第九次课后作业参考答案

May 14, 2019

必做题

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1 Ex.6.1.1

b)0011
解答：

$$
\begin{tikzcd}
(q,0011,Z_0) \arrow[d] & & & \\
(q,011,XZ_0) \arrow[r] \arrow[d] & (p,011,Z_0) & (p,11,Z_0) \arrow[r] & (p,1,\varepsilon) \\
(q,11,XXZ_0) \arrow[r] \arrow[d] & (p,11,XZ_0) \arrow[r] \arrow[ur] & (p,1,XXZ_0) \arrow[r] \arrow[dl] & (p,\varepsilon,XXXZ_0) \arrow[dl] \\
(q,1,XXZ_0) \arrow[r] \arrow[d] & (p,1,XZ_0) \arrow[r] \arrow[d] & (p,\varepsilon,XXZ_0) \arrow[dd, bend left] & \\
(q,\varepsilon,XXZ_0) \arrow[drr] & (p,1,Z_0) \arrow[r] & (p,\varepsilon,Z_0) & \\
& & (p,\varepsilon,XZ_0) \arrow[u] &
\end{tikzcd}
$$

2 Ex.6.2.5(b)

给出一个验证串abb属于$L(P)$的执行轨迹。
解答：
$(q_0,abb,Z_0) \to (q_1,bb,AAZ_0) \to (q_1,b,AZ_0) \to (q_1,\varepsilon,Z_0) \to (q_0,\varepsilon,Z_0) \to (f,\varepsilon,\varepsilon)$

3 Ex.6.2.6

3.1 a)

解答：
转换为PDA $P_1 = (\{q,p\},\{0,1\},\{Z_0,X\},\delta,q,Z_0)$，它的转移函数如下：

1. $\delta(q,0,Z_0) = \{(q,XZ_0)\}$
2. $\delta(q,0,X) = \{(q,XX)\}$
3. $\delta(q,1,X) = \{(q,X)\}$

4. $\delta(q, \varepsilon, X) = \{(p, \varepsilon)\}$

5. $\delta(p, \varepsilon, X) = \{(p, \varepsilon)\}$

6. $\delta(p, 1, X) = \{(p, XX\}$

7. $\delta(p, 1, Z_0) = \{(p, \varepsilon)\}$

8. $\delta(p, \varepsilon, Z_0) = \{(p, \varepsilon)\}$

3.2 b)

解答:
转换为PDA $P_2 = (\{p_0, p_f, q, p\}, \{0, 1\}, \{Z_0, X, X_0\}, \delta, q, Z_0, \{p_f\})$，它的转移函数如下：

1. $\delta(p_0, \varepsilon, X_0) = \{(q, Z_0X_0)\}$

2. $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$

3. $\delta(q, 0, X) = \{(q, XX)\}$

4. $\delta(q, 1, X) = \{(q, X)\}$

5. $\delta(q, \varepsilon, X) = \{(p, \varepsilon)\}$

6. $\delta(p, \varepsilon, X) = \{(p, \varepsilon)\}$

7. $\delta(p, 1, X) = \{(p, XX\}$

8. $\delta(p, 1, Z_0) = \{(p, \varepsilon)\}$

9. $\delta(q, \varepsilon, X_0) = \{(p_f, \varepsilon)\}$

10. $\delta(p, \varepsilon, X_0) = \{(p_f, \varepsilon)\}$

4 Ex.6.3.2

解答:
构造PDA为：$(\{q\}, \{a, b\}, \{a, b, S, A\}, \delta, q, S)$，其中$\delta$定义为：

$$\begin{aligned} \delta(q, \epsilon, S) &= \{(q, aAA)\} \\ \delta(q, \epsilon, A) &= \{(q, aS), (q, bS), (q, a)\} \\ \delta(q, a, a) &= \{(q, \epsilon)\} \\ \delta(q, b, b) &= \{(q, \epsilon)\} \end{aligned}$$

5 Ex.6.3.4

解答:

