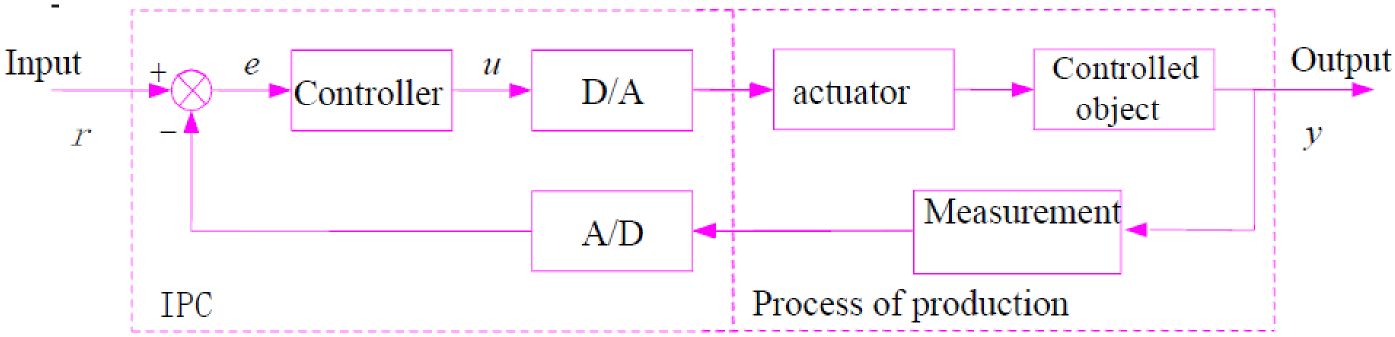
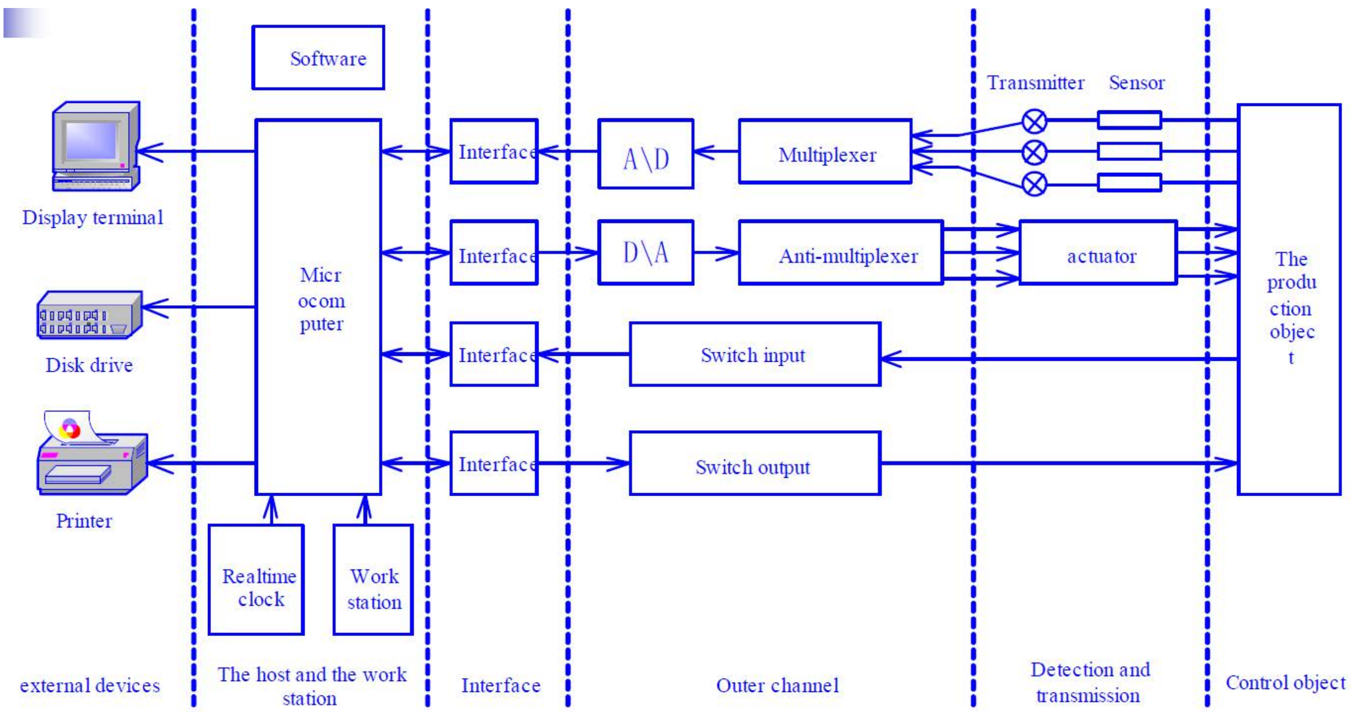
WHS

18 Computer Control Concepts





26 Typical forms of computer control system

DDC, Direct Digital Control

SCC, Supervisory Computer Control

SCC + Analog regulator

SCC + DDC

DCS, Distributed Control System

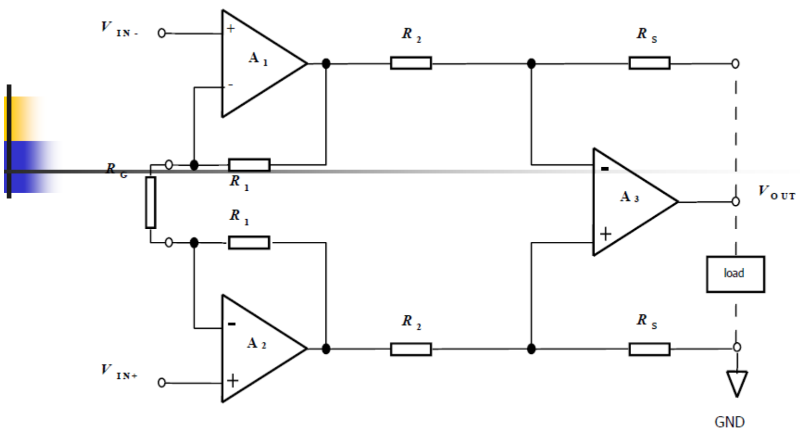
FCS, Fieldbus Control System

63 Operational amplifiers

The current signal between V + and V- points is identically zero, that is, the input impedance is infinite.

The output voltage of the amplifier is constant for a certain value, that is, the output impedance is zero.

Infinite open-loop gain



Classic preamplifier

RG external resistor, dedicated to adjust the gain of the amplifier. Therefore, the gain G of the amplifier has a close relationship with the external resistor RG. Gain formula

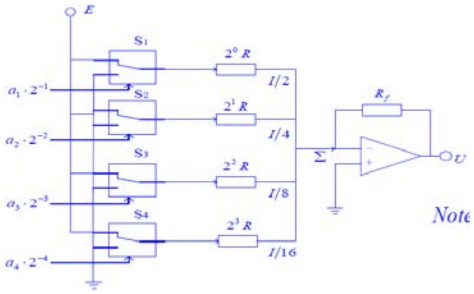
$$G = \frac{V_{OUT}}{V_{IN+} - V_{IN-}} = \frac{R_S}{R_2}\left(1 + \frac{2R_1}{R_G}\right)$$

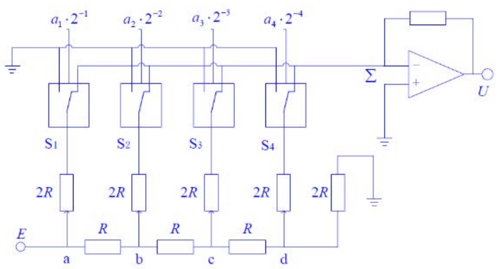
DAC

85 decoding networks

Binary weighted resistor network

T-type resistor network





111 Technical Indicators

Resolution: $\frac{1}{2^n} * 100\%$

Settling time

Output level

ADC

68 ADC Sample and Hold

Holder

- general structure
- tandem-type sample & hold

Switch

- The most common controllable switch is a relay
- CMOS analog switch is a controllable switch

Most important two components

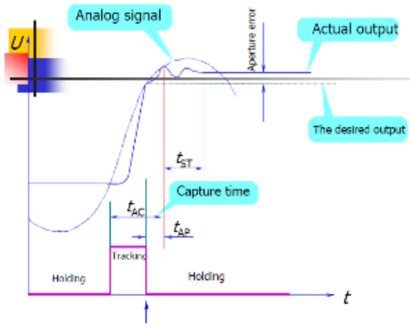
- Sample and holder
- Comparator

96 Type

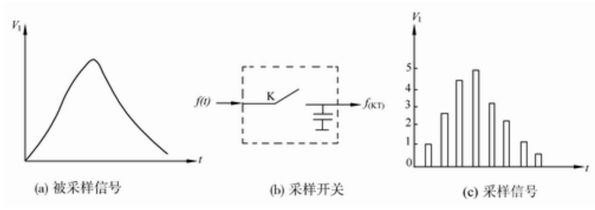
Successive Approximation Register (SAR)

