Model-Aware Nodes in Heterogeneous Networks: A Supplementary Document to Paper "Deep-Reinforcement Learning Multiple Access for Heterogeneous Wireless Networks"

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This document derives the optimal total network throughputs that can be attained by a model-aware node when it coexists with TDMA networks, ALOHA networks and a mix of TDMA and ALOHA networks. This is a supplementary document to a paper we submitted to a conference: the paper puts forth a model-free wireless MAC protocol that leverages the techniques of deep reinforcement learning (DRL) for heterogeneous networking. The throughputs achievable by the model-aware protocols, derived here, serve as benchmarks for the DRL model-free protocol described in the paper.

▪ Co-existence with a TDMA node

We first consider the co-existence of a model-aware node and a TDMA node. Thus, there are two nodes in the network. The TDMA node transmits in X specific slots within each frame of K slots in a repetitive manner. In this case, the model-aware node is a TDMA-aware node that has full knowledge of the X slots used by the TDMA node. To maximize the total network throughput, the TDMA-aware node will transmit in all the slots not used by the TDMA node. Thus, the optimal total throughput is 1.

▪ Co-existence with (N-1) q-ALOHA nodes

We next consider the co-existence of a model-aware node with $(N-1)$ *q*-ALOHA nodes. The total number of nodes in the network is therefore N. The *q*-ALOHA node transmits with a fixed transmission probability q in each time slot. The transmission probabilities of different time slots are i.i.d. The model-aware node here is referred to as a *q*-aware node. It knows the value q as well as N. Given the *q*-ALOHA nodes transmit in time slots in an i.i.d. manner, the transmission strategy of the *q*-aware node should also be identical and independent in all the time slots. Consider a particular time slot, let p be the transmission probability for the *q*-aware node and $f(p)$ be the associated total network throughput. It is given by

$$f(p)=\left(N-1\right)q\left(1-q\right)^{N-2}\left(1-p\right)+p(1-q)^{N-1}.$$

The derivative of $f(p)$ with respect to p is

$$f'(p)=-\left(N-1\right)q\left(1-q\right)^{N-2}+(1-q)^{N-1}$$

From the derivative $f'(p)$, we can see when $q > 1/N$, $f'(p) < 0$; when $q < 1/N$, $f'(p) > 0$. Therefore, to maximize $f(p)$, the optimal transmission probability for the *q*-aware node is

$$p^* = \begin{cases} 0 & \text{if } q > 1/N \\ 1 & \text{if } q \le 1/N \end{cases} \tag{1}$$

or

$$p^* = \begin{cases} 0 & \text{if } q \ge 1/N \\ 1 & \text{if } q < 1/N \end{cases} \tag{2}$$

The corresponding optimal total throughput $f(p^*)$ is

$$f\left(p^*\right) = \begin{cases} \left(N-1\right)q\left(1-q\right)^{N-2} & \text{if } q \ge 1/N \\ \left(1-q\right)^{N-1} & \text{if } q < 1/N \end{cases}$$

▪ Co-existence with a fixed-window ALOHA node

Here, we consider the co-existence of a model-aware node with a fixed-window ALOHA (FW-ALOHA). Thus, there are two nodes in the network. The FW-ALOHA node generates a random backoff counter value C in the range of $\left[0, W-1\right]$ after it transmits in a time slot. It then waits for C slots before its next transmission. The value of C is decremented at the end of each time slot. When C reaches 0, the FW-ALOHA node transmits in the next time slot. The model-aware node is named FW-aware node and it knows the transmission scheme of the FW-ALOHA node. Also, the FW-aware node knows the value of W and whether the FW-ALOHA node transmitted in past time slots by listening to the channel.

A subtlety about model-aware nodes needs to be pointed out here. Being aware of the MAC protocol used by the other node is different from knowing the instantaneous state of the MAC of the other node. In this case, although the FW-aware node knows the other node uses FW-ALOHA, it does not know the backoff counter value C of the FW-ALOHA node most of the time, except at the end of a time slot in which the FW-ALOHA node just transmitted (*the fact that* $C = 0$ *in that time slot could be derived from listening to the channel (in case the FW-aware node did not transmit) or from the lack of an ACK from the receiver of the FW-aware node (in case the FW-aware not transmitted an there was a collision)).* In particular, the FW-aware node does not know the exact C value at the beginning of a time slot, although it knows its range. For example, if the FW-aware node notices that the FW-ALOHA node did not transmit in the past *n* time slot, then it knows that the C value in the next time slot is equally likely to be 0, 1, …, $(W-1-n)$.