先将PDA P转化成与之等价的以空栈形式接受的PDA P_1，$P_1 = (\{q, p, p_0, q_0\}, \{0, 1\}, \{Z_0, X, X_0\}, \delta_1, q_0, X_0)$，转移函数$\delta_1$定义如下：

$$
\begin{aligned}
&\delta_1(q_0, \varepsilon, X_0) = \{(q, Z_0X_0)\}\\
&\delta_1(q, 0, Z_0) = \{(q, XZ_0)\}\\
&\delta_1(q, 0, X) = \{(q, XX)\}\\
&\delta_1(q, 1, X) = \{(q, X)\}\\
&\delta_1(q, \varepsilon, X) = \{(q, \varepsilon)\}\\
&\delta_1(p, \varepsilon, X) = \{(p, \varepsilon)\}\\
&\delta_1(p, 1, X) = \{(p, XX)\}\\
&\delta_1(p, 1, Z_0) = \{(p, \varepsilon)\}\\
&\delta_1(p, \varepsilon, Z_0) = \{(p_0, \varepsilon)\}\\
&\delta_1(p, \varepsilon, X) = \{(p_0, \varepsilon)\}\\
&\delta_1(p, \varepsilon, X_0) = \{(p_0, \varepsilon)\}\\
&\delta_1(p_0, \varepsilon, Z_0) = \{(p_0, \varepsilon)\}\\
&\delta_1(p_0, \varepsilon, X) = \{(p_0, \varepsilon)\}\\
&\delta_1(p_0, \varepsilon, X_0) = \{(p_0, \varepsilon)\}
\end{aligned}
$$

根据P_1构造CFG $G = (V, \{0, 1\}, P, S)$，其中$V = \{S\} \cup \{[rYs] | r, s \in \{p_0, p, q, q_0\} \wedge Y \in \{Z_0, X, X_0\}\}$，产生式集合$P$定义如下：

$$
\begin{aligned}
&S \to [q_0\ X_0\ q] | [q_0\ X_0\ p] | [q_0\ X_0\ q_0] | [q_0\ X_0\ p_0]\\
&[q_0\ X_0\ r_2] \to [q\ Z_0\ r_1][r_1\ X_0\ r_2]\\
&[q\ Z_0\ r_2] \to 0[q\ X\ r_1][r_1\ Z_0\ r_2]\\
&[q\ X\ r_2] \to 0[q\ X\ r_1][r_1\ X\ r_2]\\
&[q\ X\ r_1] \to 1[q\ X\ r_1]\\
&[q\ X\ p] \to \varepsilon\\
&[p\ X\ p] \to \varepsilon\\
&[p\ X\ r_2] \to 1[p\ X\ r_1][r_1\ X\ r_2]\\
&[p\ Z_0\ p] \to 1\\
&[p\ Z_0\ p_0] \to \varepsilon\\
&[p\ X\ p_0] \to \varepsilon\\
&[p\ X_0\ p_0] \to \varepsilon\\
&[p_0\ Z_0\ p_0] \to \varepsilon\\
&[p_0\ X\ p_0] \to \varepsilon\\
&[p_0\ X_0\ p_0] \to \varepsilon
\end{aligned}
$$

其中，$r_1, r_2 \in \{p_0, p, q, q_0\}$。

6 Ex.6.3.5(c)

$\{0^n1^m | n \le m \le 2n\}$

解答：

构造CFG $G = (\{S\}, \{0, 1\}, P, S)$，其中产生式集合$P$的定义如为：$S \to \varepsilon | 0S1 | 0S11$

根据CFG G构造以空栈方式接受同样语言的PDA $P = (\{q\}, \{0, 1\}, \{S, 0, 1\}, \delta, q, S)$，其中转移函数$\delta$定义如下：

$$
\begin{aligned}
&\delta(q, \varepsilon, S) = \{(q, \varepsilon), (q, 0S1), (q, 0S11)\}\\
&\delta(q, 0, 0) = \{(q, \varepsilon)\}\\
&\delta(q, 1, 1) = \{(q, \varepsilon)\}
\end{aligned}
$$