Dual-slope Integration

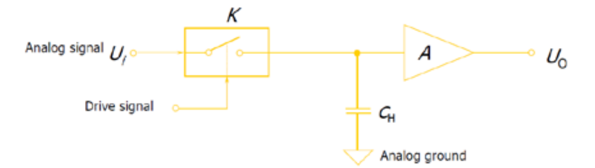
Voltage / Frequency Type

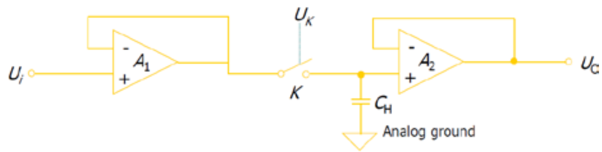


Keep the instructions issued a moment

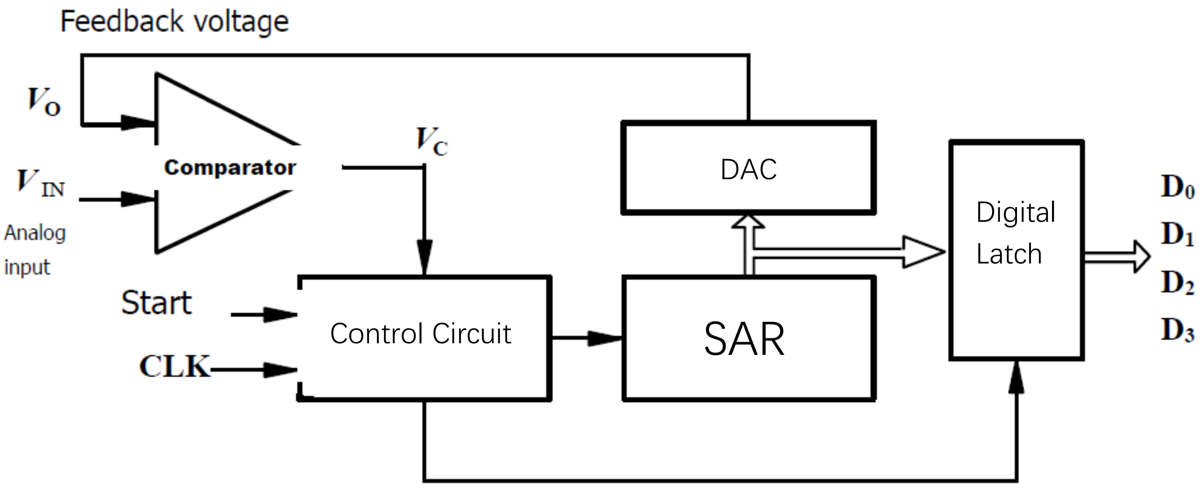


General structure

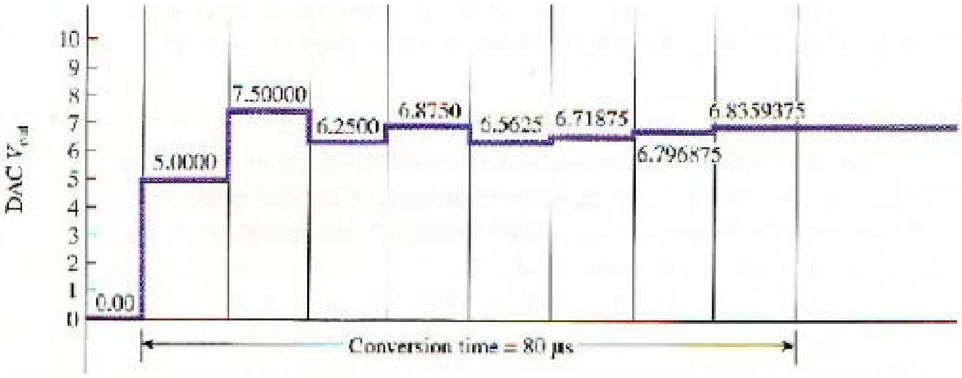




The structure of the tandem-type sample & hold



Example Vin = 6.84 V 8 bit



$$\Delta V = \frac{10}{2^8} = 0.0390625$$

108 Technical Indicators

Resolution: $\frac{1}{2^n} * 100\%$

Range: voltage

Accuracy: $\pm\frac{V}{2^{n+1}}$ (absolute) $\frac{1}{2^{n+1}} * 100\%$ (relative)

Conversion time

Output logic levels

Working temperature

Reference power supply requirements

Interfaces

129 ADC

Unipolar

Bipolar

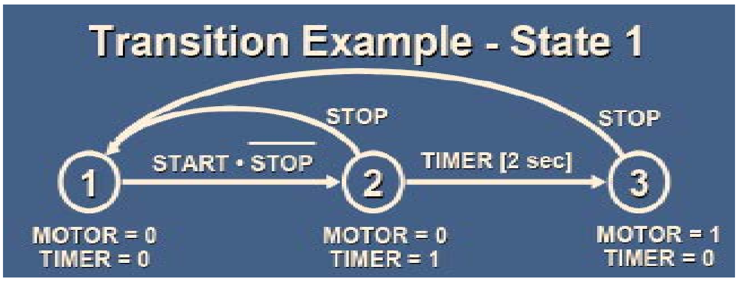
161 DAC

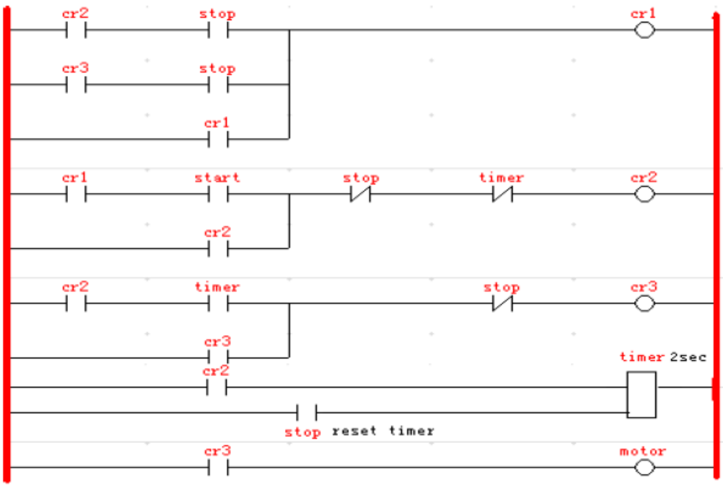
Direct mode

Single buffer mode

Double buffering mode

252 PLC





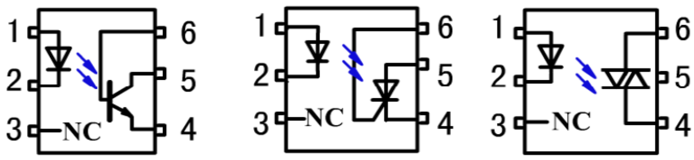
Process Channel

175 Opto-Coupler Isolation: most important anti-jamming technology with hardware in computer control system

Triode type

One-way thyristor type

TRIAC type



179 Digital input channels

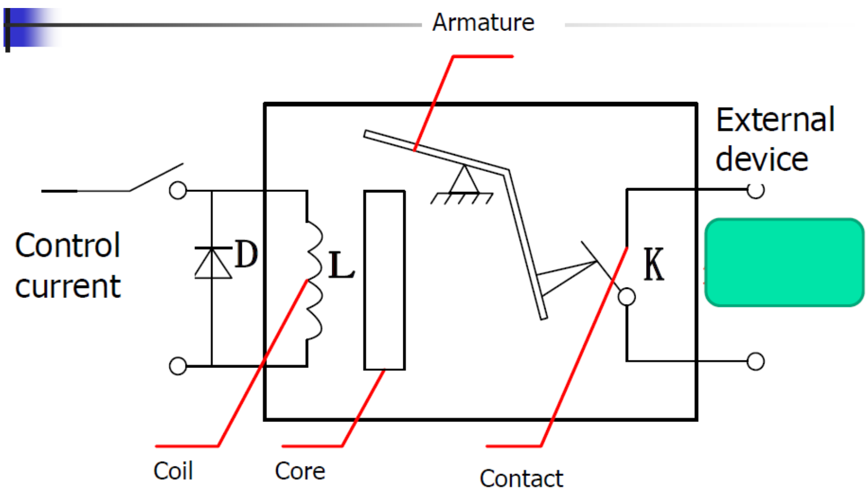
182 Digital output channels

Transistor drive circuit

Thyristor drive circuit

Relay drive circuit →

Solid-state relay drive circuit



Interference

196 Source

External

Internal

198 Way of transmission

Electrostatic coupling

Electromagnetic coupling

Common impedance coupling

204 How it works

Series mode interference

superimposed on the measured signal interference

Common mode interference

added in the gauge interference between any input and ground

Long-term transmission interference

wave reflection when high-frequency signals are transmitted in long wires

WD

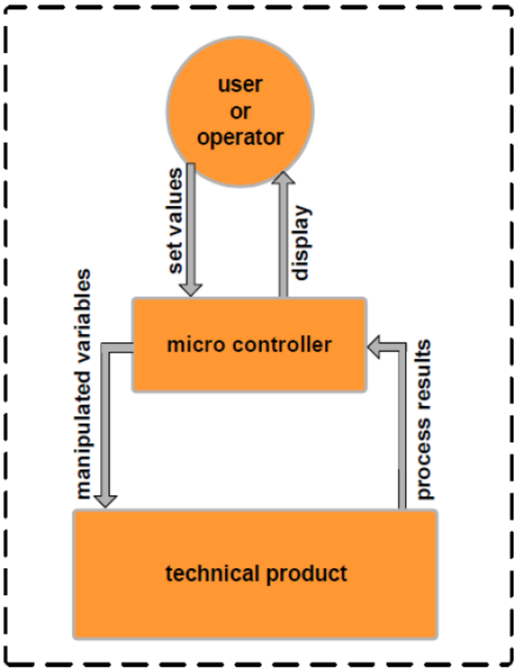
8 Features of a real-time system

Hardware/software system

Data reception, data processing, data delivery to the process within a given time interval

