STAT5120, Final Exam, Allen Baumgarten

1. [40 POINTS] ACME University administrators want to predict the total number of credit hours a student will complete based on their majors in college, their ACT (standardized admission test) scores, their high school GPAs, and their heights in inches. Data on twelve randomly sampled students from last year are below.

StudentID	Major	ACT	Height.inches	Credit.Hours
1	Math	32	66	121
2	Math	35	69	144
3	Math	28	70	137
4	Math	26	71	152
5	Math	29	64	122
6	Math	34	74	145
7	Chem	27	65	120
8	Chem	26	63	128
9	Chem	29	71	120
10	Chem	25	68	122
11	Chem	24	67	132
12	Chem	31	67	125

(a) [5 POINTS] Write down the model matrix for these data (the X matrix) for finding the least squares coefficient estimates for a linear model for predicting the total number of credit hours a student will take based on major (Math or Chem), ACT score, and height in inches.

Our X matrix would be:

One's	X_1	X_2	X_3
1	Math	32	66
1	Math	35	69
1	Math	28	70
1	Math	26	71
1	Math	29	64
1	Math	34	74
1	Chem	27	65
1	Chem	26	63
1	Chem	29	71
1	Chem	25	68
1	Chem	24	67
1	Chem	31	67

(b) [5 POINTS] Write down the Y vector for these data used to find the least-squares coefficient estimates.

The Y vector would be:

121
144
137
152

122
145
120
128
120
122
132
125

Based on this output, AND based on common sense, which variable(s) do you suggest we keep in the model and which could possibly be discarded? Explain. Remember our goal is to be able to predict the number of credit hours a student will take in college.

Using R, we enter this data into the console and run the lm() function to obtain a basic least squares model as follows:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	24.0662	59.4368	0.405	0.6962
question1$MajorMath	11.1286	6.1092	1.822	0.1060
question1$ACT	-0.7349	0.8985	-0.818	0.4371
question1$Inches	1.7996	0.8827	2.039	0.0758 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.7 on 8 degrees of freedom
Multiple R-squared: 0.5646, Adjusted R-squared: 0.4014
F-statistic: 3.458 on 3 and 8 DF, p-value: 0.07128

My common sense (or lack thereof) notwithstanding, examination of our preliminary results above shows that there is only somewhat statistically significant predictor credit ours based on its somewhat low p-value. This significant predictor is to keep Major. The predictor Height in Inches has a lower p-value but height, insofar as we know, as no bearing on credit hours taken (unless those credit hours for the college basketball team). We should keep Major as its p-value is somewhat significant at .106

(d) [5 POINTS] The pair-wise correlations for all three variables, including the response, are

	ACT	Height.inches	Credit.Hours
ACT	1.0000000	0.3405461	0.2247371
Height.inches	0.3405461	1.0000000	0.6195331
Credit.Hours	0.2247371	0.6195331	1.0000000

Based on this, are there signs of collinearity in the predictor variables? Explain.
Yes, there may be somewhat moderate collinearity between these predictors, specifically, between Height in Inches and Credit Hours at .6195

(e) [5 POINTS] Use the full model output from R in part (c) above to predict the total number of credit hours a student will take in college who is 70 inches tall, had an ACT score of 30, and who majors in math. Using the full model, we would say the predicted outcome for y would be based on:

y = 24.07 + 11.13(Major) + -.735(ACT) + 1.8(Inches)
y = 24.07 + 11.13(Major) + -.735(30) + 1.8(70)
y = 24.07 + 11.13 – 22.05 + 126
y = 139.15

(f) [5 POINTS] Use the full model output from R in part (c) above to predict the total number of credit hours a student will take in college who is 66 inches tall, had an ACT score of 28, and who majors in chemistry.

y = 24.07 + 11.13(Major) + -.735(ACT) + 1.8(Inches)
y = 24.07 + 11.13(Chem) + -.735(28) + 1.8(60)
y = 24.07 + 0 – 20.58 + 108
y = 111.49