To maximize the total network throughput, the FW-aware node has two optimal strategies that draw on results (1) and (2) of the q-ALOHA case in the previous section:

❖ Optimal Strategy 1

In both optimal strategies, the time slots immediately following a transmission by the FW-ALOHA node serve as "renewal points". In optimal strategy 1, after observing a transmission of the FW-ALOHA node in a particular time slot, the FW-aware node transmits in all the subsequent time slots except after observing consecutive $(W-1)$ idle slots of the FW-ALOHA node (i.e., the FW-ALOHA node did not transmit in the next $(W-1)$ slots after its previous transmission), in which case the FW-aware node refrains from transmission in the next time slot. If the FW-ALOHA node transmits again before $(W-1)$ idle time slots (this leads to a collision which we assume to be known by the FW-aware node through feedback mechanisms from its receiver; i.e., the lack of an ACK), then the time slot following the new transmission is a renewal point for the above strategy of the FW-aware node.

The reason the above strategy is optimal is as follows. Immediately after a transmission by the FW-ALOHA node, the probability of the FW-ALOHA node transmitting in the next time slot is $1/W$ (i.e., it will transmit only when the counter value C is 0). This is equivalent to the single q-ALOHA case where $q = 1/W$ (with N=2 in (1)). Thus, if $W \geq 2$, the FW-aware node should transmit (see (1)). If the FW-ALOHA node does not transmit, the probability it will transmit in the time slot after next is $1/(W-1)$. Again, according to (1), the FW-aware node should transmit unless $1/(W-1) = 1$. Reasoning it this way, we see that, the FW-aware node will only transmit after it observes $(W-1)$ idle time slots of the FW-ALOHA node.

The throughputs of the FW-ALOHA node, the FW-aware node, and the total throughput of the FW-ALOHA node and the FW-aware node can be derived using a renewal process. We present the derivation as follows.

We define one round as the number of slots it takes for the FW-ALOHA node backoff counter to go to zero. We have

$$E\left[\text{number of slots in one round}\right] = \frac{1}{W}\left[1+2+\ldots+(W-1)+W\right] = \frac{W+1}{2} \tag{3}$$

$$E\left[\text{number of total successuful slots in one round}\right] = \frac{1}{W}\left[0+1+2+\ldots+(W-2)+W\right] = \frac{W^2-W+2}{2W} \tag{4}$$

$$E\left[\text{number of FW-ALOHA successful slots in one round}\right] = \frac{1}{W}\bullet 1 = \frac{1}{W} \tag{5}$$

$$E\left[\text{number of FW-aware successful slots in one round}\right] = \frac{1}{W}\left[0+1+2+\ldots(W-2)+(W-1)\right] = \frac{W-1}{2} \tag{6}$$

The throughputs are therefore as follows:

$$\text{FW-ALOHA throughput} = \frac{(5)}{(3)} = \frac{2}{W(W+1)}$$

$$\text{FW-aware throughput} = \frac{(6)}{(3)} = \frac{W-1}{W+1}$$

$$\text{Total throughput} = \frac{(4)}{(3)} = \frac{W^2 - W + 2}{W(W+1)}$$

❖ **Optimal Strategy 2**

In optimal strategy 2, after observing a transmission of the FW-ALOHA node, the FW-aware node transmits in all subsequent time slots except after observing consecutive $(W-2)$ idle slots of the FW-ALOHA node, in which case the FW-aware node refrains from transmission in the next 2 time-slots. The difference between this strategy and the previous strategy is that the previous strategy follows (1) while this strategy follows (2).