External events

Processing priority



9 Requirements on Real-Time Systems

Timeliness: reaction right on time

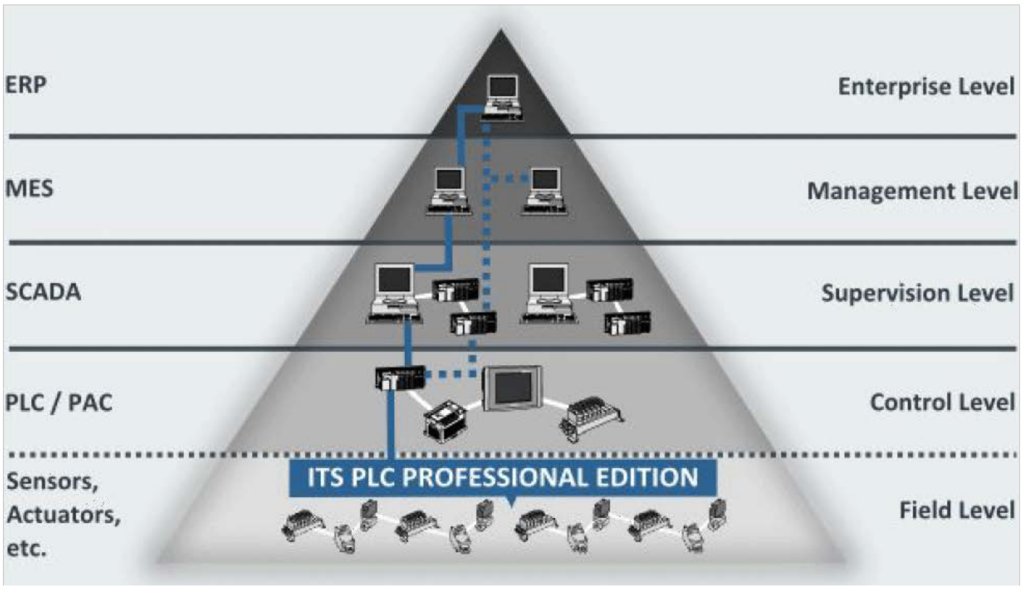
Simultaneousness: simultaneous reaction to various events

Dependability: reliable, safe, available

Predictability: all reactions must be predictable and deterministic

17 Product Automation System Structure Simple →

25 Time Requirements on the different levels in ISA-95 Standard



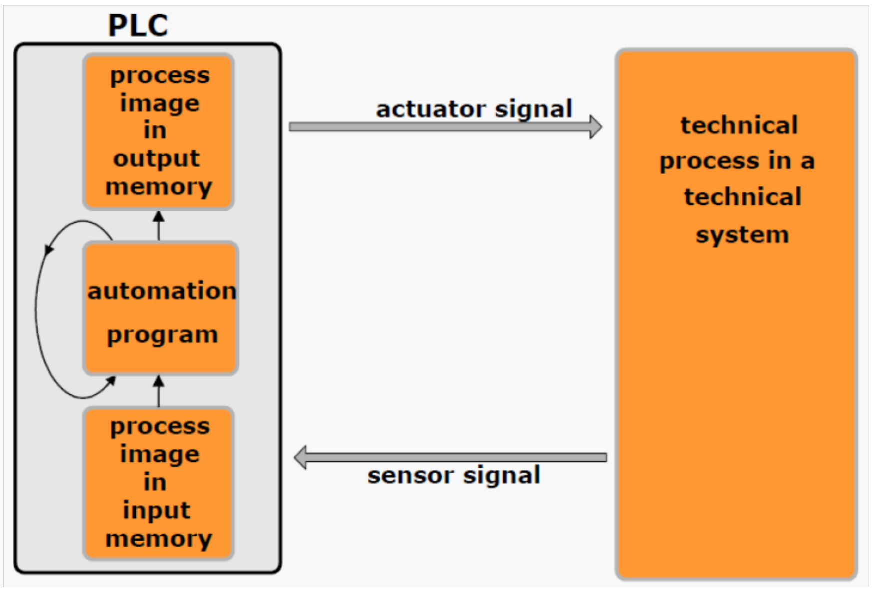
Effects ranging between months-years

Effects ranging between days-weeks-months

Effects ranging between minutes-hours

microseconds-seconds

29 Cyclic Execution of PLCs



Advantage: simple programming through cyclic operational mode

Disadvantage: maximum reaction time on events in the technical process is equal to two program cycles

Program execution time:

– **cycle time is not constant**

– **1 ms per 1000 instructions**

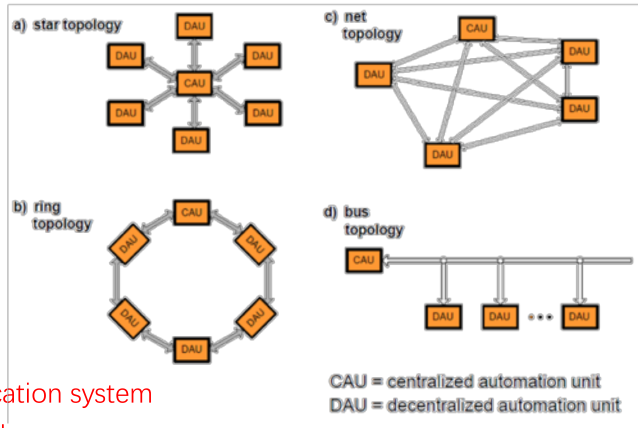
34 Microcontroller Components

CPU = Central Processing Unit micro processor
RAM = Random Access Memory data memory
EPROM/PROM/ROM = Erasable/ Programmable/ Read Only Memory fixed memory
I/O = parallel or serial input/output components process and data periphery
Counter/Timer = generation of clock pulses
Interrupt Controller = handling of hardware interrupts

38 System Communication

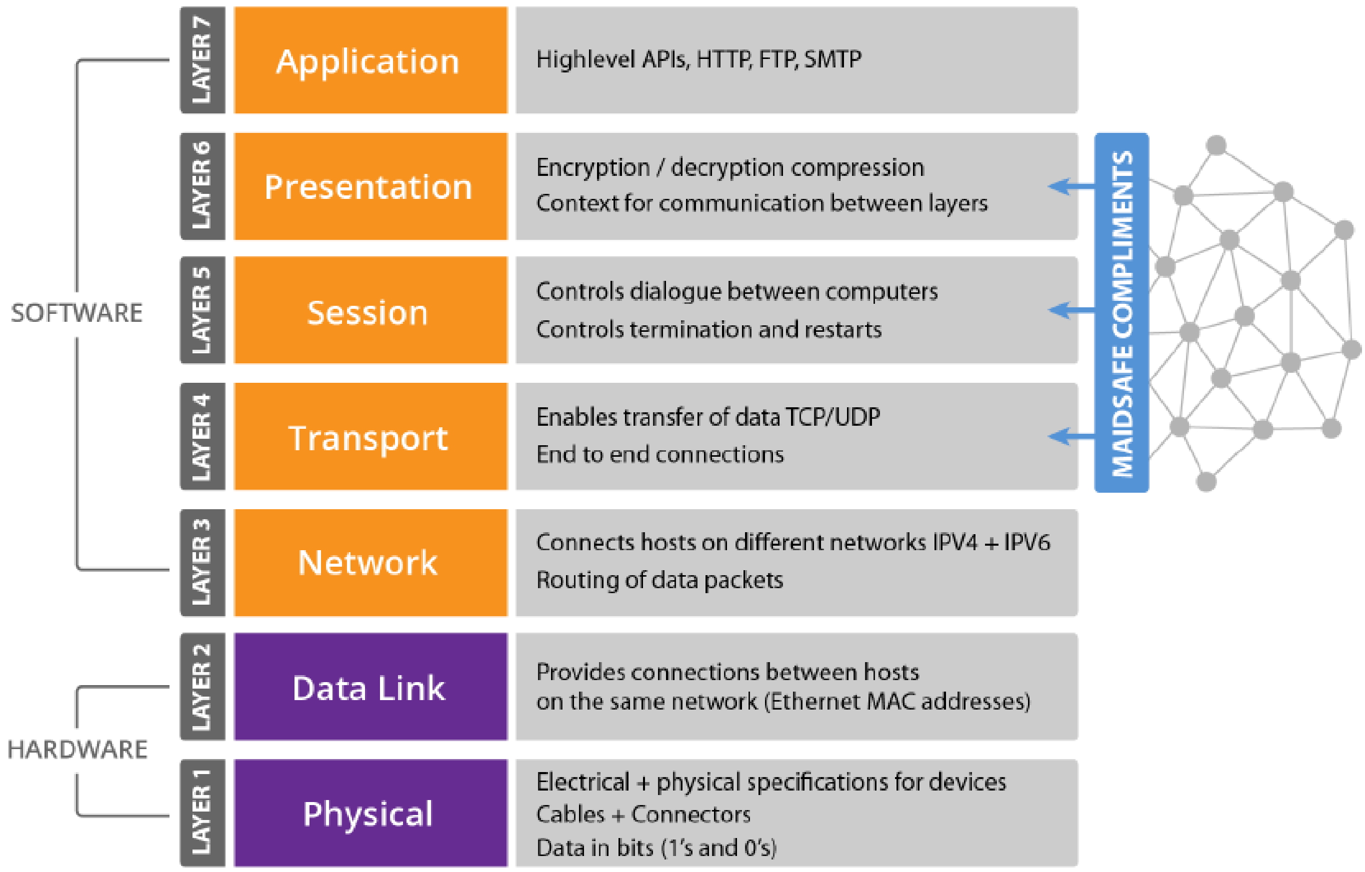
Operation level: large data, short reaction time
Process level: middle data, middle reaction time
Field level: small data, long reaction time