(g) [5 POINTS] A new variable, the number of extra curricular activities in high school (measured in years) is discovered, and the output for a new model is given below:

```
> out2 <- lm(Credit.Hours ~ Major + ACT + Height.inches + Extra.Cur.Yrs, data)
> summary(out2)

Call:
lm(formula = Credit.Hours ~ Major + ACT + Height.inches + Extra.Cur.Yrs,
    data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-4.1485 -1.4460 -0.1312  2.1128  3.5405

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  105.22989   24.23350   4.342 0.003386 **
MajorMath     -0.68923    2.73980  -0.252 0.808605
ACT            0.25898    0.35295   0.734 0.486937
Height.inches -0.02086    0.40513  -0.051 0.960374
Extra.Cur.Yrs  1.86433    0.25417   7.335 0.000158 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

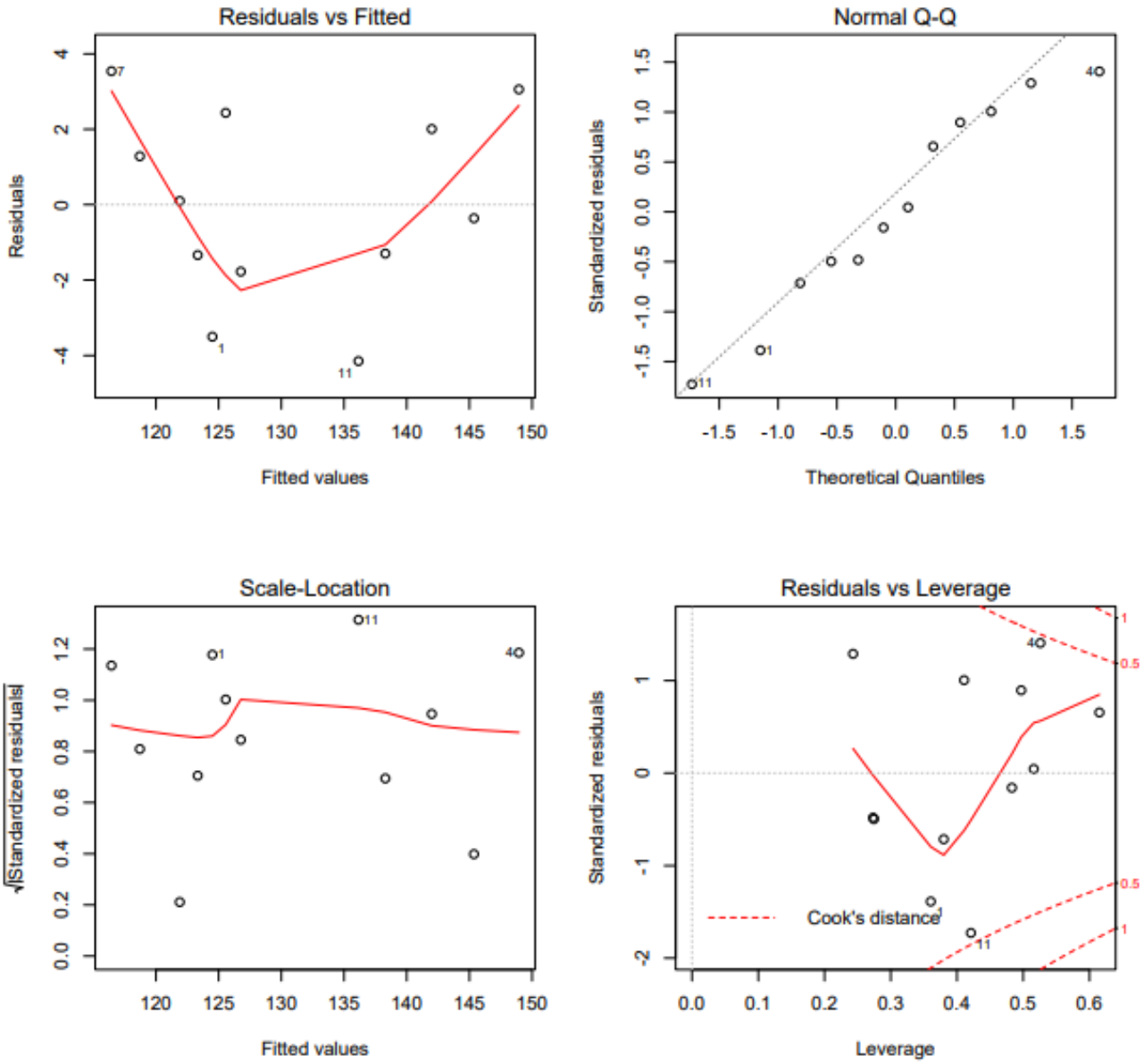
Residual standard error: 3.156 on 7 degrees of freedom
Multiple R-squared:  0.9499,    Adjusted R-squared:  0.9212
F-statistic: 33.16 on 4 and 7 DF,  p-value: 0.0001219
```

