Nonlinear Optimal Control for Electric Vehicles

M270 Final Report

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1. Abstract

In this report, an optimal controller based on Nonlinear Model Predictive Control (NMPC) is developed for Electric Vehicles (EVs). The nonlinear optimal controller is designed using a finite horizon optimal control problem to minimize a given cost function. The objective is to minimize the quadratic errors in velocity and state of charge (SoC) of the battery. Additionally, nonlinear constraints are considered and satisfied to adhere to the operational limitations of the EV. Finally, employing a developed simulation model of an EV, online simulation is utilized to evaluate the effectiveness and real-time performance of the optimal controller.

2. Introduction

MPC is renowned for its capability to handle constraints and predict future events, enabling the determination of control inputs accordingly. In this report, we develop and analyze an NMPC controller with the aim of achieving optimality in reference tracking and energy management. The model of an electric vehicle is constructed using an equation-based approach, and operational limitations are addressed by incorporating nonlinear constraints into the controller design.

The remainder of this report is organized as follows: the *Problem Formulation* Section formulates an equation-based model for the electric vehicle, considering physical constraints. The proposed solution method is explained in the *Methodology* Section. Simulation and numerical results are presented and discussed in the *Results* Section. Finally, conclusions are drawn in the *Conclusions* Section.

3. Problem Formulation

Vehicle Model

The dynamics of a vehicle can be modeled as follows:

$$\dot{v} = \frac{1}{m}(F_T - \frac{C_A}{2}v^2 - R_r - mgsin\theta)$$

, where v and $\dot{v}$ are velocity and time derivative of velocity, respectively. m is the mass of the vehicle, C_A is the drag coefficient to account for the air drag force, R_r is rolling resistance, g is gravity, θ is road inclination, thus $mgsin\theta$ accounts for the gravity force acting on an object on a ramp.

F_T is tractive force, and it can be expressed as the summation of positive tractive force, F_P, and net brake force, F_{BN}. In addition, F_P can be further broken down, which gives us the following equation:

$$F_T = F_P + F_{BN} = (T_M - T_L)\frac{G}{R_w} + F_{BN}$$

, where T_M is motor torque, T_L is spin loss, R_w is the radius of the wheels, and G is the gear ratio. Moreover, T_M can be expressed as the difference between positive motor torque and regenerative motor torque, which can be utilized to recharge the battery. Also, T_R and F_{BN} can be expressed as a function of brake force, F_B, and regenerative fraction, C_R ,as follows:

$$T_M = T_p - T_R = T_p - C_R\frac{R_w}{G}F_B$$

$$F_{BN} = (1 - C_R)F_B$$

Therefore, F_T can be rewritten as:

$$F_T = \left(T_p - T_L\right)\frac{G}{R_w} + (1 - 2C_R)F_B$$

Then, we move on to the battery's model. The dynamics of SoC and the current, I, of a battery can be expressed as:

$$S\dot{o}C = -\frac{V_{oc}}{C_B}I$$

$$I = \frac{V_{oc} - \sqrt{{V_{oc}}^2 - 4R_{int}P_I}}{2R_{int}}$$

, where V_{oc} is open-loop circuit voltage, R_{int} is the internal resistance of the battery, C_B is the capacity of the battery, and P_I is the total power input to the mechanical system of the vehicle, and it can be calculated as the sum of the power output, P_o ,and power loss, P_{Loss}. They can be expressed as follows:

$$P_I = P_o + P_{Loss}$$

$$P_o = W_{ac} + T_M\omega_M$$

$$P_{Loss} = K_c{T_M}^2 + K_w{\omega_M}^3 + K_i\omega_M + C$$

$$\omega_M = \frac{G}{R_w}|v|$$

, where ω_M is motor speed, W_{ac} is accessory load, K_c, K_w, K_i, C are coefficients for motor power loss.

From the equations above, positive torque, T_p, and break force, F_B, are selected as the control inputs. In other words, control inputs, $u = [u_1, u_2]^T = \left[T_p, F_B\right]^T$; while state, $x = [x_1, x_2]^T = [v, SoC]^T$.

