1\right]}_{\text{number of 0 is x, number of 1 is W-x}}(***) = \frac{W-x}{W}(3)$$

Note that in $(*)(**)(***)$, the addends correspond to the number of model-aware node's successful ALOHA-slots, the number of FW-ALOHA's successful ALOHA-slots and the number of total ALOHA-slots for $c = 0, 1, 2, \ldots, W-1$, respectively. Also, we point that when $c < x$ the number of model-aware node's successful ALOHA-slots is c and the number of FW-ALOHA's successful ALOHS-slots is 0; when $c \geq x$, the number of model-aware node's successful ALOHA-slots is $x + (c-x)\frac{R-1}{R}$ c and the number of FW-ALOHA's successful ALOHS-slots is 1.

Therefore, the throughput of the model-aware node is $(2)/(1)$, the throughput of the FW-ALOHA node is $(3)/(1)$, and the sum throughput is $(2)/(1) + (3)/(1)$.

Now we derive the value x that maximizes the sum throughput.

$$(2)/(1) + (3)/(1)$$
$$= \left\{\frac{1}{W}\left[\frac{x(x-1)}{2} + (W-x)x + \frac{(W-x)(W-x-1)}{2}\bullet\frac{R-1}{R}\right] + \frac{W-x}{W}\right\} / \left(\frac{W+1}{2}\right)$$
$$= \frac{1}{W(W+1)}\left[-\frac{1}{R}x^2 + \frac{2W-(2R+1)}{R}x + \frac{R-1}{R}\left(W^2 - W\right) + 2W\right](4)$$

Because $-\frac{1}{R}<0$, (4) is maximized when $x=-\frac{\frac{2W-(2R+1)}{R}}{2\left(-\frac{1}{R}\right)}=W-R-\frac{1}{2}$. Since $x\in\{0,1,\ldots,W\}$, we can conclude that when $W\le R$, $x=0$ maximizes (4); when $W\ge R+1$, $x=W-R$ or $x=W-R+1$ maximizes (4).

As an example, when $R=4$, for different W we have the following results:

W	2	3	4	5	6	7
x	0	0	0	0 or 1	1 or 2	2 or3

Coexistence with an exponential-backoff ALOHA node

We now consider the coexistence of a model-aware node and an exponential-backoff ALOHA (EB-ALOHA) node. EB-ALOHA is a variant of FW-ALOHA. As in FW-ALOHA, after each packet transmission without collision, an EB-ALOHA node randomly chooses a backoff counter value c in the range of $[0,W-1]$. However, EB-ALOHA doubles its window size each time when its transmission encounters a collision, until a maximum window size $2^m W$ is reached, where m is called the "maximum backoff stage". Upon a successful transmission, the windows size is reset to the initial value W.

We define the stage of the EB-ALOHA node to be the number of previous collisions experienced by the current packet, i.e., stage 0, stage 1, …, stage m. The model-aware node knows the transmission scheme of EB-ALOHA node. In particular, it knows the initial window size W and the maximum backoff stage m of the EB-ALOHA node. The model-aware node knows whether the EB-ALOHA node transmitted in past slots by listening to the channel (when the EB-ALOHA node did not transmit) and the lack of ACK from its receiver (when the model-aware node transmitted and there was a collision). Thus, the model-aware node knows the stage of the EB-ALOHA node by counting the number of collisions the current EB-ALOHA packet has encountered. Similar to the FW-aware node, the EB-aware node has no knowledge of the backoff counter value c of the EB-ALOHA node at each stage, although the EB-aware node knows the stage value of the EB-ALOHA node.

To analyze the optimal strategy of the model-aware node when $m=2$ (we only consider $m=2$ in this document), we construct a 3-state (state 0, state 1 and state 2) "coarse" Markov chain where the state is the stage of the EB-ALOHA node. By "coarse", we mean we do not care how many ALOHA-slots has transpired in each stage, but only focus on the stage the EB-ALOHA is in. For each stage, the model-aware node adopts a similar strategy as it does when coexisting with the FW-ALOHA node, i.e., between two transmissions of the EB-ALOHA node, the model-aware node first adopts the greedy strategy in $x_i\ (i=0,1,2)$ ALOHA-slots and then adopts the polite strategy after that, where i here refers to the state (stage) the EB-ALOHA node is in. We can draw the coarse Markov chain, as illustrated in Fig. 3.

