$V[x] = E[x^2] - (E[x])^2$

$E[x^2] = \int_0^1 x^2 f_x(x)\,dx = \int_0^1 x^2(x+\frac{1}{2})\,dx = \int_0^1 (x^3 + \frac{1}{2}x^2)\,dx = \frac{x^4}{4} + \frac{1}{2}\frac{x^3}{3}\Big|_0^1 =$

$\frac{x^4}{4} + \frac{1}{6}x^3\Big|_0^1 =$

$\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

$\therefore Var[x] = \frac{5}{12} - (\frac{7}{12})^2 = \frac{5}{12} - \frac{49}{144} = \frac{60}{144} - \frac{49}{144} = \frac{11}{144} = .076$

$Var[Y] = .076$

(c) Compute the $Cov(X,Y) = E[XY] - E[X]E[Y]$

Fist get $E[XY] = \int_0^1 dx \int_0^1 xy f_{xy}(xy)\,dx$

$= \int_0^1 dx \int_0^1 xy(x+y)\,dy = \int_0^1 dx \int_0^1 (x^2y + xy^2)\,dy =$

$\int_0^1 dx \left(\frac{x^2y^2}{2} + \frac{xy^3}{3}\right)\Big|_{y=0}^{y=1} = \int_0^1 dx\left(\frac{x^2}{2} + \frac{x}{3}\right) = \frac{x^3}{6} + \frac{x^2}{6}\Big|_0^1 =$

$\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$Cov(X,Y) = \frac{1}{3} - \frac{7}{12}\cdot\frac{7}{12} = \frac{1}{3} - \frac{49}{144} = \frac{48-49}{144} = \frac{-1}{144}$

(d) Compute $E[(2X-Y)^2]$

$E[4X^2 + Y^2 - 4XY] = E[4X^2] + E[Y^2] - E[4XY]$

$= 4E[X^2] + E[Y^2] - 4E[XY]$

$= 4\frac{5}{12} + \frac{5}{12} - 4\frac{1}{3} = \frac{5}{3} + \frac{5}{12} - \frac{4}{3} = \frac{1}{3} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4} = .75$

4. Let $X(\mu=1, \sigma^2=16)$, $Y(\mu=3.5, \sigma=2)$; $Z(\mu=-2, \sigma^2=9)$; $Cov[X,Y]=1.0$; $Corr(X,Z)=-0.5$;
$Cov(Y,Z) = 0$

(a) $E[X+Z] = E[X] + E[Y] = 1 + (-2) = -1$

(b) $Var[Y.Z] = Var[X] + Var[Z] + 2\,Cov[Y,Z] = 16 + 9 + 2*0 = 25$

(c) $Cov[(X+Y-Z)(-X+2Y-3Z)] = -1\,Var[X] + 2\,Cov(X,Y) - 3\,Cov[X,Z] + 2\,Var[Y]$
$- 1\,Cov(X,Y) + Cov[X,Z]$ $- 3Cov[X,Z]$
$+3\,3\,Var[Z]$ $- 2Cov(Y,Z)$

$-Var[X] + 2\,Var[Y] + 3\,Var[Z] + 6\,Var[X,Y] - 2Cov[X,Z] - 5Cov[Y,Z]$

$-16 + 2(4) + 3(9) + 6(16*4) - 2(16*9) - 5(4*9)$

$= -16 + 8 + 27 + 384 - 288 - 180 = -65$