Constraints

The physical constraints of the system can be modeled as following inequality:

$$0 \le u_1 = T_p \le \min\left(Max\ Torque\ Output, \frac{Max\ Power\ Output}{Motor\ Speed}\right)$$

$$-10^4 \le u_2 = F_B \le 0$$

$$T_R = \begin{cases} 0, & if\ |v| < 5mph \\ \max\left(C_R\frac{R_w}{G}F_B, \frac{-1}{2}\min\left(Max\ Torque\ Output, \frac{Max\ Power\ Output}{Motor\ Speed}\right)\right), & otherwise \end{cases}$$

$$-10^4 \le F_{BN} \le 0$$

$$5(\%) \le SoC(\%) \le 100(\%)$$

The values of the parameters are shown in the table below.

Accessary load, W_{ac}	600 [W]	C	628.2974
Regenerative fraction, C_R	0.55	K_c	0.045237
Gear ratio, G	3.55	K_i	0.0167
Spin loss, T_L	6 [Nm]	K_w	5.06640055940796E-5
Energy capacity, C_B	12.6 [kWh]	Internal Resistance, R_{int}	0.1 [ohm]
Vehicle Mass, m	2392 [kg]	Voltage open loop, V_{oc}	340 [V]
Wheel radius, R_w	0.34 [m]	Road incline, θ	0 [radian]
Rolling Resistance, R_r	225.63 [N]	Drag coefficient, C_A	1 [Nsec^2/m^2]
Maximum Torque Output	500 [Nm]	Maximum Power Output	1e5 [W]

Table 1. Parameters of the Vehicle Model

Cost Function

The optimal control problem is to find a sequence of control inputs to minimize a cost function, J. The cost function is defined as follows:

$$J(t) = \sigma \int_0^t Q(v - v_r)^2 + (SoC - SoC_r)^2 d\tau$$

, where v_r and SoC_r are velocity and state of charge reference, respectively, and σ is a constant to make J more readable.

4. Methodology

To apply NMPC to this problem, the cost function is discretized, and this problem is turned into a finite horizon control problem to make it applicable to real-time simulation. That is, $J_k = \sum_{i=k}^{N+k-1} Q\left(v_{i|k} - v_{r_{i|k}}\right)^2 + (SoC_{i|k} - SoC_{r_{i|k}})^2$. And the optimization problem with the constraints can be reformulated as

$$\min_z z^T H z \ \ subject\ to. \begin{cases} Fz \leq G \\ F_{eq}(z) = 0 \end{cases}$$, where $z = [(X - X_r)^T \ (U - U_r)^T]^T$

and $(X - X_r)^T = [\left(x_{k|k} - x_{r_{k|k}}\right)^T, \left(x_{k+1|k} - x_{r_{k+1|k}}\right)^T \dots, \left(x_{k+N-1|k} - x_{r_{k+N-1|k}}\right)^T]^T$

$$(U - U_r)^T = [\left(u_{k|k} - u_{r_{k|k}}\right)^T, \left(u_{k+1|k} - u_{r_{k+1|k}}\right)^T \dots, \left(u_{k+N-1|k} - u_{r_{k+N-1|k}}\right)^T]^T$$

H is a weighting matrix for each component of z; F and G are used to impose constraints on the state and control, so that they fulfill the operational limitations. Finally, F_{eq} is used to enforce the dynamics of the system.

In this control problem, v_r is from the US06 drive cycle and SoC_r is fixed to 100% to reduce the amount of energy consumption as much as possible. Then, the minimization problem with nonlinear constraints can be solved by sequential quadratic programming (SQP).

5. Results

In this NMPC problem, the parameters are chosen to be N=10, $\sigma = 1e^{-4}$. And we compare two NMPC controllers having different weights on velocity error, i.e., Q1=100, Q2=1000; while the Q values in J remains 1000 in the calculation of cost functions for comparison. In addition, the performance of a PID controller is also included for comparison.