Remark on which variables now seem important, what has changed and why. Specifically address

(1) the change in p-values for the t-tests, The t-tests and corresponding p-value changed dramatically for MajorMath variable. Earlier it was somewhat significant with a p-value of .106 but now has change to a p-value of .808, indicating no significance. The t-test and p-value remain mostly unchanged for the ACT variable while the Height in Inches variable climbed from a formerly somewhat significant p-value to a quite insignificant p-value of .96. The final new variable has the significant p-value.

(2) the new Multiple/Adjusted R-squared values, and (3) the values of the beta estimates. Also (4) which (if any) variables would you suggest dropping from the model? The former multiple and adjusted R-square statistics were somewhat helpful in accounting for about 56% and 40% of the variation in y, respectively. Now, in this new model they are extremely high, accounting for the variation in y by 95% and 92%, respectively. I would say based on this new data, we should drop the first three available predictors and keep the final new predictor Extra.Cur.Yrs with a p-value of .000158.

(h) [5 POINTS] Use the diagnostic plots below to remark on whether or not the assumptions have been adequately met. If anything seems wrong, be sure to suggest some kind of appropriate remedial measure.



Looking at these four plots, our Residuals vs. Fitted values shows signs of a curvilinear relationship between the predictors and the outcome variable. A transformation to straighten this out would be

suggested. Perhaps a square-root transformation would help. The Normal QQ plot also shows an outlier in the upper right area of the plot. This outlier should be investigated and if it truly belongs in the data, perhaps a transformation to center and standardize the data may be in order.

2. [5 POINTS] Write down the general matrix equation for finding the least-squares coefficient estimates. Don't write any data values here: write the matrix formula.

$(Y-X_b)^T(Y-X_b)$

3. [5 POINTS] Write down the general matrix equation for the hat matrix H in terms of the X matrix.

$X^T * -X^T$

4. [5 POINTS] Write down the matrix equation for the residuals in terms of the hat matrix H and the matrix Y.

$(Y-X_b)^T - (Y-X_b)$

5. [5 POINTS] Write down the formula for the linear correlation coefficient r when there is just one predictor variable.

$\Sigma (x - \overline{x})(y - \overline{y}) / (x - \overline{x})^2$

6. [5 POINTS] Write down the simple little formula (in terms of r and sample standard deviations) for the simple linear regression slope estimate when there is just one predictor variable.

We need for a linear model 4 things to derive our two beta coefficients:

1. $\overline{x}$
2. $\overline{y}$
3. $(x - \overline{x})(y - \overline{y})$
4. $(x - \overline{x})^2$

Now, the betas are:

5. $\beta_1 = [(x - \overline{x})(y - \overline{y})] / (x - \overline{x})^2$
6. $\beta_0 = \overline{y} + \beta_1(\overline{x})$

7. [5 POINTS] What are the assumptions that need to be checked when constructing a linear model?

We assume that the errors (residuals) have a mean of zero with a standard deviation s^2. We also assume independent random data.

8. [15 POINTS] Complete the following lack-of-fit ANOVA table:

Source	df	SS	MS	F*	p-value
Regression	2	34.783	17.3915	10.52	.021
Residual	3	4.957	1.6523		
Lack-of-Fit	13	39.74			
Pure Error	16	2.110			
Total	21	41.85			

9. [5 POINTS] Which is used in the case of nonconstant error variance when the errors are uncorrelated?
(a) Durbin-Watson test (b) Shapiro-Wilk test (c) weighted least-squares (d) none of these
weighted least-squares

10. [5 POINTS] Which is to be used when errors are found to be correlated, and can also help mitigate nonconstant error variance?
(a) Durbin-Watson (b) Shapiro-Wilk (c) generalized least-squares (d) none of these
none of these