Similar to optimal Strategy 1, these throughputs of optimal strategy 2 are also derived using a renewal process as follows:

$$E[\text{number of slots in one round}] = \frac{1}{W}\left[1 + 2 + \ldots + (W-1) + W\right] = \frac{W+1}{2} \tag{7}$$

$$E[\text{number of total successuful slots in one round}] = \frac{1}{W}\left[0 + 1 + 2 + \ldots + (W-3) + (W-1) + (W-1)\right] = \frac{W^2 - W + 2}{2W} \tag{8}$$

$$E[\text{number of FW-ALOHA successful slots in one round}] = \frac{1}{W}\bullet 2 = \frac{2}{W} \tag{9}$$

$$E[\text{number of FW-aware successful slots in one round}] = \frac{1}{W}\left[0 + 1 + 2 + \ldots(W-2) + (W-2)\right] = \frac{W^2 - W - 2}{2W} \tag{10}$$

$$\text{FW-ALOHA throughput} = \frac{(9)}{(7)} = \frac{4}{W(W+1)}$$

$$\text{FW-aware throughput} = \frac{(10)}{(7)} = \frac{W-2}{W}$$

$$\text{Total throughput} = \frac{(8)}{(7)} = \frac{W^2 - W + 2}{W(W+1)}$$

We note that although the two optimal strategies of the FW-aware node are different, they achieve the same total throughputs.

▪ Co-existence with an exponential-backoff ALOHA node

Now, we consider the co-coexistence of a model-aware node and an exponential-backoff ALOHA (EB-ALOHA) node. Thus, there are two nodes in the network. EB-ALOHA is a variant of FW-ALOHA. Specifically, after each packet transmission without collision, an EB-ALOHA node randomly chooses a backoff counter value c in the range of $[0, W-1]$. EB-ALOHA node doubles its window size each time when its transmission encounters a collision, until a maximum window size $2^m W$ is reached, where m is called the “maximum backoff stage”. Upon a successful transmission, the window size is reset to the initial value W.

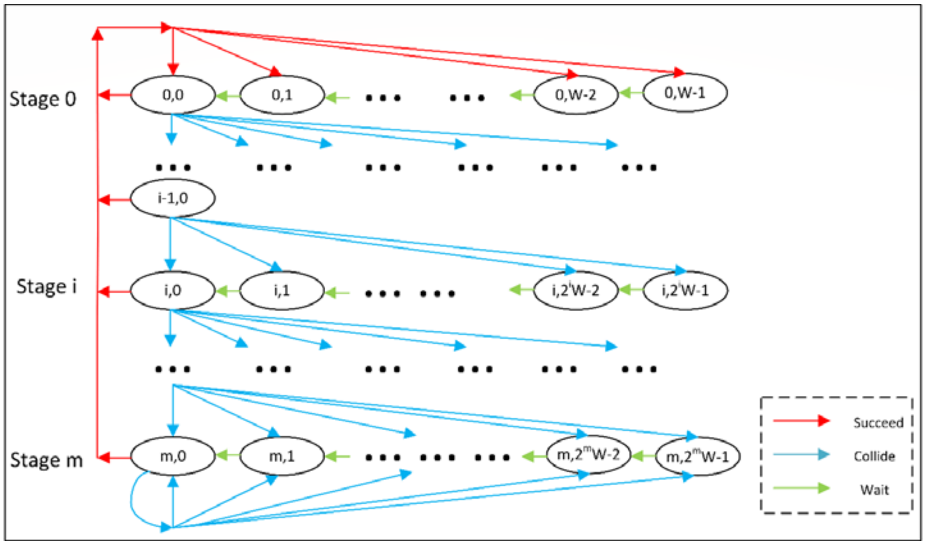

Fig. 1 Markov chain for backoff process of the EB-ALOHA node.

We define the *stage* of the EB-ALOHA node to be the number of previous collisions experienced by the current packet, i.e., stage 0, stage 1, …, stage m. We define the *state* of the EB-ALOHA node to be (i, j_i), where $i = 0,1,2,\ldots,m$ is the backoff stage and $j_i \in \left[0, 2^i W - 1\right]$ is the backoff counter value. The backoff process of the EB-ALOHA node can be modelled by a two-dimensional Markov chain as in Fig. 1 (the transition probabilities are omitted in the figure). The red arrows correspond to the EB-ALOHA node transmitting successfully and its random selection of a new backoff counter value in the range of $\left[0, W-1\right]$; the blue arrows correspond to the EB-ALOHA node encountering a collision and its random selection of a backoff counter value in the next stage (or in the last stage if the state number is already m); the green arrows correspond to the EB-ALOHA node not transmitting and decrementing its backoff counter by 1.