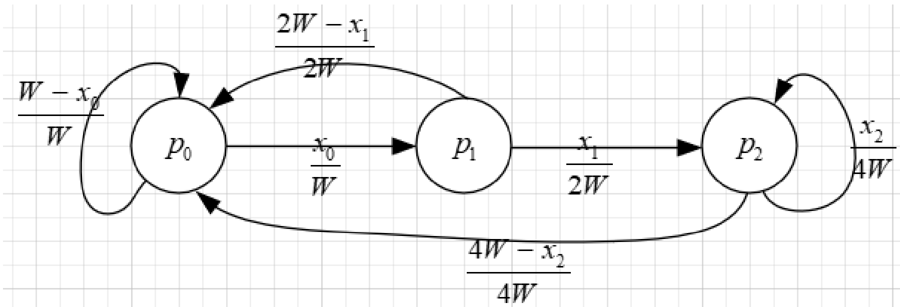

Fig. 3. A coarse Markov chain for the strategy of the model-aware node when coexisting with an EB-ALOHA node.

We can calculate the stationary probability of each state using the following equations:

$$\begin{cases} p_0 = \dfrac{W-x_0}{W} p_0 + \dfrac{2W-x_1}{2W} p_1 + \dfrac{4W-x_2}{4W} p_2 \\ p_1 = \dfrac{x_0}{W} p_0 \\ p_2 = \dfrac{x_1}{2W} p_1 + \dfrac{x_2}{4W} p_2 \\ p_0 + p_1 + p_2 = 1 \end{cases}$$

There are four possibilities in calculating the sum throughput, given as follows:

(i) $x_0 = 0$ (no matter what x_1, x_2 are)

(ii) $x_0 \neq 0, x_1 = 0$ (no matter what x_2 is)

(iii) $x_0 \neq 0, x_1 \neq 0, x_2 = 0$

(iv) $x_0 \neq 0, x_1 \neq 0, x_2 \neq 0$

For (i)(ii)(iii), we can get $p_0 = \dfrac{1}{1 + \dfrac{x_0}{W} + \dfrac{2x_0 x_1}{W\left(4W - x_3\right)}}, p_1 = \dfrac{x_0}{W} p_0, p_2 = \dfrac{2x_0 x_1}{W\left(4W - x_2\right)} p_0$. Therefore, we can use the method in FW-ALOHA case to calculate throughputs.

$$E\left[\text{number of total ALOHA-slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (5)$$

$$E\left[\text{number of model-aware successful ALOHA-slots in one round}\right]$$

$$= p_0 \bullet \frac{1}{W}\left[\frac{x_0\left(x_0-1\right)}{2} + \left(W-x_0\right)x_0 + \frac{\left(W-x_0\right)\left(W-x_0-1\right)}{2} \bullet \frac{R-1}{R}\right]$$

$$+ p_1 \bullet \frac{1}{2W}\left[\frac{x_1\left(x_1-1\right)}{2} + \left(2W-x_1\right)x_1 + \frac{\left(2W-x_1\right)\left(2W-x_1-1\right)}{2} \bullet \frac{R-1}{R}\right]$$

$$+ p_2 \bullet \frac{1}{4W}\left[\frac{x_2\left(x_2-1\right)}{2} + \left(4W-x_2\right)x_2 + \frac{\left(4W-x_2\right)\left(4W-x_2-1\right)}{2} \bullet \frac{R-1}{R}\right] \quad (6)$$

$$E\left[\text{number of EB-ALOHA successful ALOHA-slots in one round}\right] = p_0 \bullet \frac{W-x_0}{W} + p_1 \bullet \frac{2W-x_1}{2W} + p_2 \bullet \frac{4W-x_2}{4W} \quad (7)$$

Therefore, the throughput of the model-aware node is $(6)/(5)$, the throughput of the FW-ALOHA node is $(7)/(5)$, and the sum throughput is $(6)/(5)+(7)/(5)$.

For 4), $p_0 = 0, p_1 = 0, p_2 = 1$. This corresponds to a greedy strategy in FW-ALOHA case with a fixed window size of $4W$. The throughput of the model-aware node is $\frac{4W-1}{4W+1}$, the throughput of the EB-ALOHA node is 0, and the sum throughput is $\frac{4W-1}{4W+1}$.

When R and W is fixed, we can do an exhaustive searching of every combination of x_0, x_1, x_2 and find the optimal sum throughput. For example, when $R=4$, we can conclude that when $W=2,3$, $x_0 = 0\left(x_1 = 0, x_2 = 0\right)$, the polite strategy is optimal; when $W>3$, $x_0 = W, x_1 = 2W, x_2 = 4W$, the greedy strategy is optimal.

Coexistence with a mix of a TDMA node and a *q*-ALOHA node

To be continue …