11. [5 POINTS] How can you check for correlated residuals?
(a) Durbin-Watson (b) run cor() on 'neighboring' residuals (c) plot residuals vs. fits (d) all of these
plot residuals vs. fits

12. [5 POINTS] How can you check for normality of residuals?
(a) Anderson-Darling (ad.test) (b) Shapiro-Wilk (shapiro.test) (c) normal probability plot (qqnorm) (d) all of these
all of these

13. [5 POINTS] How can you check for equal variance of the residuals?
(a) plot residuals vs. fits; (b) Levene's test on a partitioning of the data; (c) regress p |residuals| vs. predictor variable and check p-value associated with the slope; (d) all of these.
all of these

14. [5 POINTS] True or False: Cook distances measure the changes in fitted values when removing data points and so are used to identify potential influential observations. TRUE

15. [20 POINTS] The four problems on the next page refer to the following R output.
(a) [5 POINTS] In the output on the previous page, circle the condition numbers.
These are the condition numbers:

```
> sqrt(e$val[1]/e$val)

[1]    1.00000   17.85504   25.15256   60.78472 1647.47771
[6] 5751.21560
```

(b) [5 POINTS] Do the condition numbers seem to indicate anything about some of the variables in particular? Explain. Other than the fact that they are in good condition (sorry) the final two numbers of 1647 and 5751 indicate collinearity in the data.

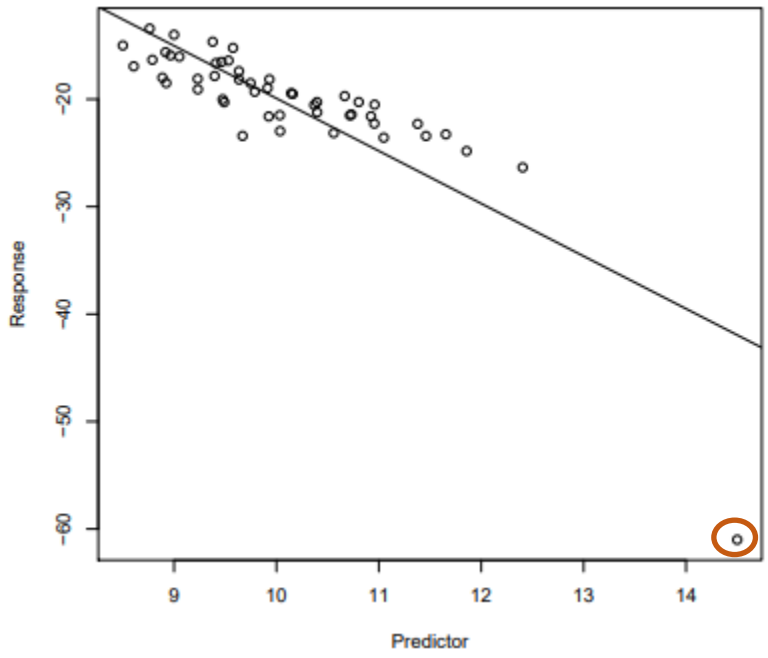
(c) [5 POINTS] In the output on the previous page, draw a rectangle around the variance inflation factors. These are the variance inflation factors:

```
> vif(x)

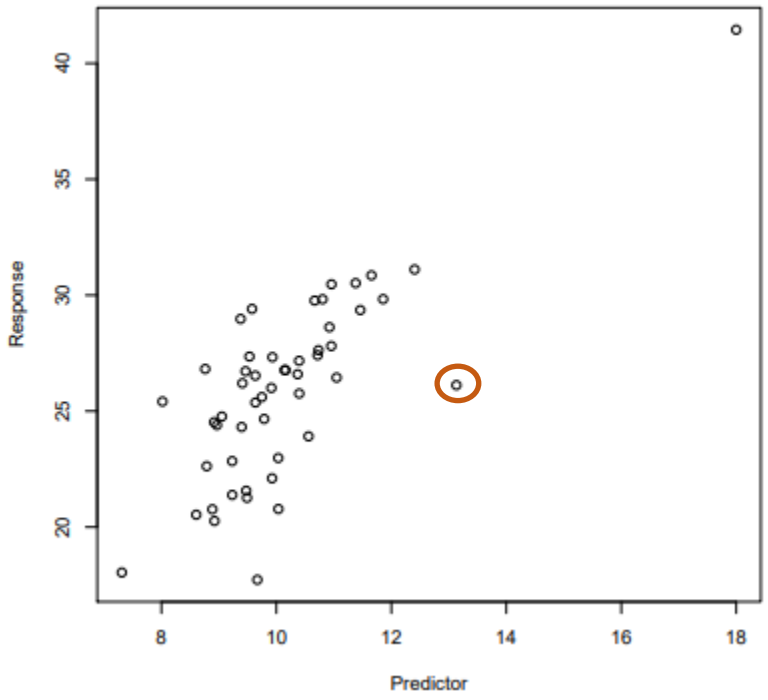
NP.deflator          GNP   Unemployed Armed.Forces
   135.53244   1788.51348     33.61889      3.58893
 Population         Year
   399.15102    758.98060
```