44 Communication in Distributed Automation Systems



Star: failure of the central unit causes failure of the communication system
Ring: each unit can only transmit messages to its direct neighbors
Net: parallel information transmission, short reaction time, many interfaces, high cabling costs
Bus: only one participant at a time is able to send, simultaneous information reception from all participants

45 Open System Interconnection (OSI)



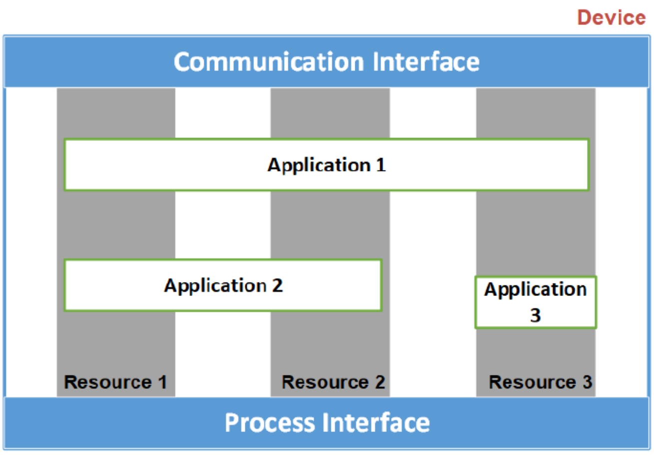
46 Redundancy

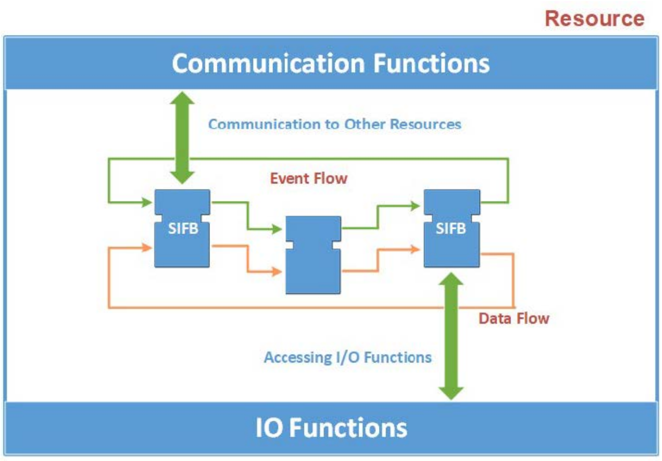
- Hardware redundancy
  - Goal: Detection of hardware failures
  - Operation principle: m-of-n-redundancy majority ruling
  - Effect: no faults, until multiple defects occur
- Software redundancy
  - Objective: detection of errors in software
  - Starting point: software has errors
  - Redundant software alternatives are executed one by one and are compared with a voter (× real-time systems)
- Measured value redundancy
  - redundant measured value
  - dependent measured value
- Time redundancy
  - multiple inquiry of the same measured value in certain intervals
- Levels of fault-tolerance
  - complete fault-tolerance
  - reduced operational capacity
  - transition to a safe state

57 The IEC 61499 Standard Goals

- Reusability
- Portability
- Reconfigurability
- Interoperability

61 IEC 61499 Device / Resource Models





65 IEC 61499 Function Block Models

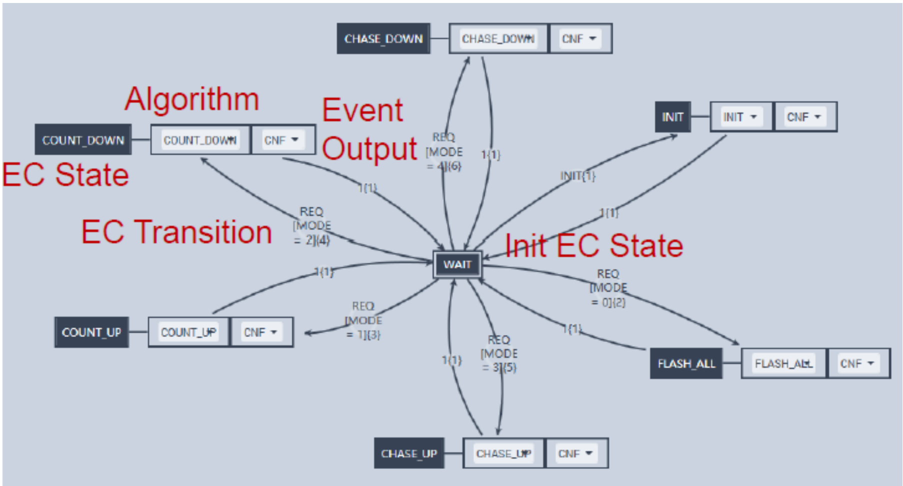
- Atomic function block model
  - Basic function block (BFB)
  - Service interface function block (SIFB)
- Composite function block model
  - Composite function block
  - Sub-application

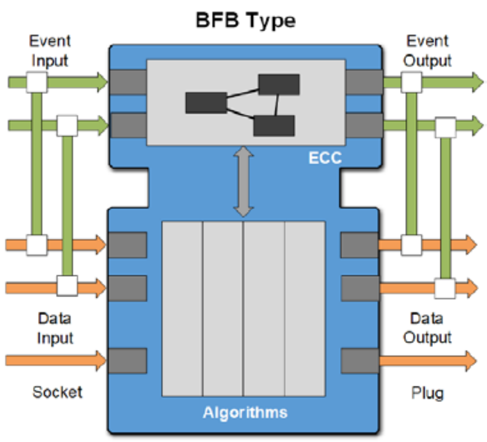
69 IEC 61499 Execution Control Chart

In an ECC, there will be one initial state (START) with some EC states. Switching between EC states is called EC transition. An EC transition consists of two parts: event and guard condition (must at least have one). When the EC transition condition is true, the ECC will jump to the next state, execute the algorithm and emit event output. If more than one EC transition condition is satisfied, the ECC will jump to the highest priority EC state labelled with {priority}.

The execution semantics of the ECC are defined as:

- The ECC will be executed once when one input event is activated.
- Both the event and the guard condition must be satisfied before an EC state can jump to another EC state.
- Once the ECC enters the new state, it will execute the algorithm, emit the output event.
- Then the ECC will check all EC transitions from the new EC state. If any EC transition condition is met, the ECC will jump to that state. As there is no new incoming event during this time, EC transitions with guard condition only will be considered.
- If the event condition is not set in the EC transition condition, this means any event will satisfy the requirement.
- If the EC transition condition is set to 1, this means unconditionally true.





80 IEC 61499 Execution Models

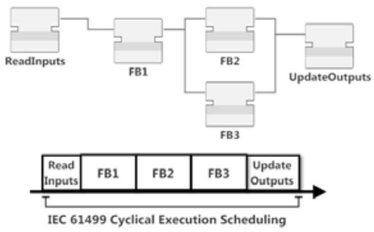
Cyclic

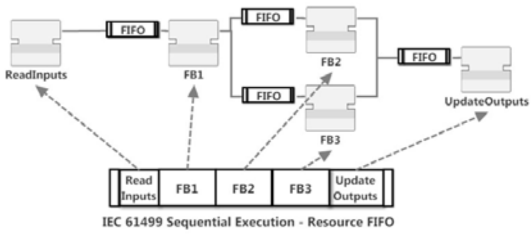
Buffered Sequential

Non-preempted Multi-threading

Synchronous

Time-stamped Discrete-event





85 Design IEC 61499

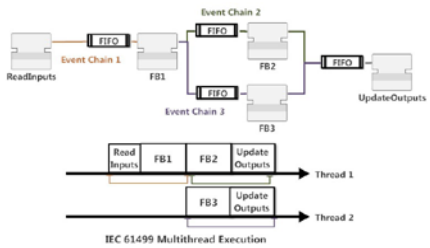
Design the FB Interface

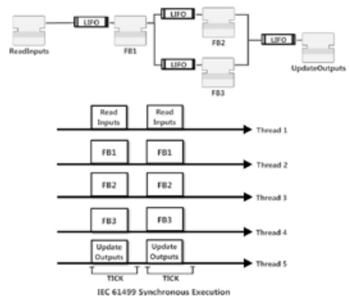
Create Algorithms (PLC Ladder Diagram)

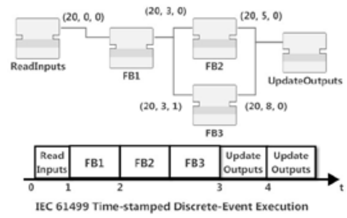
Design ECC (Execution Control Chart)

Design HMI (Human Machine Interface)

Put All FBs Together and Deploy to Runtime





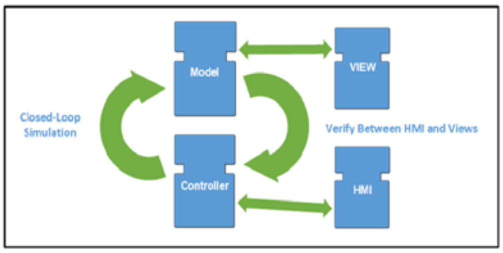


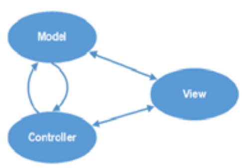
88 IEC 61499 Model-View-Controller (MVC)

A closed-loop between controller block and physical model block.

A view block could be attached to both controller and model.