思考题

7 Ex.6.2.1

7.1 b)

所有由0和1构成的使得任何前缀中1的个数都不比0的个数多的串的集合。
解答:
PDA为：$P = (\{p\}, \{0,1\}, \{Z_0, A, B\}, \delta, p, Z_0, \{p\})$，

$$\delta(p, 0, Z_0) = (p, AZ_0)$$
$$\delta(p, 0, A) = (p, AA)$$
$$\delta(p, 1, A) = (p, \varepsilon)$$

7.2 c)

所有0和1个数相同的0和1的串的集合。
解答:
PDA为：$P = (\{p\}, \{0,1\}, \{Z_0, A, B\}, \delta, p, Z_0)$，

$$\delta(p, 0, Z_0) = (p, AZ_0)$$
$$\delta(p, 1, Z_0) = (p, BZ_0)$$
$$\delta(p, 0, A) = (p, AA)$$
$$\delta(p, 1, B) = (p, BB)$$
$$\delta(p, 0, B) = (p, \varepsilon)$$
$$\delta(p, 1, A) = (p, \varepsilon)$$
$$\delta(p, \varepsilon, Z_0) = (p, \varepsilon)$$

8 Ex.6.2.2(b)

所有0的个数是1的个数的两倍的串的集合。
解答:
构造文法：

$$Z \quad \rightarrow \quad Z0Z0Z1Z|Z0Z1Z0Z|Z1Z0Z0Z|\varepsilon$$

PDA为：$P = (\{p\}, \{0,1\}, \{Z, 0, 1\}, \delta, p, Z)$，

$$\delta(p, \varepsilon, Z) = (p, Z0Z0Z1Z)$$
$$\delta(p, \varepsilon, Z) = (p, Z0Z1Z0Z)$$
$$\delta(p, \varepsilon, Z) = (p, Z1Z0Z0Z)$$
$$\delta(p, \varepsilon, Z) = (p, \varepsilon)$$
$$\delta(p, 0, 0) = (p, \varepsilon)$$
$$\delta(p, 1, 1) = (p, \varepsilon)$$

9 Ex.6.2.3(b)

所有不是ww形式的a和b的串的集合，也就是所有不是一个串重复两遍的串的集合。
解答:
构造文法：

$$\begin{aligned} Z &\rightarrow A \mid B \mid AB \mid BA \\ A &\rightarrow CAC \mid 0 \\ B &\rightarrow CBC \mid 1 \\ C &\rightarrow 0 \mid 1 \end{aligned}$$

(A表示以0为中心的长度为奇数的串，B表示以1为中心的长度为奇数的串，AB和BA表示长度为偶数的串。)

PDA为：$P=(\{p\},\{0,1\},\{Z,A,B,C,0,1\},\delta,p,Z)$，

$$\begin{aligned}
\delta(p,\varepsilon,Z)&=(p,A)\\
\delta(p,\varepsilon,Z)&=(p,B)\\
\delta(p,\varepsilon,Z)&=(p,AB)\\
\delta(p,\varepsilon,Z)&=(p,BA)\\
\delta(p,\varepsilon,A)&=(p,CAC)\\
\delta(p,\varepsilon,A)&=(p,0)\\
\delta(p,\varepsilon,B)&=(p,CBC)\\
\delta(p,\varepsilon,B)&=(p,1)\\
\delta(p,\varepsilon,C)&=(p,0)\\
\delta(p,\varepsilon,C)&=(p,1)\\
\delta(p,0,0)&=(p,\varepsilon)\\
\delta(p,1,1)&=(p,\varepsilon)
\end{aligned}$$

10 Ex.6.3.7

解答：

构造的CFG的变元都是$[s\ X\ s']$的形式，其中s,s'是状态，X是堆栈符号。所以CFG中变元数目的紧上界为：$1+s\times t\times s=s^2t+1$。