(d) [5 POINTS] Do the variance inflation factors seem to indicate anything about some of the variables in particular? Explain. Yes, the GNP has outliers greatly influencing the model.

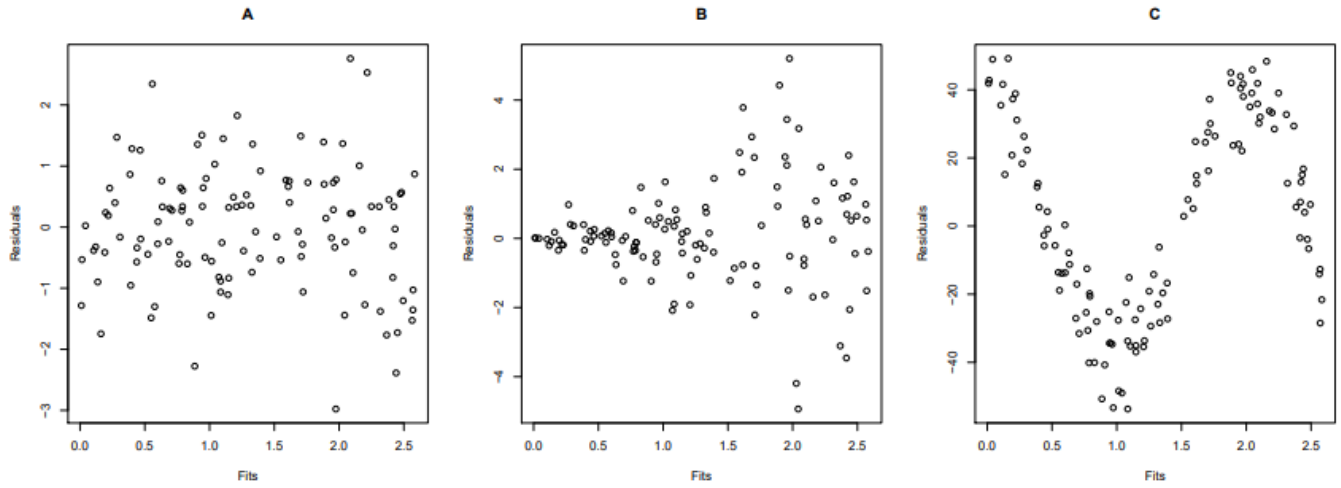
16. [5 POINTS] Circle the best candidate for an influential observation.



17. [5 POINTS] Circle the best candidate for a leverage point.



18. [20 POINTS] The following three plots were made in conjunction with three different linear models of the form lm(y ~ x1 + x2).



Use the plots to answer these questions.

(a) [5 POINTS] Which plot (A, B, or C) most indicates constant error variance?
(a) A (b) B (c) C

(b) [5 POINTS] Which plot (A, B, or C) most indicates the error variance is increasing in the fitted values?
(a) A (b) B (c) C

(c) [5 POINTS] Which plot (A, B, or C) most indicates a nonlinear model might be more appropriate than the one made with lm(y ~ x1 + x2)?
(a) A (b) B (c) C

(d) [5 POINTS] How would you recommend correcting for the problem in plot B? Weighted Least squares might be an option, or a Box-Cox transformation.

19. [5 POINTS] When doing principle components analysis (PCA) or principle components regression (PCR), why is it typically important to standardize the predictor variables? This is used to correct for unequal variance between the predictors or other problems.

