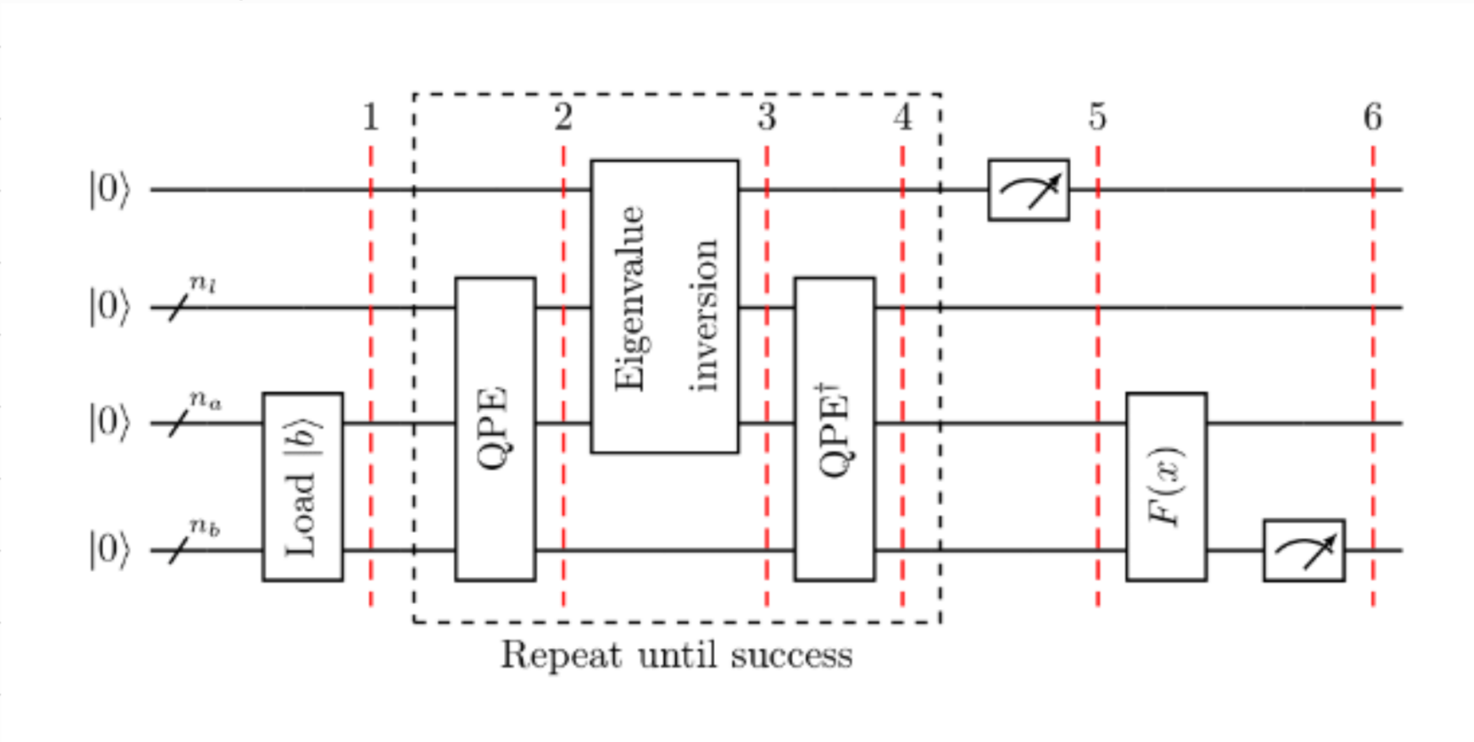
< HHL Algorithm에 대한 정리 >



< 0. Background for HHL Algorithm >

(1) fundamental concept.

