Thinking it through

Whenever you have some complicated governing equations, it's a good idea to try to explain why they make sense. To the greatest extent possible, explain each term in your DE's.

Let's look first at the equation for $\ddot{r}$. The first term on the RHS side, $-g\sin\theta$, is a classic "block on a ramp" term: if the ramp is slanted, the block tends to accelerate down the ramp. The second term, $r\dot{\theta}^2$, is relatively easy to make sense of as well – if the ramp is spinning at some velocity, the block's speed along the ramp will increase in an outward direction. For example, imagine a bead on a spinning wire: the bead will get flung off the end of the wire. This looks like a so-called "centrifugal force" (which of course is not a real force, but a result of the fact that the block would have to accelerate in order to stay at a given r on the ramp!).

Now, if we examine $\ddot{\theta}$, there are three things that require interpretation: the two terms in the numerator, and the denominator. The first term in the numerator, $-mgr\cos\theta$, looks like the traditional torque due to a gravitational normal force – pretty easy to buy. The second term,$-2mr\dot{r}\dot{\theta}$, takes some more thought: it says that if the block is moving along the ramp, and the ramp is rotating, there is an additional torque that tends to slow down the ramp. This takes some thought, but makes sense if you think about it. Finally, the denominator, $mr^2 + I$, is pretty easy to interpret: it's the total moment of inertia of the system (ramp + skateboarder).