清華大學

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Week 1

1. ①找到一一映射 $f: S \to T$:

$$\begin{cases} f(a)=y \\ f(b)=x \\ f(c)=z \end{cases}$$

② 证明 $\forall a,b \in S$, $f(a*b)=f(a)\circ f(b)$:

$f(a*a) = f(a) = y$
$= f(a)\circ f(a) = y\circ y = y$

$f(a*b) = f(b) = x$
$= f(a)\circ f(b) = y\circ x = x$

$f(b*a) = f(b) = x$
$= f(b)\circ f(a)$
$= x\circ y = x$

$f(a*c) = f(c) = z$
$= f(a)\circ f(c) = y\circ z = z$

$f(c*a) = f(c) = z$
$= f(c)\circ f(a) =$
$= z\circ y = z$

$f(b*b) = f(c) = z$
$= f(b)\circ f(b) = x\circ x = z$

$f(c*c) = f(b) = x$
$= f(c)\circ f(c) = z\circ z = x$

$f(b*c) = f(a) = y$
$= f(b)\circ f(c) = x\circ z = y$

$f(c*b) = f(a) = y$
$= f(c)\circ f(b)$
$= z\circ x = y$

∴ 得证

① 证明同态:

只需证明 $\forall x, y \in A^*$,

$$f(x \cdot y) = f(x) + f(y)$$

(1) x, y 均有偶数个1(包括0个).

$x \cdot y$ 也有偶数个1

$$f(x \cdot y) = f(x) = f(y) = 0 = f(x) + f(y)$$

(2) x, y 均有奇数个1

$x \cdot y$ 也有偶数个1

$f(x \cdot y) = 0$ $\quad f(x \cdot y) = f(x) + f(y)$

$f(x) = f(y) = 1$

(3) x, y 有一个奇数个1
另一个偶数个1

$$f(x) + f(y) = 1 + 0 = 1$$

~~故~~ $x \cdot y$ 有奇数个1

$$f(x \cdot y) = 1 = f(x) + f(y)$$

∴ 同态

② 非同构: 是满射: $100 \in A^*$, $000 \in A^*$

$f(100) = 1$ $\quad f(000) = 0$ $\Rightarrow$ f 不是一一对应
不同构

不是 ~~双射~~ 单射 $\quad 100, 10011 \in A^*$

$f(100) = f(10011) = 1$

3. $L^0 = \{\varepsilon\}$, $L^1 = \{ab, aa, baa\}$ (记为 s_1, s_2, s_3) , $L^2 = \{\cancel{abaa}, \cancel{abbaa}, abab, \cancel{aaab}, \cancel{aaaa}, aabaa, baaab, \cancel{baaaa}, baabaa\}$

~~$L' = \{abab, abaa, abbaa$~~

① abaabaaabaa

$= ab \cdot aa \cdot baa \cdot ab \cdot aa \in L^5$ ✓

② aaaabaaaa $= aa \cdot aa \cdot baa \cdot aa \in L^4$ ✓

④ baaaaabaa

$= baa \cdot aa \cdot ab \cdot aa \in L^4$ ✓

③ $\boxed{baa}$ aaab aa a$\boxed{ab}$ 不属于：证明：若其属于 L^*，其必可表示为 $s_{i_1} \cdot s_{i_2} \cdots s_{i_k}$ $(1 \le i_1, \dots, i_k \le 3$, $i_1 \dots i_k \in Z)$

则第一项必为baa，最后一项必为ab

$\underbrace{}_{s_3}$ $\underbrace{}_{s_1}$

则中间aaabaaa必可表示为 $s_{i_2} \cdot s_{i_3} \cdot s_{i_4}$ ⇒ 其长为7，必为2个 s_2 (1个 s_3) ⇒ $aaa \cdot s_3 \cdot a$ ⇒ 无法(此式矛盾