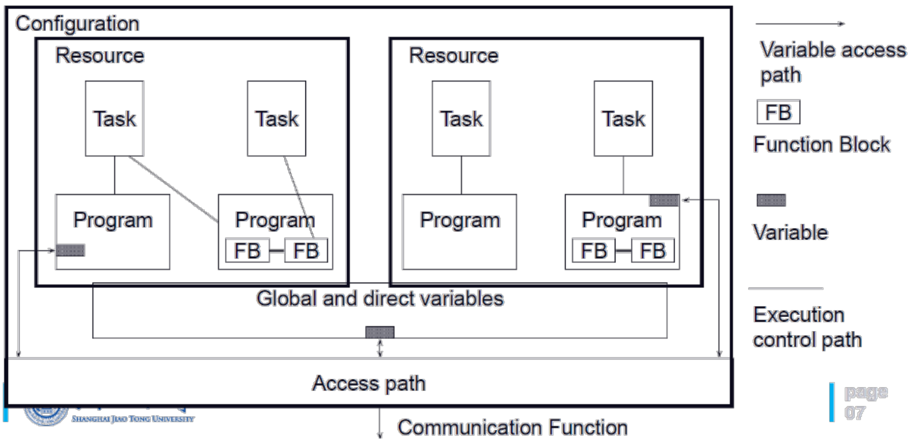
The model block could be replaced with the physical plant directly.





96 IEC 61131-3 language

Ladder Diagram (LD)
Structure Text (ST)
Instruction List (IL)
Function Block Diagram (FBD)
Sequential Function Chart (SFC)

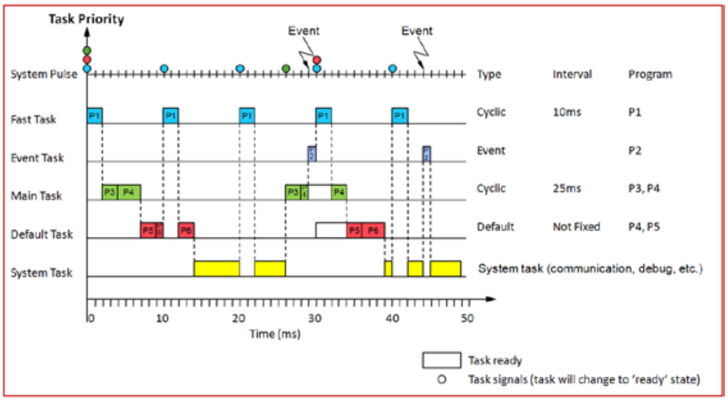


99 Software Model in IEC 61131-3 →

100 IEC 61131-3 Tasks

Class
- Continuous Tasks
- Periodic Tasks
- Interrupt Tasks

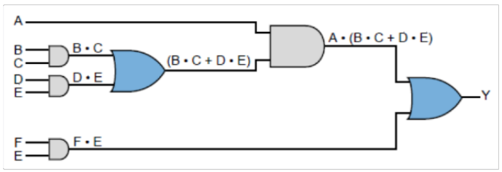
Priority
- System Pulse
- Fast Task
- Event Task
- Main Task
- Default Task
- System Task

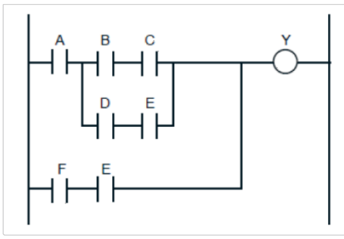


101 IEC 61131-3 Programming Organization Units

Function
Function Block
Program

which can be minimized, using Boolean algebra's distributed rule, to
Y = A · (B ·C +D ·E) + (F ·E)





111 Ladder Diagram Logic →

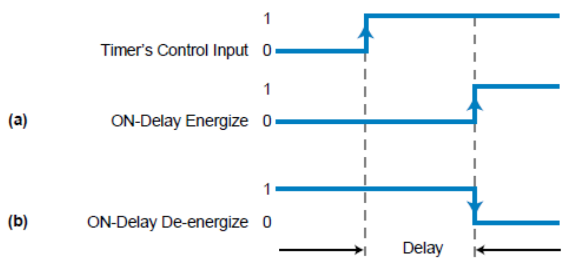
112 Ladder Relay Instructions

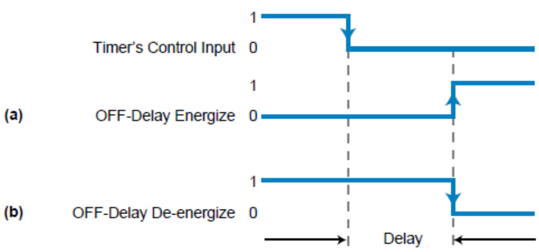
Ladder Relay Instructions
(Purpose: To provide hardwired relay capabilities in a PLC)

Instruction	Symbol	Function
Examine-ON/Normally Open	–| |–	Tests for an ON condition in a reference address
Examine-OFF/Normally Closed	–|/|–	Tests for an OFF condition in a reference address
Output Coil	–( )–	Turns real or internal outputs ON when logic is 1
NOT Output Coil	–(/)–	Turns real or internal outputs OFF when logic is 1
Latch Output Coil	–(L)–	Keeps an output ON once it is energized
Unlatch Output Coil	–(U)–	Resets a latched output
One-Shot Output	–(OS)–	Energizes an output for one scan or less
Transitional Contact	–|↑|–	Closes for one scan when its trigger contact makes a positive transition

Timer Instructions
(Purpose: To provide hardware timer capabilities in a PLC)

Instruction	Symbol	Function
ON-Delay Energize Timer	TON	Energizes an output after a set time period when logic 1 exists
ON-Delay De-energize Timer	TON	De-energizes an output after a set time period when logic 1 exists
OFF-Delay Energize Timer	TOF	Energizes an ouput after a set time period when logic 0 exists
OFF-Delay De-energize Timer	TOF	De-energizes an output after a set time period when logic 0 exists
Retentive ON-Delay Timer	RTO	Energizes an output after a set time period when logic 1 exists and then retains the accumulated value
Retentive Timer Reset	RTR	Resets the accumulated value of a retentive timer





159 SCADA Key Features

Input / Output task: interface between the control and monitoring system and the plant floor
Alarm task: detecting digital alarm points and compare the values of analog alarm points to alarm thresholds
Trends task: collects data to be monitored over time
Reports task: periodic, event triggered or activated by the operator and acquired from plant data
Display task: manages all data to be monitored by the operator and all control actions requested by the operator

161 SCADA Real-Time Constraints

Display: Display of value from RTU (1 to 2 seconds maximum)
Display: Display of entire new display (1 second)
Trend: Retrieval of historical trend and display (2 seconds)
Alarm: Acknowledge of alarm on operator screen (1 second)
RTU: Control request from operator to RTU (1 second critical; 3 seconds noncritical)
RTU: Sequence of events logging at RTU of critical events (1 millisecond)

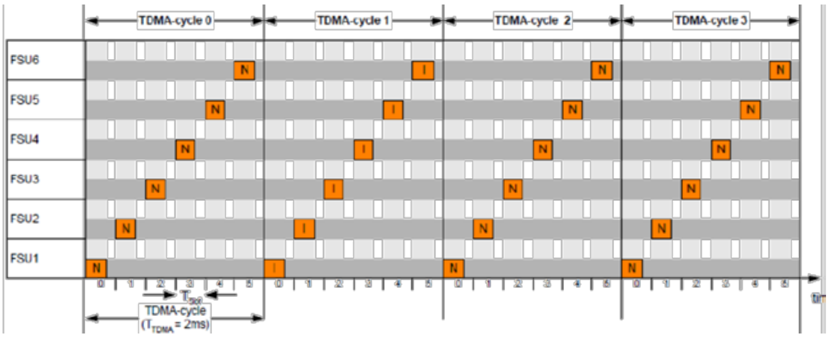
165 Bus Access Methods

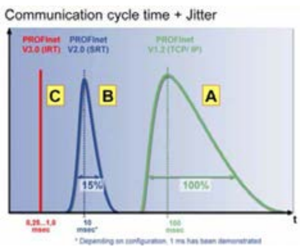
Deterministic bus access

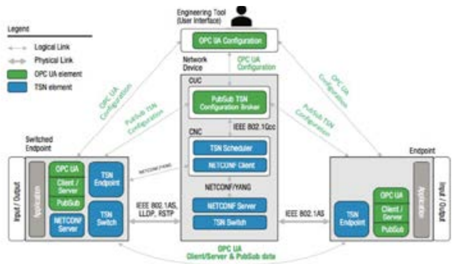
- determined transmission rights
- predictable response time
- permanent monitoring of the bus
- event driven communication
- low/middle bus load
- E.g. master/slave, Token-Passing, TDMA

Random bus access

- random transmission rights
- non-predictable response time
- E.g. CSMA (CSMA/CD, CSMA/CA)







168 TDMA →

Procedure: Within a period (TDMA-cycle) each participant is assigned one or several time slots of determined length.
Principle: distributed shift registers
Advantages: short and constant cycle time, low protocol overhead
Disadvantages: √ synchronization of participants, × autonomous participants, low flexibility, no dynamic adaptation

169 CSMA

Procedure: Each participant checks if bus is free/occupied (carrier sense). If bus is free, it starts transmission. In case of collisions, try transmission once again.
Principle: each participant has bus access without explicit broadcasting right dispatching. (Multiple Access)
CSMA-CD (Collision Detection, Recognition of collisions through data comparison, Transmission repetition after participant specific waiting period)
CSMA-CA (Collision Avoidance, Avoidance of collisions through priority rules)