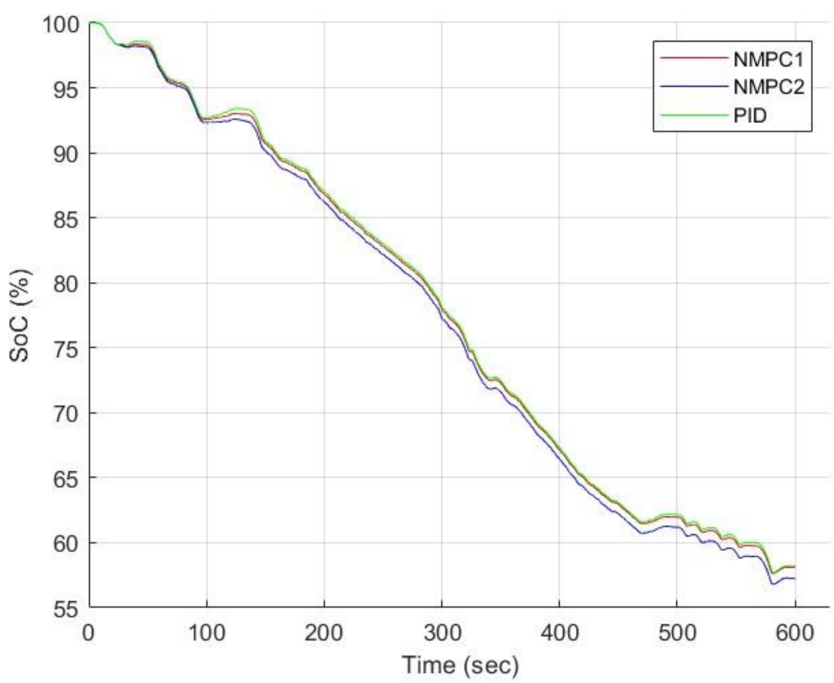

Figure 1. State of Charge over time for different controllers

From figure 1, we can see the state of charge of the battery is higher for NMPC1 than NMPC2 because Q1 is less than Q2, which means NMPC1 has relatively more penalty on the SoC drop than NMPC2. This results in less energy consumption for NMPC1, and therefore higher SoC level than its counterpart.

