Soup to Simulation: The Skateboard Ramp

Modeling and Simulation

Instructions

A number of you identified in your feedback that the lectures were not working very well for you, and that you would find it helpful to see a more in-depth example. This exercise is an attempt to address both of these issues.

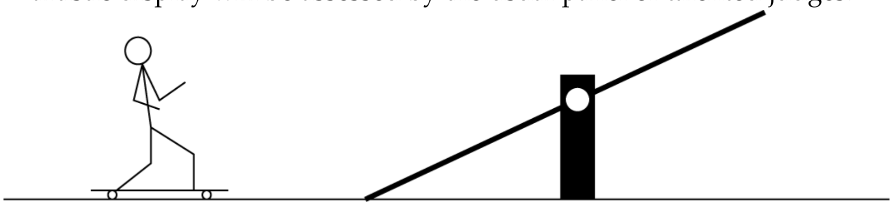
The exercise presents a set of questions to answer about a modeling a physical system: a "see-saw" skateboard ramp. You should attempt the questions on your own in studio; please feel free to talk to your colleagues and your instructor for ideas on how to proceed on each part.

If you're really stuck, *or* when you complete a section, talk to a ninja or an instructor – when you are stuck or or finished with a section, they will provide you with a *possible* solution for that section. Of course, other approaches are possible, so the solutions they provide exemplify one approach, which may not be the best approach (although it is might the simplest one, which makes it the best FIRST approach!).

If after reviewing the solutions for a given part you feel uncertain how the solution was obtained, or you feel that you could not do something similar on a somewhat simpler problem, **identify the questions and concerns you have**. There will be an opportunity on Wednesday to talk through questions about the solutions.

Announcement from World Skateboarding Federation

We are pleased to announce that the federation has just voted to include a new skateboard ramp competition in the next World Games. Unlike a typical skateboard ramp, this one is free to pivot about a support point. Skateboarders approach the ramp on a flat surface and then coast up the ramp; they are not allowed to put their feet down while on the ramp. If they go fast enough, the ramp will rotate and they will gracefully ride down the rotating ramp. Technical and artistic display will be assessed by the usual panel of talented judges.



We are now looking for a team of engineers to design a new ramp which will allow a rider to accomplish the task outlined above. There are two design requirements:

- A simple model that allows an estimate of the length of ramp and mass of ramp for a typical rider approaching at a typical velocity.
- A dynamic simulation of the system, based on a model with enough detail to predict the minimum velocity a rider needs to get over the ramp and down the other side.

Key Frames

Create a set of key frames for this system that help you think through what the different parts of the system are, and how they move with respect to one another. Be sure to try to capture the different types of behavior you think might occur! These key frames should come in handy below both when you try to identify state variables, and when you try to do some estimation!

Abstraction Class, State Variables and Parameters

Given the system, the question, and your key frames, what class of abstraction do you expect to use for this system? In other words, how many different parts will you deal with? Will you be treating these parts as rigid bodies or as particles? Are the parts constrained in their motion, or unconstrained? Make a diagram and a table that define the state variables you will need to keep track of in order to model the dynamics of this system, as well as the relevant parameters for the system . Your table should include both a symbol for each state variable or parameter, a description of what the state variable or parameter tells you, and why it is important.

Relevant Forces (and Torques)

Identify all of the different forces and torques that might be important in this system. Be sure to think about forces between parts of the system, as well as forces between the system and the rest of the universe. Make a table of the symbols, descriptions, and dependencies of the forces and torques. In your dependencies, identify both state variable and parameter dependencies!

Graphical Abstraction: Free Body Diagrams

Draw free body diagrams for each particle or rigid body that you have identified as being a necessary component of your abstraction. Please use notation that is consistent with your force table!

Simple Estimates

It is often a good idea to use your brain before you get really stuck into the detailed modeling. Can you set some bounds on this problem? Are there conditions under which it is obvious that the skateboarder would not make it over the ramp? Are there conditions under which it is obvious that the skateboarder will make it over?

Equations of Motion

Using the principles of conservation of momentum and conservation of angular momentum, come up with second order differential equations that describe the time evolution of the system.

Thinking it through

Whenever you have some complicated governing equations, it's a good idea to try to explain why they make sense. To the greatest extent possible, explain each term in your DE's.

From second order to first order

Once you have your differential equations, you'll want to transform them into a set of first order DE's that will be compatible with an ODE solver. Do so!

Implement

Create the core functions for building a simulation of the system – in other words, write an appropriate "slope" function that can be used with `ode45`.