172 CAN (Controller Area Network) Bus

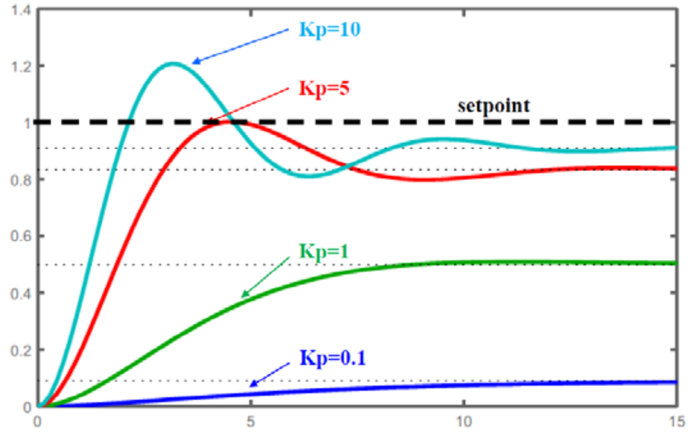
Message oriented addressing | Multi-master bus access method
In case of access conflict: bus arbitration according to priority like CSMA/CA
Short length of messages (0...8 byte) | Fast Transmission (1Mbit/s)
Various error recognition mechanisms | Self-test through error counter
ProfiNet ↑, Ethernet/IP, Ethernet Powerlink, Modbus TCP, OPC UA, OPC UA TSN ↑

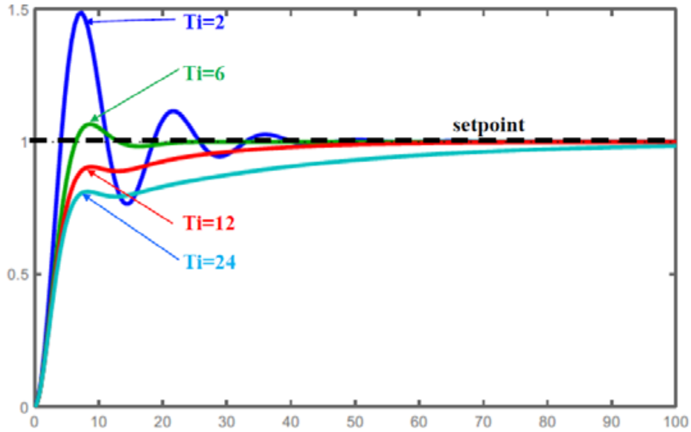
ZSY

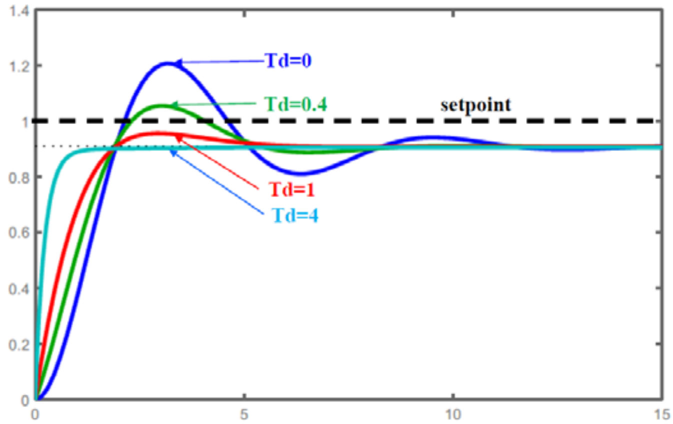
L6 PID Control

Analog PID

23 Parameters







EFFECTS OF INDEPENDENT P, I, AND D TUNING

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady-State Error	Stability
Increasing K_P	Decrease	Increase	Small Increase	Decrease	Degrade
Increasing K_I	Small Decrease	Increase	Increase	Large Decrease	Degrade
Increasing K_D	Small Decrease	Decrease	Decrease	Minor Change	Improve

Digital PID

34 Forms

$$\boxed{u_k = K_\mathrm{p}\left(e_k + \frac{T}{T_\mathrm{i}}\sum_{j=1}^{k} e_j + \frac{T_\mathrm{d}}{T}(e_k - e_{k-1})\right)}$$

$$\boxed{\begin{aligned}\Delta u_k &= K_\mathrm{p}\left(e_k - e_{k-1} + \frac{T}{T_\mathrm{i}}e_k + \frac{T_\mathrm{d}}{T}(e_k - 2e_{k-1} + e_{k-2})\right) \\ &= d_0 e_k + d_1 e_{k-1} + d_2 e_{k-2}\end{aligned}}$$

$$\boxed{u_k = P_\mathrm{p}(k) + P_\mathrm{i}(k) + P_\mathrm{d}(k)}$$

$$d_0 = K_\mathrm{p}\left(1 + \frac{T}{T_\mathrm{i}} + \frac{T_\mathrm{d}}{T}\right),\ d_1 = -K_\mathrm{p}\left(1 + 2\frac{T_\mathrm{d}}{T}\right),\ d_2 = K_\mathrm{p}\frac{T_\mathrm{d}}{T}$$

$$P_\mathrm{p}(k) = K_\mathrm{p} e_k$$

$$P_\mathrm{i}(k) = K_\mathrm{p}\frac{T}{T_\mathrm{i}}\sum_{j=1}^{k} e_j = P_\mathrm{i}(k-1) + K_\mathrm{p}\frac{T}{T_\mathrm{i}}e_k$$

$$P_\mathrm{d}(k) = K_\mathrm{p}\frac{T_\mathrm{d}}{T}(e_k - e_{k-1})$$

Practical Issues Related to Implementation

37 Selecting the sampling period

Sample rule modification: $T \le \frac{1}{Mf_{max}}$ $\quad M = 5\sim10$

In control algorithm: $\frac{t_s}{15} \le T \le \frac{t_s}{6}$ $\quad t_s$ *is settling time*

42 Proportional kick

Setpoint weighting: $w_k \to \beta w_k \quad 0 \le \beta \le 1$

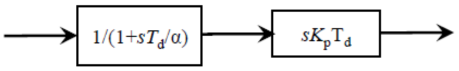
38 Derivative kick

Filtering the D action

Suppressing large changes resulting from setpoint changes

Setpoint weighting

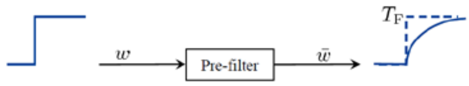
$3 \le \alpha \le 20$



Low-pass filter Ideal D control

In $P_d^m(k)$

$e_k \rightarrow -y_k$



$\bar{w}_i = \sigma \bar{w}_{i-1} + (1-\sigma) w_i$

where $\sigma = \exp(-T/T_F)$, T_F is the time constant of the pre-filter.

- Modified digital PID controller

$$u_k = P_p^m(k) + P_i(k) + P_d^m(k)$$

$P_d^m(k) = K_p(\beta(\sigma \bar{w}_{k-1} + (1-\sigma) w_k) - y_k)$	**(SW)** for P control to reduce the maximum overshooting of the process output and D control
$P_i(k) = K_p \frac{T}{T_i} \sum_{j=1}^{k} e_j = P_i(k-1) + K_p \frac{T}{T_i} e_k$	No change
$P_d^m(k) = \frac{T_d}{T_d + \alpha T} P_d^m(k-1) + \frac{K_p T_d \alpha}{T_d + \alpha T}[-y_k + y_{k-1}]$	**(F)** to limit the amplification of high-frequency noise and **(S)** to damp the response to the setpoint change

44 Integral windup caused by control saturation in position PID

Switch off the I action when the control error is large

$$u_k = P_p(k) + \chi_{e_k} P_i(k) + P_d(k)$$

$$\chi_{e_k} = \begin{cases} 0, & \text{if } |e_k| > \epsilon \\ 1, & \text{if } |e_k| \le \epsilon \end{cases}$$

$$u_k = P_p(k) + P_i(k) + P_d(k) \quad P_i(k) = P_i(k-1) + K_p \frac{T}{T_i} e_k \chi_{e_k}$$

where

$$\chi_{e_k} = \begin{cases} 0, & \text{if } u_{k-1} > u_{\max} \& e_k \ge 0 \text{ or } u_{k-1} < u_{\min} \& e_k \le 0 \\ 1, & \text{otherwise} \end{cases}$$

Stop summation if control signal saturates, control variable sign = control error sign

Re-calculate the error

If the actuator saturates at k, i.e.,

$$u_k > u_{\max} \quad \text{or} \quad u_k < u_{\min}$$

then calculate the effective error

$$e_k \leftarrow \frac{\frac{1}{K_p} u^* - \frac{T}{T_i} \sum_{j=1}^{k-1} e_j + \frac{T_d}{T} e_{k-1}}{1 + \frac{T}{T_i} + \frac{T_d}{T}}$$

where $u *= u_{\max}$ or $u_{\min}$.