The mode-aware node in this case is called the EB-aware node and it knows the transmission scheme of the EB-ALOHA node, i.e., it knows that the EB-ALOHA node will double its window size upon a collision. Also, the initial window size W and the maximum backoff stage m are known to the EB-aware node. The EB-aware node knows whether the EB-ALOHA node transmitted in past time slots by listening to the channel (when the EB-aware node did not transmit) and from the lack of ACK from its receiver (when the EB-aware transmitted and there was a collision). Thus, the EB-aware node knows the stage of the EB-ALOHA node by counting the number of collisions the current EB-ALOHA packet has encountered. Similar to the FW-aware node, the EB-aware node has no knowledge of the backoff counter value c of EB-ALOHA node at each stage, although the EB-aware node knows the stage value.

One possible strategy for the EB-aware node is to behave like the FW-aware node, i.e., after observing a transmission of the EB-ALOHA node in a particular time slot, the EB-aware node knows the stage of the EB-ALOHA node. If there was no collision, the EB-ALOHA node will be in stage 0; if there was a collision, the stage of the EB-aware node will increase by 1. If the EB-ALOHA is in stage i, then its counter value at that moment in time is equally likely to be 0, 1, …, $2^i W - 1$. The EB-aware node could behave like a FW-aware node when the FW-aware node interacts with a FW-ALOHA node with window size $2^i W$: i.e., the EB-aware node will transmit in all subsequent time slots except after $2^i W - 1$ idle time slots of the EB-ALOHA node.

It turns out that the optimal strategy is not always given by such dynamic emulation of the EB-aware node that depends on i. To see this, consider the case with a large m. A better strategy could be to cause lots of collisions with the EB-ALOHA node so that it seldom transmits. In particular, the EB-aware node could transmit greedily in all time slots, causing the EB-ALOHA nodes to get stuck in stage *m*. When *m* is large, the EB-ALOHA node then seldom transmits. The success probability of the EB-aware node will be close to 1 when *m* is large, allowing the throughput to approach the upper bound of 1. In general, the optimal strategy depends on *m* and *W*.

Next, we analyze the optimal strategy for the EB-aware node when $m = 2$ (used in our experiment).

After a transmission of the EB-ALOHA node, the EB-aware node knows the EB-ALOHA node is at stage i. Let $g(i)$ be the number of observed consecutive idle slots of EB-ALOHA node at stage i ($g(i) \in [0, 2^i W - 1]$) after the EB-ALOHA node moves to stage i. When $g(i) < 2^i W - 1$, the EB-aware node always transmits in the next time slot (as in optimal strategy 1 of the FW-aware node). When $g(i) = 2^i W - 1$ at the end of a time slot, the EB-aware node needs to decide to transmit or not to transmit in the next slot. When $m = 2$, there will be 8 possible strategies for the EB-aware node as summarized in Table 1 and the optimal strategy is one of the strategies. Next, we analyze the 8 strategies to find out the optimal strategy.

Table 1 Strategies for the EB-aware node

Stage i	g(i)	After observing g(i) idle slots, transmit or not (EB-aware node)?							
0	W-1	No	Yes	No	Yes	No	Yes	No	Yes
1	2W-1	No	No	Yes	Yes	No	No	Yes	Yes
2	4W-1	No	No	No	No	Yes	Yes	Yes	Yes
Strategy		NNN	YNN	NYN	YYN	NNY	YNY	NYY	YYY

First, we note that there is no difference between strategies NNY, YNY, NYY and YYY over the long term, because once the system moves to stage 2, it will never escape from stage 2 and this corresponds to the FW-ALOHA case with a window size of $4W$. We use Strategy xxY to represent these four strategies.

Next, we examine strategy xxY, together with the remaining four strategies: NNN, YNN, NYN and YYN.

- **Strategy xxY**

As analyzed above, Strategy xxY causes the EB-ALOHA node to behave like an FW-ALOHA node with a window size of $4W$. Similarly, we case use a renewal process to analyze the throughputs of as follows:

$$E[\text{number of slots in one round}] = \frac{1}{4W}\left(1 + 2 + \ldots + (4W-1) + 4W\right) = \frac{4W+1}{2} \tag{11}$$

$$E\left[\text{number of EB-aware successful slots in one round}\right]=\frac{1}{4W}\left(0+1+2+\ldots(4W-2)+(4W-1)\right)=\frac{4W-1}{2} \quad (12)$$

$$\text{EB-ALOHA throughput} = 0$$

$$\text{EB-aware throughput} = \frac{(12)}{(11)}=\frac{4W-1}{4W+1}$$

$$\text{Total throughput} = \frac{4W-1}{4W+1}$$

❖ Strategy NNN

To analyze Strategy NNN, we construct a 3-state (state 0, state 1 and state 2) coarse Markov chain and "state" here corresponds to "stage" in Fig. 1. By saying "coarse", we mean we do not care how many time slots are spent in each stage. The coarse Markov chain is illustrated in Fig. 2, where p_0, p_1 and p_2 denote the probability of EB-ALOHA node at state 0, state 1 and state 2, respectively. The transition probabilities (as a function of initial window size W) are given in Fig. 2.