20. [5 POINTS] What is the objective function to be minimized in ridge regression?
$\lambda|e|$

21. [5 POINTS] What is the objective function that is to be minimized in lasso regression?
$-\lambda|e|$

22. [5 POINTS] How are the tuning parameters found when doing ridge and lasso regression? We minimize the lambda function

23. [5 POINTS] Match the following link functions to their names:

$\eta(p) = \Phi^{-1}(p)$ This is the Probit link

$\eta(p) = \log(-\log(1-p))$ This is the complimentary log-log

$\eta(p) = \log(p/1-p)$ This is the logit function

24. [5 POINTS] Match the following goodness-of-fit measures to their names.

$1 - \frac{n-1}{n-p-1}\frac{SSE_p}{SSTO}$ This is adjusted R^2

$(y - Xb)^T(y - Xb)$ This is SSE

$SSE_p/MSE + 2p - n$ This is Mallow's C_p

$n\log(SSE_p) - n\log n + p\log n$ This is BIC

$n\log(SSE_p) - n\log n + 2p$ This is AIC

25. [5 POINTS] What is the null deviance and what should you do if it is large? The null deviance is the deviance of the null model with no predictors, only the intercept term included. If it is large, our model does not fit well.

26. [5 POINTS] What is the residual deviance and what should you do if it is large? The residual deviance is the fit of the model errors based on a chi-square distribution with n-1 df. If it is much larger than the 1, the model may have problems.

27. [5 POINTS] If the response is truly a binomial random variable, and the n_i are relatively large, what is the approximate distribution of the deviance? A chi-square distribution.

28. [20 POINTS] Data and R output on the incidence of respiratory disease in infants by sex and feeding method are below.

	Bottle Only	Some Breast with Supplement	Breast Only
Boys	77/458	19/147	47/494
Girls	48/384	16/127	31/464

```
Call:
glm(formula = cbind(disease, nondisease) ~ sex + food, family = binomial,
    data = babyfood)

Deviance Residuals:
      1       2       3       4       5       6
 0.1096  -0.5052  0.1922  -0.1342  0.5896  -0.2284

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -1.6127     0.1124 -14.347  < 2e-16 ***
sexGirl      -0.3126     0.1410  -2.216   0.0267 *
foodBreast   -0.6693     0.1530  -4.374 1.22e-05 ***
foodSuppl    -0.1725     0.2056  -0.839   0.4013
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 26.37529  on 5  degrees of freedom
Residual deviance:  0.72192  on 2  degrees of freedom
AIC: 40.24

Number of Fisher Scoring iterations: 4
```

(a) [5 POINTS] Use the output to predict the probability that a baby girl who only bottle feeds will contract a respiratory disease.
Y = -1.61 -.3126(x) -.6693(x) - .1725(x)
Y = 0.199 - .732 - .512 - .84
All things being equal, a baby girl who feeds has a -.0732 odds of contract respiratory diseases

(b) [5 POINTS] Use the output to predict the probability that a baby boy who only breast feeds will contract a respiratory disease.
Y = -1.61 -.3126(x) -.6693(x) - .1725(x)

Y = 0.199 - .732 - .512 - .84
All things being equal, a baby girl who feeds has a -.0732 odds of contract respiratory diseases