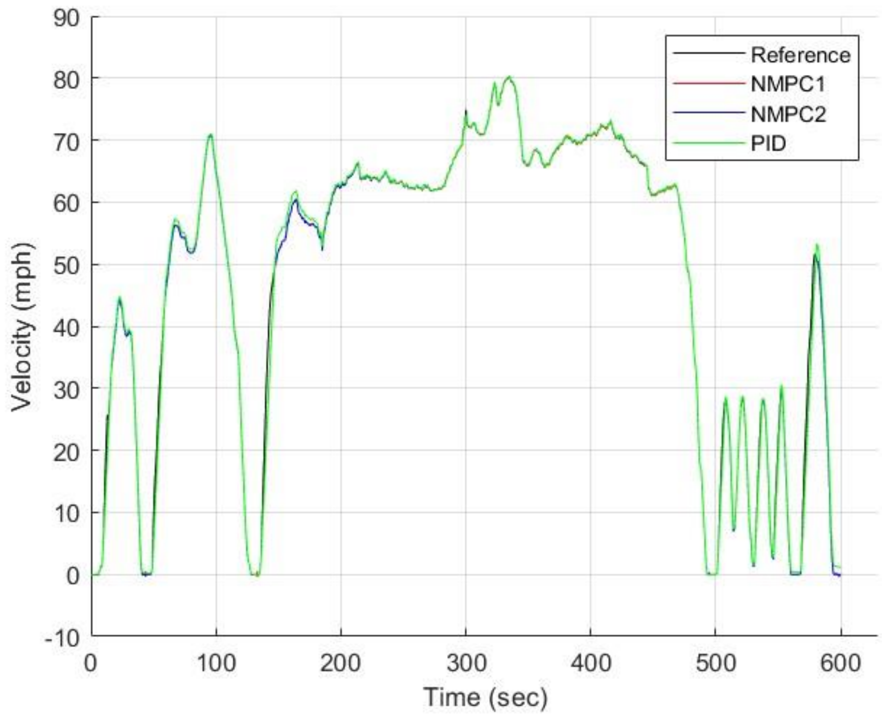

Figure 2. Velocity tracking performances for different controllers

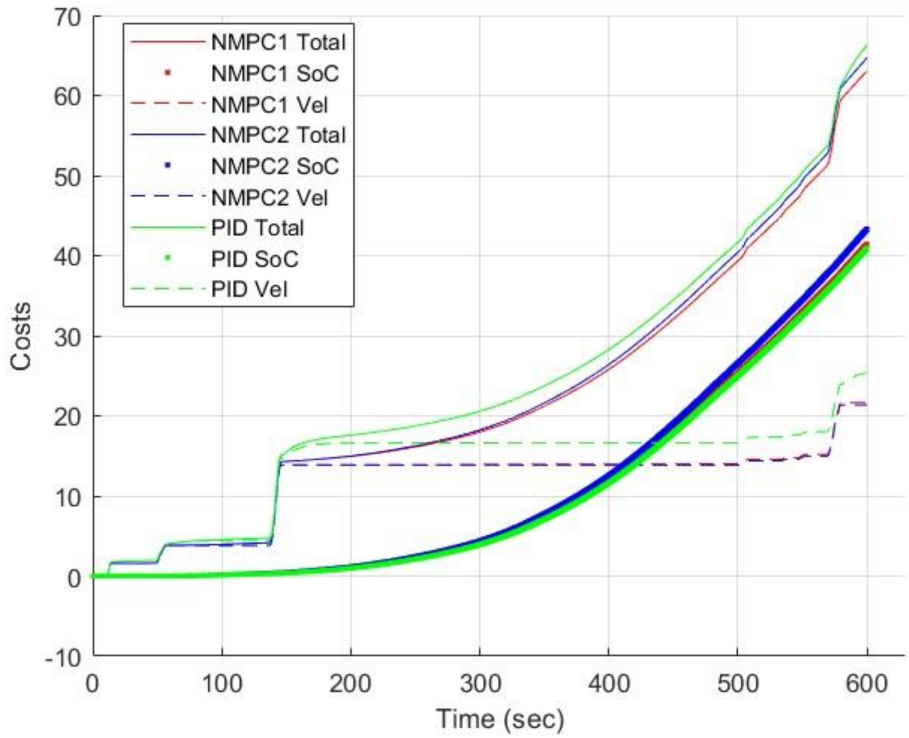

Figure 3. Total Costs, velocity costs, and SoC Costs for different controllers

From figures 2 and 3, we can see NMPC controllers can track velocity reference better than a PID controller from the velocity costs (dash lines) without compromising the SoC level significantly. (Note that the values of the costs are scaled by a constant $\sigma = 1e^{-4}$ for better readability). Furthermore, as the value of Q1 is less than Q2, NMPC2 has better velocity tracking performance than NMPC1, resulting in larger velocity cost value for NMPC1.