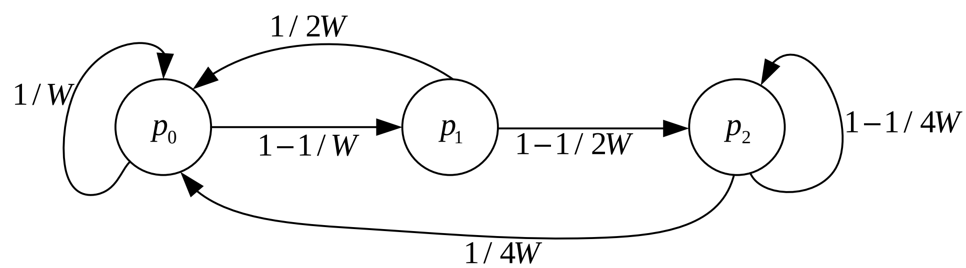

Fig. 2 Coarse Markov Chain for strategy NNN.

p_0, p_1 and p_2 can be calculated as follows:

$$\begin{cases} \frac{1}{W}p_0+\frac{1}{2W}p_1+\frac{1}{4W}p_2=p_0 \\ \left(1-\frac{1}{W}\right)p_0=p_1 \\ \left(1-\frac{1}{2W}\right)p_1+\left(1-\frac{1}{4W}\right)p_2=p_2 \\ p_0+p_1+p_2=1 \end{cases} \Rightarrow p_0=\frac{W}{(2W-1)^2},\, p_1=\frac{W-1}{(2W-1)^2},\, p_2=\frac{2(W-1)}{2W-1}$$

Then, the throughputs can also be derived using a renewal process (The results in (3)-(6) are used in the following derivation):

$$E\left[\text{number of slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (13)$$

$$E\left[\text{number of total successuful slots in one round}\right] = p_0 \bullet \frac{W^2 - W + 2}{2W} + p_1 \bullet \frac{(2W)^2 - (2W) + 2}{2(2W)} + p_2 \bullet \frac{(4W)^2 - (4W) + 2}{2(4W)} \quad (14)$$

$$E\left[\text{number EB-ALOHA successful slots in one round}\right] = p_0 \bullet \frac{1}{W} + p_1 \bullet \frac{1}{2W} + p_2 \bullet \frac{1}{4W} \quad (15)$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{4W-1}{2} \quad (16)$$

$$\text{EB-ALOHA throughput} = \frac{(15)}{(13)}$$

$$\text{EB-aware throughput} = \frac{(16)}{(13)}$$

$$\text{Total throughput} = \frac{(14)}{(13)}$$

The derivations of strategies YNN, NYN and YYN are similar to strategy NNN. We only give the coarse Markov chain and the final results for each strategy below without detailed description.

❖ **Strategy YNN**

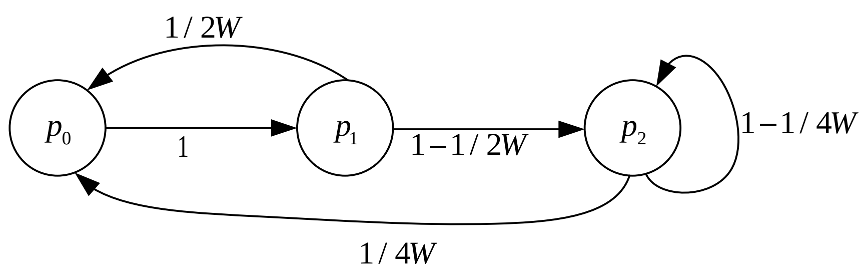

Fig. 3 Coarse Markov Chain for strategy YNN.