51 Noise

Exponential filter

$$\bar{e}_k = \frac{\tau_F}{T + \tau_F} \bar{e}_{k-1} + \frac{T}{T + \tau_F} e_k$$

Moving average filter (λ = 1, or Introducing the forgetting factor λ < 1)

$$\bar{e}_k = \bar{e}_{k-1} + \frac{1}{N}[e_k - e_{k-N}] \quad \Big| \quad \overline{e_k} = \lambda \overline{e_{k-1}} + \frac{1-\lambda}{1-\lambda^N}(e_k - \lambda^N e_{k-N})$$

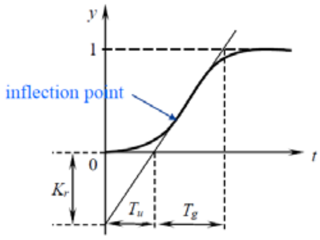
Noise-spike filter (rate-of-change filter)

$$\bar{y}_F(k) = \begin{cases} y_k, & \text{if } |y_k - \bar{y}_F(k-1)| \le \Delta \\ \bar{y}_F(k-1) - \Delta, & \text{if } y_k - \bar{y}_F(k-1) < -\Delta \\ \bar{y}_F(k-1) + \Delta, & \text{if } y_k - \bar{y}_F(k-1) > \Delta \end{cases}$$

Tuning

63 Methods

- Analytical methods
- Heuristic methods
- Frequency response methods
- Optimization methods
- Adaptive tuning methods



Controller	K_p	T_i	T_d
P	$1/K_r$	–	–
PI	$0.8/K_r$	$3T_u$	–
PID	$1.2/K_r$	$2T_u$	$0.42T_u$

Controller parameters for Z-N step response method

69 Ziegler-Nichols Step Response Method

Obtain the step response $y(t) = L^{-1}(\frac{H(s)}{s})$

Find the point with maximum gradient $\dot{y}_{max}\ in\ (t_0, y_0)$ $(\ddot{y} = 0)$

Calculate the auxiliary line $y - y_0 = \dot{y}_{max}(t - t_0)$

$let\ t = 0, we\ get\ -K_r$

$let\ y = 0, we\ get\ T_u$

s^3	1	23	
s^2	9	$15 + Kp$	auxiliary equation
s^1	$\frac{192 - K_p}{9}$	0	critical equation
s^0	$15 + Kp$		

Controller	K_p	T_i	T_d
P	$0.5K_r$	–	–
PI	$0.45K_r$	$0.85T_r$	–
PID	$0.6K_r$	$0.5T_r$	$0.12T_r$

72 Ziegler-Nichols Frequency Method

Obtain the critical gain K_r by Routh-Hurwitz condition in system's $\mathrm{den} + K_p num = 0$

Solve the auxiliary formula (right above the critical formula) to get T_r

L7 Digital Controller Design

88 Z transformation

Time function y(t)	Time sequence y(kT)	z transform Y(z)	Laplace transform Y(s)
$\delta(t)$	$\begin{cases}1, & k=0\\0, & k>0\end{cases}$	1	1
$1_{t\geq 0}$	$1_{k\geq 0}$	$\frac{1}{1-z^{-1}}$	$\frac{1}{s}$
t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$	$\frac{1}{s^2}$
$\frac{1}{2}t^2$	$\frac{1}{2}(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}$	$\frac{1}{s^3}$
e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$	$\frac{1}{s+a}$

- Linearity: $\mathcal{Z}[af(k)+bg(k)] = aF(z)+bG(z)$
- Multiplication by a^k: $\mathcal{Z}[a^k y(k)] = Y(z/a)$
- Real translation theorem: $\mathcal{Z}[y(k-i)] = z^{-i}Y(z)$,
- Initial-value theorem:

$$y(0) = \lim_{z\to\infty} Y(z)$$

- Final-value theorem:

$$\lim_{k\to\infty} y(k) = \lim_{z\to 1}(z-1)Y(z)$$

Approximation of $s = \ln z/T$ is necessary!

$z = e^{sT} \approx 1 + sT$		$s \approx \frac{z-1}{T}$
$z^{-1} = e^{-sT} \approx 1 - sT$	Taylor expansion	$s \approx \frac{z-1}{Tz}$
$z = e^{sT} = \frac{e^{sT/2}}{e^{-sT/2}} \approx \frac{1+sT/2}{1-sT/2}$		$s \approx \frac{2}{T}\frac{z-1}{z+1}$

Indirect approach (Design in continuous domain, discretize controllers to obtain discrete-time destiny)

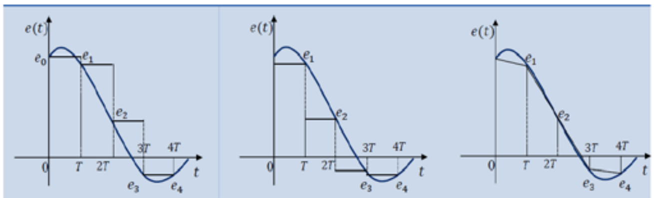
97 Emulation (Laplace domain) ↑

99 Discretization (Time domain)

Proportional	sampling	$e_k \triangleq e(kT)$
$e(t)$	e_k	
integral	backward/forward rectangle, trapezoidal	
$\int_0^t e(\tau)d\tau$		
derivative	backward/forward difference	
$\frac{\mathrm{d}e(t)}{\mathrm{d}t}$	$\frac{e_k - e_{k-1}}{T}, \frac{e_{k+1} - e_k}{T}$	

Forward backward rectangular method, trapezoidal method for integral approximation

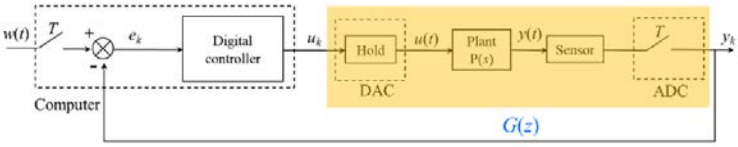
$\int_0^t e(\tau)d\tau \approx T\sum_{j=1}^{k} e_{j-1}$	$\int_0^t e(\tau)d\tau \approx T\sum_{j=1}^{k} e_j$	$\int_0^t e(\tau)d\tau \approx T\sum_{j=1}^{k} \frac{e_j + e_{j-1}}{2}$



$s \approx \frac{z-1}{T}$	$s \approx \frac{z-1}{Tz}$	$s \approx \frac{2}{T}\frac{z-1}{z+1}$

The trapezoidal method is also called the Bilinear / Tustin transformation.

Direct approach (Design in discrete domain, ZOH discretize plant model to obtain design foundation)



$$G(z) = \mathcal{Z}\left[G_{ZOH}(s)P(s)\right] = (1 - z^{-1})\mathcal{Z}\left[\frac{P(s)}{s}\right]$$

$$D(z) = \frac{U(z)}{E(z)} = \frac{a_0 + a_1 z^{-1} + \cdots + a_m z^{-m}}{b_0 + b_1 z^{-1} + \cdots + b_l z^{-l}}$$

The digital controller $D(z)$ is physically realizable **if and only if maxPowerU(z) ≤ maxPowerE(z)**

$$b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_l u_{k-l} = a_0 e_k + a_1 e_{k-1} + \cdots + a_m e_{k-m}$$

$$\Rightarrow \quad u_k = \frac{1}{b_0}\left[\sum_{i=0}^{m} a_i e_{k-i} - \sum_{j=1}^{l} b_j u_{k-j}\right]$$

Needs much memory to store all the previous data $O(m+l)$.

Define the intermediate variable $C(z) = \frac{E(z)}{\sum_{j=0}^{l} b_j z^{-j}}$, then $U(z) = C(z)\sum_{i=0}^{m} a_i z^{-i}$

$$\begin{cases} c_k = \frac{1}{b_0}\left[e_k - \sum_{i=1}^{l} b_i c_{k-i}\right] \\ u_k = \sum_{i=0}^{m} a_i c_{k-i} \end{cases}$$

Memory size is reduced to $O(\max\{m, l\})$.

110 Time Domain

Consider M time steps into the future and collect the control errors e_k and control actions into u_k

$$\{e_0, e_1, \ldots, e_M\}, \quad \{u_0, u_1, \ldots, u_M\} \qquad \min \quad J = \sum_{k=0}^{M}\left[e_k^2 + \alpha(u_k - u_\infty)^2\right]$$

Define the objective function J to measure the derivation of e_k from zero and u_k from the final value u_∞

Solve the minimization by exhaustive search, gradient descent, evolutionary algorithm ..., to obtain $\{a_i, \ b_j\}$ offline

If we are only interested in the steady-state error subject to unit-step, we get $\sum_{j=0}^{l} b_j = 0$ ↓

Recalling that $b_0 \neq 0$ (physical realizability), the simplest form is $b_0 = 1, \ b_1 = -1$

We can recover the ideal PI or PID by choosing appropriate $a_0, \ a_1, \ a_2$.

Final-value theorem to find the steady-state error

$$e(\infty) = \lim_{z \to 1}(z-1)E(z) = \lim_{z \to 1}\frac{1}{1 + D(z)G(z)}$$

In order to have $e(\infty) = 0$, it is necessary that $D(1) = \infty$, i.e., $D(z)$ has a pole at $z = 1$. This means that

Minimal Prototype Control (synthesis in z plane)

117 Raw

We mainly focus on the cases when the reference input is a step, a ramp, or an acceleration function. z transforms of such time-domain polynomial inputs can be written as

$$W(z) = \frac{A(z)}{(1 - z^{-1})^q}$$

→ q is the den power of input

$$G_w(z) = 1 - (1 - z^{-1})^q(1 + f_1 z^{-1} + \cdots + f_p z^{-p})$$

$F(z) = 1$, we obtain the **minimal prototype controller**.

$$D(z) = \frac{1}{G(z)}\frac{G_w(z)}{1 - G_w(z)}$$

$$E(z) = [1 - G_w(z)]W(z) = \sum_{k=0}^{\infty} e(kT)z^{-k}$$

$e(0) \quad e(T) \quad e(2T)$ → how many steps to follow

125 Optimization

Stability

Not only e(t), but also the control output u(t) should be stable.

Include all zeros of $G(z)$ that lie on or outside the unit circle in $G_w(z)$ as zeros

Plant model estimation deviation

Any error in the pole-zero cancellation will diverge as time elapse

Include all poles of $G(z)$ that lie on or outside the unit circle in $1\text{-}G_w(z)$ as zeros.