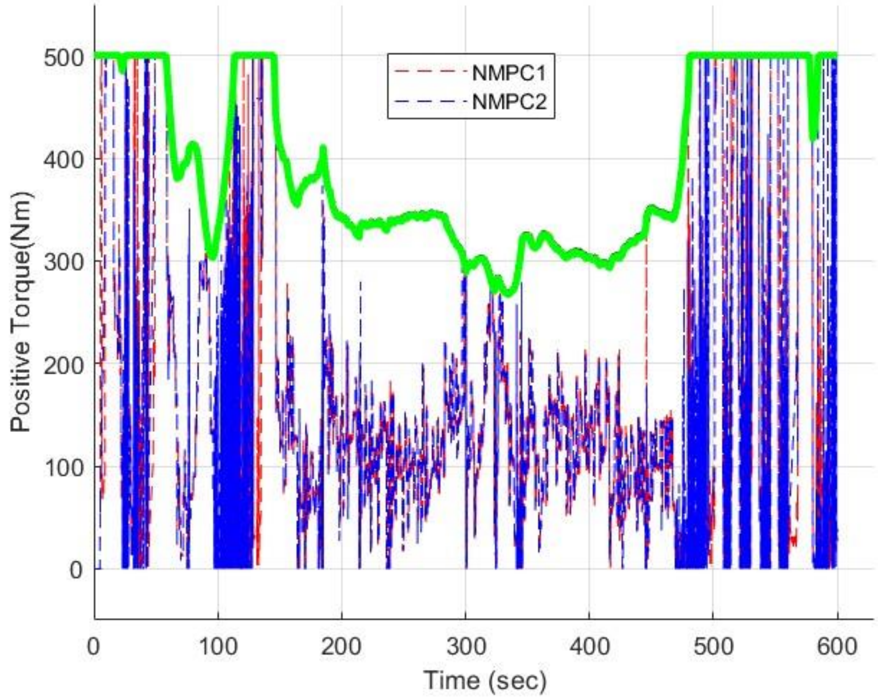

Figure 4. First input channels for the two NMPC Controllers

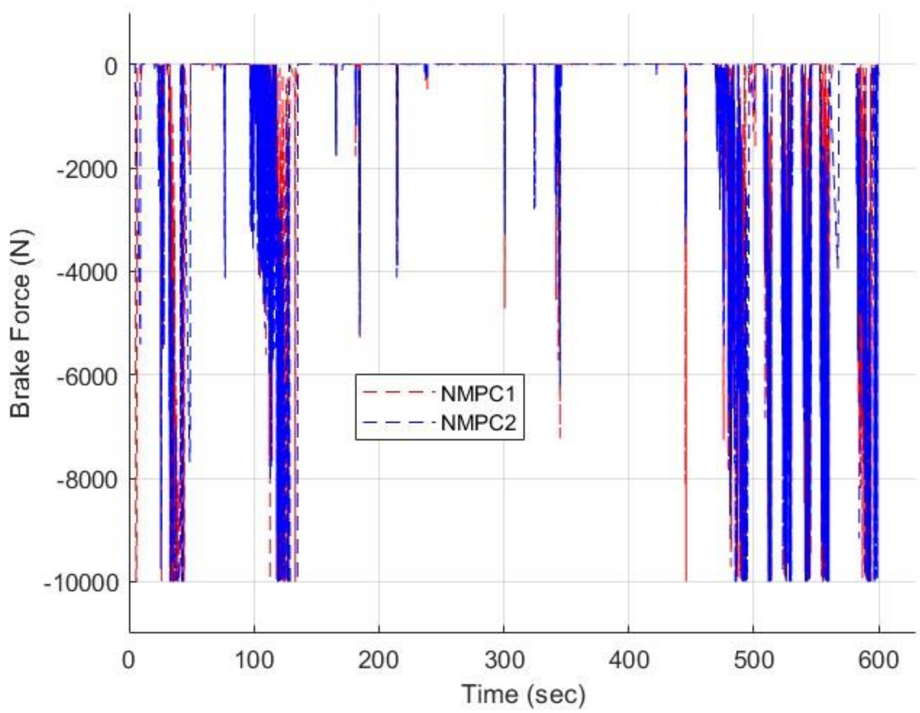

Figure 5. Second input channels for the two NMPC Controllers

From figures 5 and 6, we can see that inputs of the controllers are kept within the input constraints. For example, we can see that the positive torque for both controllers is kept below the green curve in figure 5. This indicates the positive torque is upper bounded by the maximum available output torque, which is the function of velocity and thus the two NMPC controllers have similar profile of maximum available output torque. (That's why there are supposed to be two curves as the upper bounds for the controllers, but we can barely see the second one due to overlap in the figure.)

6. Conclusions

An NMPC approach is proposed to address the minimization problem of velocity tracking errors and energy consumption in an electric vehicle. The proposed controller can generate control signals for online simulation by discretizing the objective function into a finite horizon optimal control problem, which is then solved using dynamic programming. Furthermore, simulation results demonstrate that NMPC controllers can track the reference with significantly greater accuracy compared to what a PID controller can achieve, all while minimizing additional energy consumption.

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