$$p_0 = \frac{1}{4W}, p_1 = \frac{1}{4W}, p_2 = \frac{2W-1}{2W}$$

$$E\left[\text{number of slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (17)$$

$$E\left[\text{number of total successuful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{(2W)^2 - (2W) + 2}{2(2W)} + p_2 \bullet \frac{(4W)^2 - (4W) + 2}{2(4W)} \quad (18)$$

$$E\left[\text{number EB-ALOHA successful slots in one round}\right] = p_1 \bullet \frac{1}{2W} + p_2 \bullet \frac{1}{4W} \quad (19)$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{4W-1}{2} \quad (20)$$

$$\text{EB-ALOHA throughput} = \frac{(19)}{(17)}$$

$$\text{EB-aware throughput} = \frac{(20)}{(17)}$$

$$\text{Total throughput} = \frac{(18)}{(17)}$$

- **Strategy NYN**

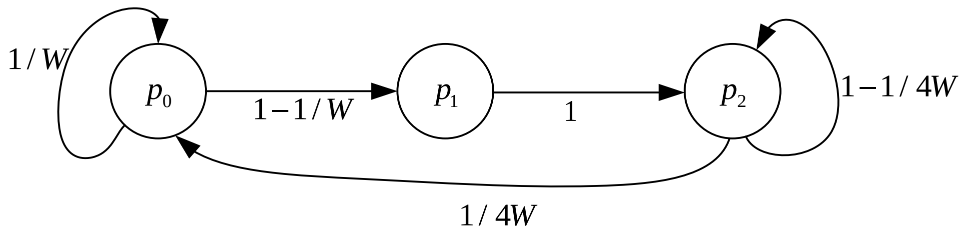

Fig. 4 Coarse Markov Chain for strategy NYN.

$$p_0 = \frac{W}{4W^2-2W-1},\ p_1 = \frac{W-1}{4W^2-2W-1},\ p_2 = \frac{4W^2-4W}{4W^2-2W-1}$$

$$E\left[\text{number of slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (21)$$

$$E\left[\text{number of total successuful slots in one round}\right] = p_0 \bullet \frac{W^2-W+2}{2W} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{(4W)^2-(4W)+2}{2(4W)} \quad (22)$$

$$E\left[\text{number EB-ALOHA successful slots in one round}\right] = p_0 \bullet \frac{1}{W} + p_2 \bullet \frac{1}{4W} \quad (23)$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{4W-1}{2} \quad (24)$$

$$\text{EB-ALOHA throughput} = \frac{(23)}{(21)}$$

$$\text{EB-aware throughput} = \frac{(24)}{(21)}$$

$$\text{Total throughput} = \frac{(22)}{(21)}$$

❖ **Strategy YYN**

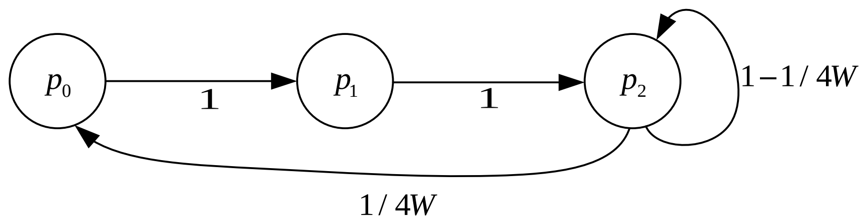

Fig. 5 Coarse Markov Chain for strategy YYN.

$$p_0 = \frac{1}{4W+2}, p_1 = \frac{1}{4W+2}, p_2 = \frac{4W}{4W+2}$$

$$E\left[\text{number of slots in one round}\right] = p_0 \bullet \frac{W+1}{2} + p_1 \bullet \frac{2W+1}{2} + p_2 \bullet \frac{4W+1}{2} \quad (25)$$

$$E\left[\text{number of total successuful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{(4W)^2 - (4W) + 2}{2(4W)} \quad (26)$$

$$E\left[\text{number EB-ALOHA successful slots in one round}\right] = p_2 \bullet \frac{1}{4W} \quad (27)$$

$$E\left[\text{number of EB-aware successful slots in one round}\right] = p_0 \bullet \frac{W-1}{2} + p_1 \bullet \frac{2W-1}{2} + p_2 \bullet \frac{4W-1}{2} \quad (28)$$

$$\text{EB-ALOHA throughput} = \frac{(27)}{(25)}$$

$$\text{EB-aware throughput} = \frac{(28)}{(25)}$$

$$\text{Total throughput} = \frac{(26)}{(25)}$$

Now we can calculate the throughputs for strategies xxY, NNN, YNN, NYN and YYN numerically for different initial window sizes and the results are given in Table 2. The optimal strategy can be concluded as follows (with ten decimal place accuracy):

