Due date : March-17-2020.

1. Find ACF and ACVF of the following processes. Also use R to plot them.
   (a) $X_t = e_t + \frac{5}{2}e_{t-1} - \frac{3}{2}e_{t-2}$ where $e_t \sim WN(0, 1)$
   (b) $X_t = e_t - \frac{1}{6}e_{t-1} - \frac{1}{6}e_{t-2}$ where $e_t \sim WN(0, 9)$
   (c) Which of these models in a and b are (i) invertible, (ii) causal and (iii) stationary ?

2. For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are $ARMA(p, q)$ (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $e_t \sim WN(0, 1)$.
   (a) $X_t + 0.81X_{t-2} = e_t + \frac{1}{3}e_{t-1}$.
   (b) $X_t - X_{t-1} = e_t - 0.5e_{t-1} - 0.5e_{t-2}$.
   (c) $X_t - 3X_{t-1} = e_t + 2e_{t-1} - 8e_{t-2}$.
   (d) $X_t - 2X_{t-1} + 2X_{t-2} = e_t - \frac{8}{9}e_{t-1}$.
   (e) $X_t - 4X_{t-2} = e_t - e_{t-1} + 0.5e_{t-2}$.
   (f) $X_t - \frac{9}{4}X_{t-1} - \frac{9}{4}X_{t-2} = e_t$.

3. For those processes in previous problem that are causal and invertiable Use R to plot the ACF and PACF function. Also compute first 10 coefficients in the stationary/causal representation of those models.

4. Show that for a MA(1) process $|\rho(1)| \leq 1/2$.

5. Let $X_t = e_t + \theta e_{t-1}$ be a MA(1) process, with $e_t \sim WN(0, \sigma_e^2)$. Find another MA(1) process with same ACVF as X_t.

6. Given ACF and PACF functions how can you identify the order of a MA and AR ?

   Extra Credit Question:

7. Show that for a MA(1) process, $X_t = e_t + \theta e_{t-1}$, partial auto correlation at lag 2 is: $\phi_{22} = \frac{-\theta^2}{1+\theta^2+\theta^4}$, where $e_t \sim WN(0, 1)$, and $|\theta| < 1$.

   For Graduate Students:

8. **Consider the process $X_t = \phi X_{t-1} + e_t$, where $e_t \sim WN(0, \sigma^2)$. Assume that the process starts at $t = 0$ and $X_0 = e_0$. Prove that (i) $Var(X_t) = \frac{\sigma^2}{1-\phi^2}[1 - \phi^{2(t+1)}]$ and (ii) $Cov(X_t, X_{t+h}) = \phi^h Var(X_t)$. Is the process stationary as $t \to \infty$?**