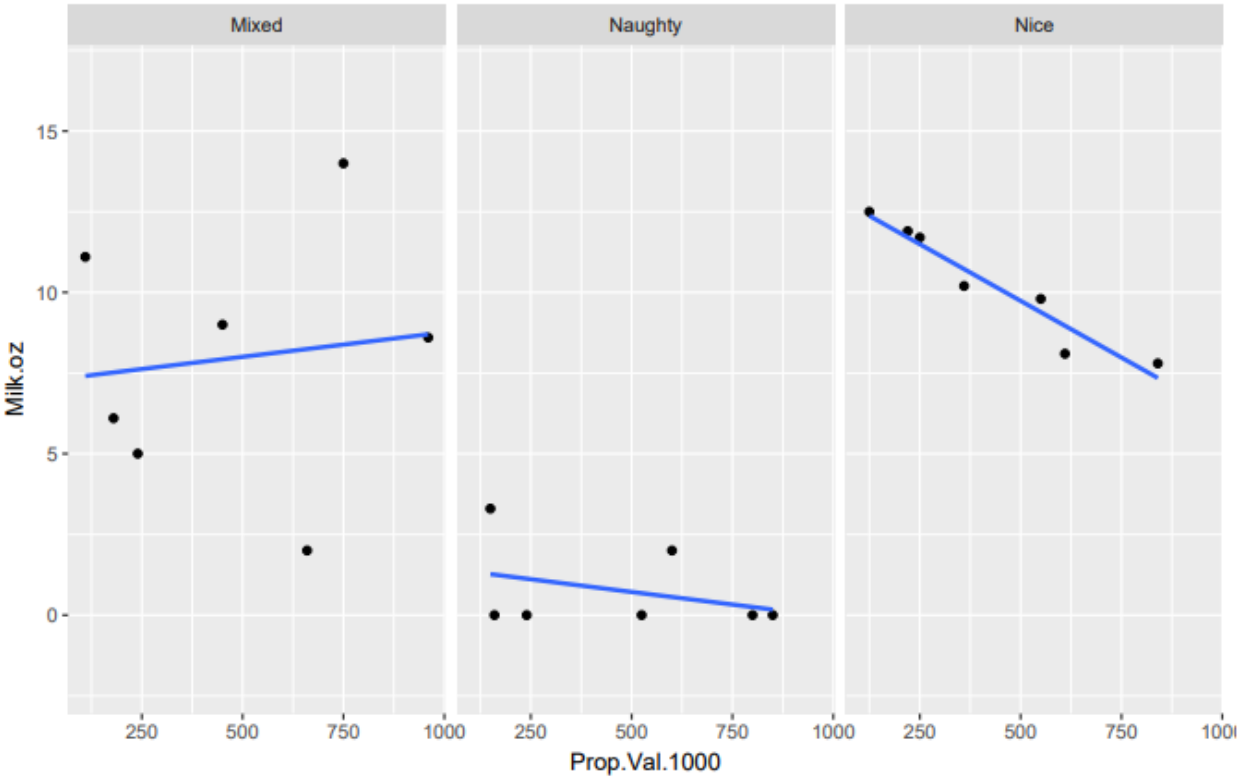
(c) [5 POINTS] Breast feeding reduces the odds of respiratory disease to % of that for bottle feeding.
.0732

(d) [5 POINTS] Is the $\chi 2$ approximation valid for obtaining a p-value for the residual deviance in this case? Explain. In this case, yes: the null deviance is much higher than the residual deviance.

29. [20 POINTS] Santa Claus wants to estimate how many ounces of milk will be left at houses he visits based on (1) the assessed property value of the residence; (2) if the children of the household were naughty, nice, or some of each (mixed). Below are some data he took last year.

(a) [5 POINTS] According to the R output on the previous page, do the interaction terms seem statistically significant? Explain. The interaction terms for BehaviorNaughty and BehaviorNice are statistically significant with very small p-values.

