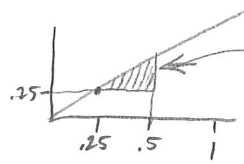
STAT 5140 Homework #1 Allen Baumgarten

Problem 1: Let X + Y be two random variables w/ a joint pdf of

$$f(x,y) = \begin{cases} 3x & \text{for } 0 \le y \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$



$A = \{x,y\}$:
$0 < x < 0.5$
$y \ge 0.25$

(a) Compute $P(0 < X < 0.5 \cap Y \ge 0.25)$

$$P(A) = \int_{0.25}^{0.5} dx \int_{0.25}^{x} 3x\, dy = \int_{0.25}^{0.5} dx \left(3xy\Big|_{0.25}^{x}\right) =$$

$$\int_{1/4}^{1/2} \left(3x^2 - \tfrac{3}{4}x\right) dx = \frac{3x^3}{3} - \frac{3}{4}\frac{x^2}{2}$$

$$= x^3 - \tfrac{3}{8}x^2 \Big|_{1/4}^{1/2}$$

$$= \left[\left(\tfrac{1}{2}\right)^3 - \tfrac{3}{8}\left(\tfrac{1}{2}\right)^2\right] - \left[\left(\tfrac{1}{4}\right)^3 - \tfrac{3}{8}\left(\tfrac{1}{4}\right)^2\right]$$

$$= \left(\tfrac{1}{8} - \tfrac{1}{32}\right) - \left(\tfrac{1}{64} - \tfrac{1}{28}\right) = (0.125 - 0.03125) - (0.0156 - 0.0357)$$

$$= 0.094 - (-0.02) = .114$$

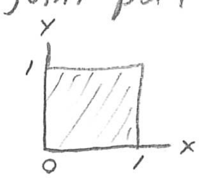
(b) Compute marginal densities of X + Y

$$f_X(x) = \int_0^x f(x,y)\, dy = \int_0^x 3x\, dy = 3xy\Big|_0^x = 3x^2$$

$$\int_0^1 3x^2 dx = \frac{\cancel{3}x^3}{\cancel{3}}\Big|_0^1 = 1$$

Problem 2: Suppose X + Y are random variables w/ a joint pdf of

(a) $f(x,y) = x + y$ for $0 \le x \le 1$ and $0 \le y \le 1$ and 0 if $0 \le x \le 1$



$$f_X(x) = \int_0^1 (x+y)\, dy = xy + \frac{y^2}{2}\Big|_0^1 = x + \tfrac{1}{2} \quad 0 \le x \le 1$$

$$f_Y(y) = \int_0^1 (x+y)\, dx = xy + \frac{y^2}{2}\Big|_0^1 = y + \tfrac{1}{2} \quad 0 \le y \le 1$$

(b) Compute $E(X)$, $E(Y)$, $Var(X)$, $Var(Y)$

$$E[X] = \int_0^1 x f_X(x)\, dx = \int_0^1 x\left(x + \tfrac{1}{2}\right) dx = \int_0^1 \left(x^2 + \tfrac{1}{2}x\right) dx = \frac{x^3}{3} + \frac{1}{2}\frac{x^2}{2}\Big|_0^1$$

$$= \tfrac{1}{3} + \tfrac{1}{4} = \tfrac{7}{12}$$

$$E[Y] = \int_0^1 y f_X(y)\, dx = \int_0^1 y\left(y + \tfrac{1}{2}\right) dx = \int_0^1 \left(y^2 + \tfrac{1}{2}y\right) dx = \frac{y^3}{3} + \frac{1}{2}\frac{y^2}{2}\Big|_0^1$$

$$= \tfrac{1}{3} + \tfrac{1}{4} = \tfrac{7}{12}$$