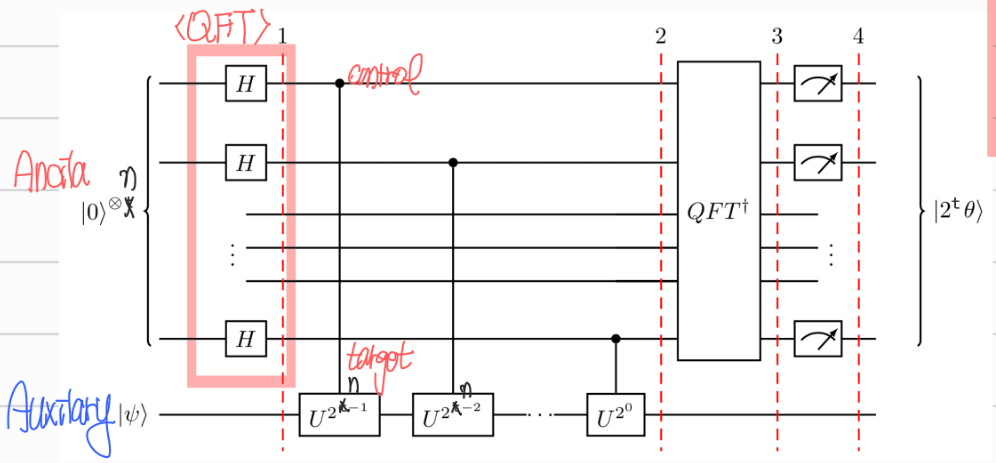
$$A|x\rangle = |b\rangle$$

$$A = \sum_{j=0}^{N-1} \lambda_j |u_j\rangle\langle u_j| \longrightarrow A^{-1} = \sum_{j=0}^{N-1} \lambda_j^{-1} |u_j\rangle\langle u_j| \quad (\text{where } \lambda_j \in \mathbb{R})$$

$$|b\rangle = \sum_{j=0}^{N-1} b_j |u_j\rangle \quad (\text{where } b_j \in \mathbb{C})$$ → 왜 complex 여야하는가?

$$\therefore |x\rangle = A^{-1}|b\rangle = \sum_{j=0}^{N-1} \lambda_j^{-1} |u_j\rangle \cancel{\langle u_j | u_j\rangle} b = \sum_{j=0}^{N-1} b \cdot \lambda_j^{-1} |u_j\rangle$$

(2) Quantum Phase Estimation.



* U gate의 eigenvalue 표기.
$U|\psi\rangle = e^{i \cdot 2\pi \cdot \theta}|\psi\rangle$

여기서 controlled-U gate로 Fourier Basis 상에서 rotation이 일어나게 됨.

QFT를 실시하는 경우, $\frac{x}{2^n} \times 2\pi \times (k+1)$로 (where k; qubit 위치, x; QFT를 하는 수)

큐빗개수 아래부터

위상값도가 주어졌다.

(QFT) $\boxed{\theta = \frac{x}{2^n}}$ $\longrightarrow$ $QFT|x\rangle = \frac{1}{\sqrt{2^n}} \bigotimes_{k=0}^{n} \left(|0\rangle + e^{\frac{2\pi i x}{2^k}} |1\rangle\right)$

$\hookrightarrow U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$; Qiskit notation.

여기서 오해하지 말아야하는 사항은 Qiskit 상에서는

$2\pi \times \frac{x}{2^n}$을 θ로 둔것이 아니라, $\theta = \frac{x}{2^n}$로 두었다.

∘ 단계별 Statevector의 표현

$|\psi_0\rangle = |0\rangle^{\otimes n} |\psi\rangle$ Auxilary qubit. (Initial state)

Ancila qubit.

$|\psi_1\rangle = \frac{1}{\sqrt{2^n}} (|0\rangle + |1\rangle)^{\otimes n} |\psi\rangle$ (Take H gate to each ancila qubit)

① Recall, $U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$ (Consecutive application of U gate)

where $\theta \equiv \frac{x}{2^n}$

Eigenvalue에 2^j번 적용됨.

