Simple Estimates

It is often a good idea to use your brain before you get really stuck into the start modeling. Can you set some bounds on this problem? Are there conditions under which it is obvious that the skateboarder would not make it over the ramp? Are there conditions under which it is obvious that the skateboarder will make it over?

Well, looking at the key frames, and thinking about this from an energy perspective, it seems pretty clear that a lower bound is set by the energy required to get up to the pivot. If the skateboarder's kinetic energy is below this, the outcome is guaranteed to be outcome 1 above.

Similarly, it seems pretty clear that if the skateboarder is going too fast, outcome 2 above will happen. One obvious example of this is if the skateboarder's kinetic energy is above the graviational potential energy of getting to the end of the (unrotated) ramp.

Thus, our first pass would at least give us a range of velocities that MIGHT work for the skateboarder:

$$mg\frac{L}{2}\sin\theta_0 < \frac{1}{2}mv^2 < mgL\sin\theta_0$$

where θ_0 is the ramp's designed angle, and L is the ramp's length.

We could put a further limit on this by thinking about the time it would take to make the ramp horizontal – i.e., to rotate the ramp through $\theta_0/2$ – by fixing the skateboarder at the far end of the ramp, and comparing this to the time for the skateboarder to roll back down to the middle of the ramp assuming the ramp was fixed. This would give us some insight into how the mass of the ramp would impact the problem – at the point when these two times are the same, you are around the "breakpoint" of the system; if $t_{horiz} \gg t_{L/2}$, you know the ramp is too heavy; if $t_{horiz} \ll t_{L/2}$, you can be pretty confident that the ramp is light enough. To do this, we'll assume constant accelerations and angular accelerations (this approach will turn this into much more of a "textbook" problem).

The moment of inertia of such a system would be given by

$$I = m(L/2)^2 + \frac{1}{12}ML^2$$

and, assuming that θ_0 is not too big, the gravitational torque would be *about*

$$\tau = -mgL/2$$

Note that this ignores the effect of angle in the torque, which is of course significant as θ gets larger. But so long as $\theta_0 < \frac{\pi}{8}$ or so, this should be an OK approximation. In this case, we'd expect that

$$\theta(t) = \theta_0 + 1/2\alpha t^2$$

where α is the angular momentum, given by

$$\alpha = \frac{\tau}{I}$$

This gives us a time to horizontal of

$$t_{horiz} \approx \sqrt{\frac{2\theta_0 I}{-\tau}} = \sqrt{\frac{2\theta_0(m(L/2)^2 + \frac{1}{12}ML^2)}{mgL/2}} = \sqrt{\frac{\theta_0 L(1+\frac{M}{3m})}{g}}$$

We could compare this to the time that it would take the skateboarder to roll back a distance $L/2$:

$$t_{L/2} \approx \sqrt{\frac{2d}{a}} = \sqrt{\frac{L}{g\sin\theta_0}}$$

and since we've assumed θ_0 is relatively small, $\sin\theta_0 \approx \theta_0$ Thus, the condition that $t_{horiz} \approx t_{L/2}$ becomes

$$\theta_0\sqrt{1+\frac{M}{3m}} \approx 1$$

or, put another way, for a given value of M and m, the maximum value of θ_0 that will potentially work is about

$$\theta_{max} \approx \frac{1}{\sqrt{1+\frac{M}{3m}}}$$

Note that this has the right limiting behavior: if $M \gg m$, $\theta_{max} \to 0$ – in other words, the ramp is too heavy for anything other than an almost horizontal initial angle. On the other hand, when $m \approx M$, θ_{max} can be quite a bit larger.

Now at this point we don't really have enough information to design the ramp, but we at least have some insight into what the bounds of the problem should be. In other words, we know that we probably want to be designing a ramp that has a height appropriate for typical skateboarder velocities – i.e.,

$$h_{pivot} < \frac{v_{max}^2}{2g}$$

and that the mass of ramp should be determined both by the mass of the skateboarder and by the initial angle of the ramp.