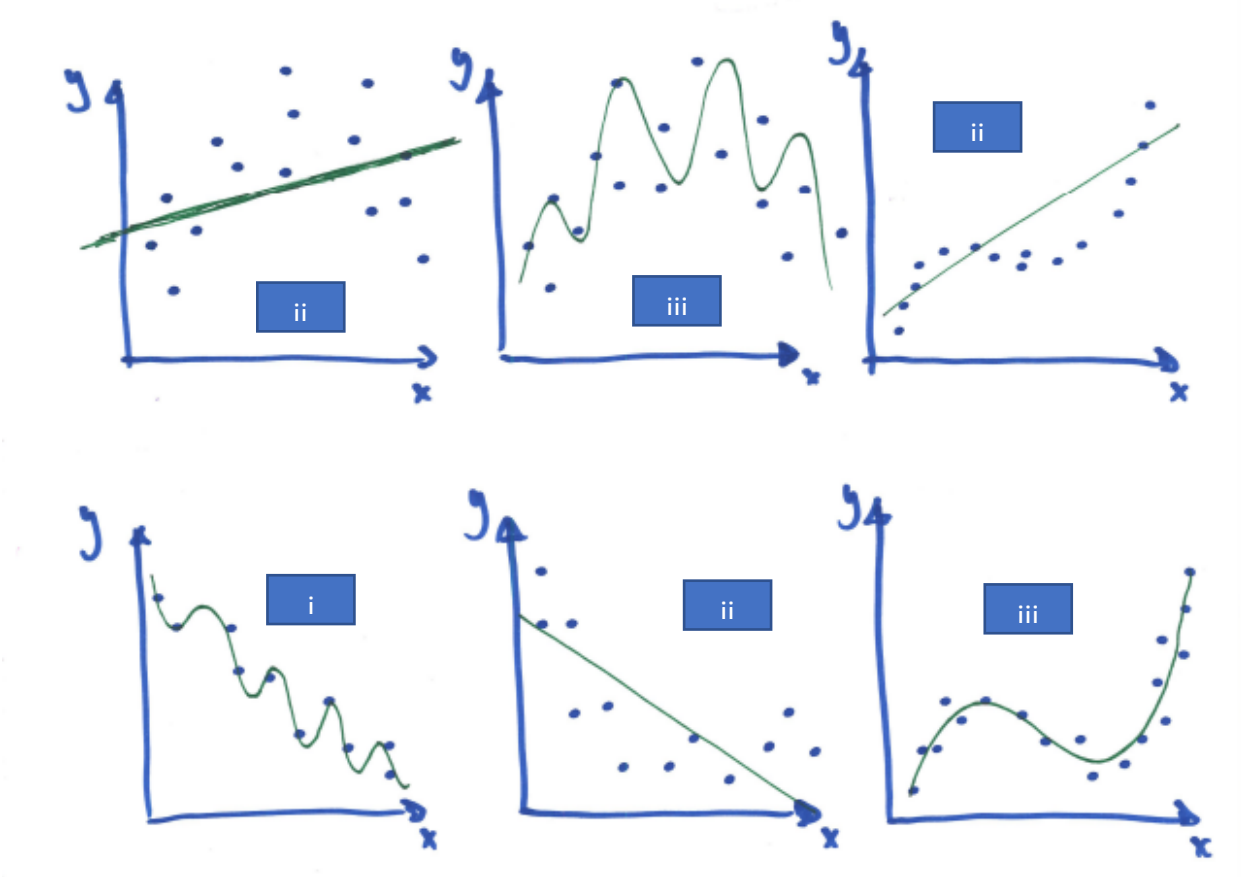
(b) [5 POINTS] Now considering the following plot of the data, do you think the interaction terms are significant? Explain. I would say that based on these nice plots from the ggplot2 package that Nice is significant.



(c) [5 POINTS] Use the model to predict the amount of milk left for Santa at a $500,000 home of a child who was partly naughty, and partly nice (mixed).
Y = 7.24 + .0015(500,000) – 5.74 + 6.01
Y = 757.5

(d) [5 POINTS] Use the model to predict the amount of milk left for Santa at a $150,000 home of a nice child.
Y = 7.24 + .0015(150,000) + 6.01
Y= 238.25

30. [10 POINTS] Which of the following modeling scenarios look (i) like a high bias scenario; (ii) like a high variance scenario; (iii) just about right? Label each of the six plots with (i) or (ii) or (iii).



31. [5 POINTS] What happens to the training error (such as SSE or MSE evaluated on the training set) as the model flexibility increases? It decreases.

32. [5 POINTS] What happens to the test or cross validation error (such as SSE or MSE evaluated on the test or cross validation set) as the model flexibility increases? It decreases.

APPENDIX: R SCRIPTS, MUSINGS ON THE MIAMI DOLPHINS, AND OTHER MINUTIAE

Question 1

```
> setwd("c:/Users/baumgaral/R")
> library(readxl)
> question1 <- read_excel("c:/Users/baumgaral/Data/Practice_Sets/question1.xlsx")
> lmod <- lm(question1$CreditHours ~ question1$Major + question1$ACT + question1$Inches)
> summary(lmod)

Call:
lm(formula = question1$CreditHours ~ question1$Major + question1$ACT +
    question1$Inches)

Residuals:
    Min      1Q  Median      3Q     Max
-10.5287  -6.3170   0.2087   5.7811  10.3511

Coefficients:
                          Estimate    Std. Error   t value  Pr(>|t|)
(Intercept)               24.0662     59.4368      0.405    0.6962
question1$MajorMath       11.1286      6.1092      1.822    0.1060
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