Due date : Feb-20-2020

1. Consider a $\{Z_t\}$ be a sequence of iid random variables such that $Z_t \sim Normal(0, \sigma^2)$. Which of the processes are stationary (weak). If the process is stationary compute the mean and autocovariance function.
   (a) $X_t = a + bZ_t + cZ_{t-2}$
   (b) $X_t = Z_t cos(ct) + Z_{t-1} sin(ct)$
   (c) $X_t = a + bZ_0$
   (d) $X_t = Z_t Z_{t-1}$
2. The time series process $\{X_t\}$ is given by:
$$X_t = e_t + \theta e_{t-1}$$
   where θ is a real number and $e_t = WN(0, \sigma^2)$.
   (a) Compute the autocovariance function for the process when $\theta = 0.8$
   (b) Compute the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$ when $\theta = 0.8$.
3. Let $\{Z_t\}$ be iid N(0,1), and define
$$X_t = \begin{cases} Z_t, & \text{if t is even} \\ \frac{1}{\sqrt{2}}(Z_{t-1}^2 - 1), & \text{if t is odd.} \end{cases}$$
   Show that X_t is a white noise (WN) process with mean 0 and variance 1. Also prove that they are not iid noise.
4. Consider a time series process as:
$$X_t = U_1 sin(2\pi\omega t) + U_2 cos(2\pi\omega t)$$
   where U_1 and U_2 are independent random variables with mean 0 and variance σ^2. Prove that the process is weak stationary.
5. Suppose we want to predict a stationary time series $\{X_t\}$ with zero mean and autocovariance function $\gamma(h)$ at some time in the future, say $t + l$ for $l > 0$. Assume we use the predictor as $\widehat{X_{t+l}} = AX_t$ for $A \in R$. Prove that the mean-square prediction error is minimized by choosing $A = \frac{\gamma(l)}{\gamma(0)}$.
6. Consider two uncorrelated stationary sequences $\{X_t\}$ and $\{Y_t\}$, show that $\{X_t + Y_t\}$ is a stationary sequence and the autocovariance function of $\{X_t + Y_t\}$ is the sum of the autocovariance functions of $\{X_t\}$ and $\{Y_t\}$.
7. Consider the random walk with a drift defined as $x_t = \delta + x_{t-1} + e_t$, for $t = 1, 2, \ldots$, with $x_0 = 0$, where $e_t \sim WN(0, \sigma^2)$.
   (a) Prove that the process can also be written as $x_t = \delta t + \sum_{k=1}^{t} e_k$.
   (b) Compute its mean, variance and auto-covariance function.
   (c) Prove that the process is non-stationary.

(d) Suggest a transformation to make the process stationary.

(e) Simulate and plot the process for positive and negative values of δ (may be $\delta = 2$ vs $\delta = -2$). Explain the effect of δ on the process.

8. Consider the time series process

$$X_t = \beta_1 + \beta_2 t e_t$$

. Show that X_t is not stationary, but the process $Y_t = X_t - X_{t-1}$ is stationary.