Intersample ripple

The control is unchanged between two consecutive sampling points.

Include all non-zero zeros of $G(z)$ in the unit circle in $G_w(z)$ as zeros.

134 Conclusion

Include non-zero zeros and z^{-1} of $G(z)$ in $G_w(z)$ and poles on/outside of unit circle of $G(z)$ in $1\text{-}G_w(z)$

Balance coefficient with $G_w'(z)$ and $F(z)$

- Consider a discrete-time $G(z)$ with l pure delays, assume it has w non-zero zeros $b_1, b_2, \dots, b_w$, and v poles $a_1, a_2, \dots, a_v$ lying on or outside the unit circle,

$$G(z) = z^{-l}\frac{(1-b_1z^{-1})\dots(1-b_wz^{-1})}{(1-a_1z^{-1})\dots(1-a_vz^{-1})}G'(z)$$

- The minimal prototype controller is determined by

$$D(z) = \frac{1}{G(z)}\frac{G_w(z)}{1-G_w(z)}$$

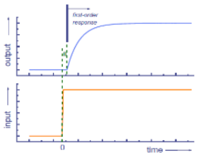
q is the power of input denominator $G_w'(z)$ and $F(z)$ are the unfixed parts with c_i and f_i carrying the other's power to balance

where

$$G_w(z) = z^{-l}(1-b_1z^{-1})\dots(1-b_wz^{-1})G_w'(z)$$
$$1-G_w(z) = (1-a_1z^{-1})\dots(1-a_vz^{-1})(1-z^{-1})^qF(z)$$
$$G_w'(z) = c_1z^{-1}+c_2z^{-2}+\dots+c_sz^{-s}$$
$$F(z) = 1+f_1z^{-1}+\dots+f_pz^{-p}$$

and $s = v + q, p = w + l$. If there are k poles at $z = 1$, it suffices to have $(1-z^{-1})^{\max\{q,k\}}$ in $1 - G_w(z)$.

135 Dahlin's Method (synthesis in z plane)



$$G_w(z) = \frac{(1-\sigma)z^{-l-1}}{1-\sigma z^{-1}}$$

$\sigma = e^{-T/\tau_c},\ l = \theta/T$

$$D(z) = \frac{1}{G(z)}\frac{(1-\sigma)z^{-l-1}}{1-\sigma z^{-1}-(1-\sigma)z^{-l-1}}$$

A negative pole near (-1, 0) on the unit circle has pronounced effect on the response, causing the controller to oscillate or ring. Dahlin suggested that ringing can be eliminated by setting $z^{-1} = 1$ in the ringing term.

$$D(z) = \frac{2.6356(1-0.7413z^{-1})}{(1+0.733z^{-1})(1-z^{-1})(1+0.3935z^{-1})}$$

138 Internal Model Control

The plant model is factored as $\tilde{G}(z) = \tilde{G}_+(z)\tilde{G}_-(z)$

$\tilde{G}_+(z)$ is the bad part, containing

time-delay term z^{-l-1}

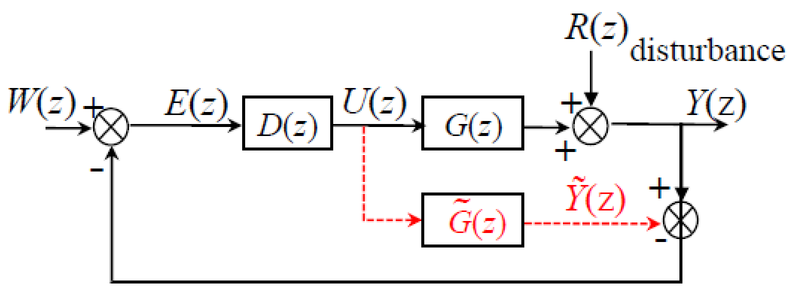
zeros that outside the unit circle

zeros that lie inside the circle near (-1, 0)

steady state gain of unity as denominator (found by setting z = 1)

The controller is obtained by inverting good part of G(z) and multiplying by a first-order filter F(z) $D(z) = \frac{F(z)}{\tilde{G}_-(z)}$

$F(z) = \frac{1-\alpha}{1-\alpha z^{-1}}, \frac{(1-\alpha)z^{-1}}{1-\alpha z^{-1}}$



L8 Model Predictive Control

163 Parameters

N model rank

T sample period

P prediction horizon

M control horizon

√ 164 154 Finite Step Response Model

$\mathbf{y(t)} = \boldsymbol{L}^{-1}(\frac{\boldsymbol{H(s)}}{\boldsymbol{s}})\quad \boldsymbol{S_i} = \boldsymbol{y(iT)}$

$$y(k) = \sum_{i=0}^{N-1} S_i \Delta u(k-i) + S_N u(k-N)$$

$$\hat{y}^o(k+j) \triangleq \sum_{i=j+1}^{N-1} S_i \Delta u(k+j-i) + S_N u(k+j-N)$$

$\boldsymbol{S}$ is the $P \times M$ **dynamic matrix**

$$\boldsymbol{S} = \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ S_2 & S_1 & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ S_M & S_{M-1} & \cdots & S_1 \\ \vdots & \vdots & \ddots & \vdots \\ S_P & S_{P-1} & \cdots & S_{P-M+1} \end{bmatrix}$$

156 Model prediction

$$\hat{\mathbf{y}}(k+1) = \boldsymbol{S}\Delta\mathbf{u}(k) + \hat{\mathbf{y}}^o(k+1)$$

$y(k+j)$ 实际输出

$y^0(k+j)$ 非受迫输出，k 时刻起不进行任何控制

$\hat{y}(k+j)$ 预测输出

$\tilde{y}(k+j)$ k 时刻实际误差作为恒定误差的修正预测输出

158 Model prediction with current error

$$\delta(k) = y(k) - \hat{y}(k) \qquad \tilde{\mathbf{y}}(k+1) = \hat{\mathbf{y}}(k+1) + \delta(k)\mathbf{1}$$

√ 160 Destiny function optimization

$$\min_{\Delta\mathbf{u}(k)} \phi = \tilde{\mathbf{e}}^T(k+1)\tilde{\mathbf{e}}(k+1)$$

$$\tilde{\mathbf{e}}(k+1) = \boldsymbol{w}(k+1) - \tilde{\mathbf{y}}(k+1) \quad = \boldsymbol{w}(k+1) - \hat{\boldsymbol{y}}^{\mathbf{0}}(k+1) - \delta(t)\mathbf{1} - \mathbf{S}\Delta\mathbf{u}(k) = \tilde{\mathbf{e}}^{\mathbf{0}}(k+1) - \boldsymbol{S}\Delta\boldsymbol{u}(k)$$

$$\frac{\partial\phi}{\partial\Delta\mathbf{u}(k)} = 0 \quad \Rightarrow \quad \Delta\mathbf{u}(k) = (\boldsymbol{S}^T\boldsymbol{S})^{-1}\boldsymbol{S}^T\tilde{\boldsymbol{e}}^o(k+1) \qquad \frac{\partial(\tilde{e}(k+1)^T\tilde{e}(k+1))}{\partial\Delta u(t)} = -2S^T\tilde{\boldsymbol{e}}^{\mathbf{0}}(k+1) + 2S^TS\Delta u(t)$$

$$\min_{\Delta\mathbf{u}(k)} \phi = \tilde{\mathbf{e}}(k+1)^T\tilde{\mathbf{e}}(k+1) + \alpha\Delta\mathbf{u}(k)^T\Delta\mathbf{u}(k) \qquad \Delta\mathbf{u}(k) = (\boldsymbol{S}^T\boldsymbol{S} + \alpha\mathbf{I})^{-1}\boldsymbol{S}^T\tilde{\mathbf{e}}^o(k+1)$$

√ Matrix derivation

$$\frac{\partial A^T x}{\partial x} = A = (\frac{\partial Ax}{\partial x})^T \qquad \frac{\partial x^T Ax}{\partial x} = (A + A^T)x \qquad (AB)^T = B^T A^T \qquad \frac{\partial f(x)}{\partial x} = \frac{\partial f^T(x)}{\partial x} \qquad \frac{\partial f(x)}{\partial x} = \frac{\partial g(x)}{\partial x}\cdot\frac{\partial f(x)}{\partial g(x)}$$

162 Implementation

$$\Delta u(k) = \boldsymbol{K}_{c1}\tilde{\boldsymbol{e}}^o(k+1) \qquad \boldsymbol{K}_{c1} \text{ is the firs row of } (\boldsymbol{S}^T\boldsymbol{S} + \alpha\mathbf{I})^{-1}\boldsymbol{S}^T \quad = \boldsymbol{K}_c \quad (M * P)$$

167 Extensions

Step response model for integrating process: replace $\hat{y}(k)$ with $\Delta\hat{y}(k)$

Step response model with known disturbances: a known disturbance variable can be included in step-response model

Reference trajectory specification: specify the reference trajectory to be the filtered setpoint

Prediction for multi-input multi-output systems: r inputs and m outputs

$$\min_{\Delta\mathbf{u}(k)} \phi = \tilde{\mathbf{e}}(k+1)^T\boldsymbol{Q}\tilde{\mathbf{e}}(k+1) + \Delta\mathbf{u}(k)^T\boldsymbol{R}\Delta\mathbf{u}(k) \qquad \Delta\boldsymbol{U}(k) = \left(\boldsymbol{S}^T\boldsymbol{Q}\boldsymbol{S} + \boldsymbol{R}\right)^{-1}\boldsymbol{S}^T\boldsymbol{Q}\tilde{\mathbf{e}}^o(k+1)$$