When $W = 2$*, the optimal strategy is strategy NNN;*

When $W = 3$*, the optimal strategies are strategy xxY, strategy NNN and strategy NYN;*

When $W \geq 4$*, the optimal strategy is strategy xxY.*

Table 2 Throughputs for different strategies of EB-aware node.

	strategy xxY			strategy NNN			strategy NYN		
W	EB-aware	EB-ALOHA	Total	EB-aware	EB-ALOHA	Total	EB-aware	EB-ALOHA	Total
2	0.7777777778	0	**0.7777777778**	0.7230769231	0.0615384615	**0.7846153846**	0.7349397590	0.0481927711	**0.7831325301**
3	0.8461538462	0	**0.8461538462**	0.8251748252	0.0209790210	**0.8461538462**	0.8284023669	0.0177514793	**0.8461538462**
4	0.8823529412	0	**0.8823529412**	0.8712220762	0.0105124836	**0.8817345598**	0.8725376593	0.0092699884	**0.8818076477**
5	0.9047619048	0	**0.9047619048**	0.8978562421	0.0063051702	**0.9041614123**	0.8985176739	0.0057012543	**0.9042189282**
6	0.9200000000	0	**0.9200000000**	0.9152957648	0.0042002100	**0.9194959748**	0.9156742839	0.0038622465	**0.9195365304**
7	0.9310344828	0	**0.9310344828**	0.9276231263	0.0029978587	**0.9306209850**	0.9278597051	0.0027899562	**0.9306496613**
8	0.9393939394	0	**0.9393939394**	0.9368066283	0.0022468754	**0.9390535037**	0.9369642622	0.0021099829	**0.9390742451**
9	0.9459459459	0	**0.9459459459**	0.9439161653	0.0017465554	**0.9456627207**	0.9440264269	0.0016516792	**0.9456781061**

	strategy YNN			strategy YYN		
W	EB-aware	EB-ALOHA	Total	EB-aware	EB-ALOHA	Total
2	0.7419354839	0.0322580645	**0.7741935484**	0.7500000000	0.0250000000	**0.7750000000**
3	0.8297872340	0.0141843972	**0.8439716312**	0.8323353293	0.0119760479	**0.8443113772**
4	0.8730158730	0.0079365079	**0.8809523809**	0.8741258741	0.0069930070	**0.8811188811**
5	0.8987341772	0.0050632911	**0.9037974683**	0.8993135011	0.0045766590	**0.9038901601**
6	0.9157894737	0.0035087719	**0.9192982456**	0.9161290323	0.0032258065	**0.9193548388**
7	0.9279279279	0.0025740026	**0.9305019305**	0.9281437126	0.0023952096	**0.9305389222**
8	0.9370078740	0.0019685039	**0.9389763779**	0.9371534196	0.0018484288	**0.9390018484**
9	0.9440559441	0.0015540016	**0.9456099457**	0.9441587068	0.0014695077	**0.9456282145**

▪ Co-existence with a mix of TDMA and q-ALOHA networks

Now we consider the coexistence of one model-aware node with one TDMA node and $(N-1)$ q-ALOHA nodes. Altogether, there are $(N+1)$ nodes in the network. The optimal strategy for the model-aware node is given as follows. The model-aware node refrains from transmission in the slots used by the TDMA node. For the slots not used by the TDMA node, the model-aware node will transmit according to the value of N and q, as per (1) and (2).

Suppose that TDMA is assigned X slots of 10 slots in a frame, the optimal throughputs are given in the following table:

Table 3 Throughputs for the co-existence of a model-aware node and a mix of a TDMA and (N-1) q-ALOHA nodes.

Throughput	TDMA	q-ALOHA	Mode-aware	Total Throughput
$q \geq 1/N$	$\frac{X}{10}(1-q)^{N-1}$	$\left(1-\frac{X}{10}\right)q(1-q)^{N-2}$	0	$\frac{X}{10}(1-q)^{N-1}+\left(1-\frac{X}{10}\right)q(1-q)^{N-2}$
$q < 1/N$	$\frac{X}{10}(1-q)^{N-1}$	0	$\left(1-\frac{X}{10}\right)(1-q)^{N-1}$	$(1-q)^{N-1}$