ALGEBRA OF EXPECTATIONS

E(X) ≡ Mean of X **=** $\boldsymbol{\sum_X \sum_Y xp(x, y)} = \sum_X [x \sum_Y p(x, y)]$ **=** $\boldsymbol{\sum_X xp_x(x)}$

E(aX+b) = $\sum_X (ax + b)p_X(x) = \sum_X (axp_X(x) + bp_X(x)) =$

$\sum_X axp_X(x) + \sum_X bp_X(x) = a\sum_X xp_X(x) + b\sum_X p_X(x)$ **= aE(X) + b**

E(X+Y) = $\sum_X \sum_Y (x + y)p(x, y) = \sum_X \sum_Y xp(x, y) + \sum_X \sum_Y yp(x, y)$ **=E(X)+E(Y)**

ALGEBRA OF VARIANCES

VAR(X) = E([X –E(X)]^2) = $\boldsymbol{\sum_X \sum_Y [x - E(X)]^2 p(x, y)}$ **=** $\boldsymbol{\sum_X [x - E(X)]^2 p_x(x)}$

VAR(aX+b) = E([aX+b –E(aX+b)]^2) = E([aX+b –aE(X)-b)]^2) =E([aX–aE(X))]^2) =

E(a^2[X-E(X)]^2) = a^{2}E([X-E(X)]^2) **= a^{2}Var(X)**

COV(aX,bY) = E([aX-E(aX)][bY-E(bY)] =E(ab[X-E(X)][Y-E(Y)]) = **abCov(X,Y)**

VAR(X+Y) = E([X+Y – E(X+Y)]^2) = E([X+Y – E(X)-E(Y)]^2) =

E([{X– E(X)} + {Y - E(Y)}]^2) =

E([X-E(X)]^2+[Y-E(Y)]^2+2[X-E(X)][X-E(Y)])=

E([X-E(X)]^2)+E([Y-E(Y)]^2)+2E([X-E(X)][Y-E(Y)]) =

VAR(X) + VAR(Y) + 2COV(X,Y)

And by similar steps,

VAR(X+Y+Z) = VAR(X) + VAR(Y) + VAR(Z) + 2COV(X,Y) +2COV(X,Z) + 2COV(Y,Z)

and: **VAR(**$\boldsymbol{\sum_{i=1}^{n} X_i}$**) =** $\boldsymbol{\sum_{i=1}^{n} VAR_i(X_i)}$ **+ 2**$\boldsymbol{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} COV(X_i, X_j)}$

= $\boldsymbol{\sum_{i=1}^{n} VAR_i(X_i)}$**+** $\boldsymbol{\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} COV(X_i, X_j)}$

The Algebra of Expectations Mantras

The Expected Value of a constant times a random variable is the constant times the expected value of the random variable: E(aX) = aE(X).

The Expected Value of a constant plus a random variable is the constant plus the expected value of the random variable: E(b+X) = b + E(X).

The Expected Value of a sum is the sum of the expected values:
$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$**.**

The Algebra of Variances Mantras

The Variance of a constant times a random variable is the constant squared times the variance of the random variable: $VAR(aX) = a^2 VAR(X)$**.**

The Variance of a constant plus a random variable is the variance of the random variable: VAR(b+X) = VAR(X).

The Variance of a sum is the sum of the variances plus two times all the covariance terms: $VAR(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} VAR(X_i)$ **+ 2*(all the covariance terms).**

The Covariance between a constant, a, times a random variable, X, and a constant, b, times another random variable is ab times the covariance of X and Y: Cov(aX,bY) = abCov(X,Y).