$\therefore U^{2^j}|\psi\rangle = U^{2^{j-1}} \cdot (e^{2\pi i\theta})|\psi\rangle = e^{2\pi \cdot 2^j i\theta}|\psi\rangle$

② Recall, Controlled-rotation에서

$CROT\,|0, x_k\rangle = |0\, x_k\rangle$

$CROT\,|1, x_k\rangle = \exp\left(\frac{2\pi i}{2^k} x_j\right)|1 x_j\rangle$

control target

와 같은 역할.

$\Rightarrow$ Controlled-unitary (CU)또한 응용하기 적용, $0 \le j \le n-1$

(n = Ancila qubit의 수)

$$|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \left(|0\rangle + e^{2\pi i \cdot 2^{n-1}\theta}|1\rangle\right) \otimes \cdots \otimes \left(|0\rangle + e^{2\pi i 2^0\theta}|1\rangle\right) \otimes |\psi\rangle$$

control qubit이 1일 경우만 CROT이 적용되므로!

$$= \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{2\pi i\theta k}|k\rangle \otimes |\psi\rangle$$ (where k: Integer representation of n-bit binary num.)

(check!) $n=3$인 경우

$$|\psi_2\rangle = \frac{1}{\sqrt{2^3}} \left(|0\rangle + e^{2\pi i \cdot 2^2\theta}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i \cdot 2^1\theta}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i \cdot 2^0\theta}|1\rangle\right) \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2^3}} \Big[|000\rangle + e^{2\pi i \cdot \overset{=1}{2^0}\theta} \overset{=|1\rangle}{|001\rangle} + e^{2\pi i \cdot 2^1\theta}|010\rangle + e^{2\pi i \cdot (2^1+2^0)\theta}|011\rangle$$

$$+ e^{2\pi i \cdot 2^2\theta}|100\rangle + e^{2\pi i \cdot (2^2+2^0)\theta}|101\rangle + \cdots + e^{2\pi i \cdot (2^2+2+2^0)\theta}|111\rangle \Big) \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2^3}}\left[\, |0\rangle + e^{2\pi i\theta}|1\rangle + e^{2\pi i\cdot 2}|2\rangle + \cdots + e^{2\pi i\cdot 8\theta}|8\rangle \right] \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2^3}} \sum_{k=1}^{2^n-1} e^{2\pi i k\theta} |k\rangle \otimes |\psi\rangle$$

Recall. <QFT의 표현>

$$QFT|x\rangle = \frac{1}{\sqrt{2^n}}\left(|0\rangle + e^{2\pi i x/2}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i x/2^2}|1\rangle\right) \otimes \cdots \otimes \left(|0\rangle + e^{2\pi i x/2^n}|1\rangle\right)$$

$x = 2^n\theta$를 대입하면, $|\psi_2\rangle$와 동일한 상태가 됨.

$$QFT|2^n\theta\rangle = \frac{1}{\sqrt{2^n}} \sum_{k=1}^{2^n-1} e^{2\pi i k\cdot \frac{x}{2^n}} |k\rangle \otimes |\psi\rangle \qquad \frac{x}{2^n} \to \theta$$

$$|\psi_3\rangle = \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{2\pi i k\cdot\theta} |k\rangle \otimes |\psi\rangle \xrightarrow{QFT^{-1}} \frac{1}{2^n} \sum_{x=0}^{2^n-1}\sum_{k=0}^{2^n-1} e^{-\frac{2\pi i k}{2^n}(x-2^n\theta)} |x\rangle \otimes |\psi\rangle$$

$$\Rightarrow \left(2\pi i k\theta - 2\pi i k\frac{x}{2^n}\right)$$

QPE에서 θ는 $\frac{x}{2^n}$로 추정되며, 근사의 정확도는 n에 달려있음.

복소평면에서 생각해보면,
$x = 2^n\theta$에서 1로 collapse하므로,
$x = 2^n\theta$가 가장 높은 확률을 갖는다.

pf) $\displaystyle\sum_{k=0}^{2^n-1} e^{\frac{2\pi i}{2^n}k} = 0$ ····· ⊛

0에 대한 증명 : let $w = 2\pi i/n'$, $w^{n'} = 1$

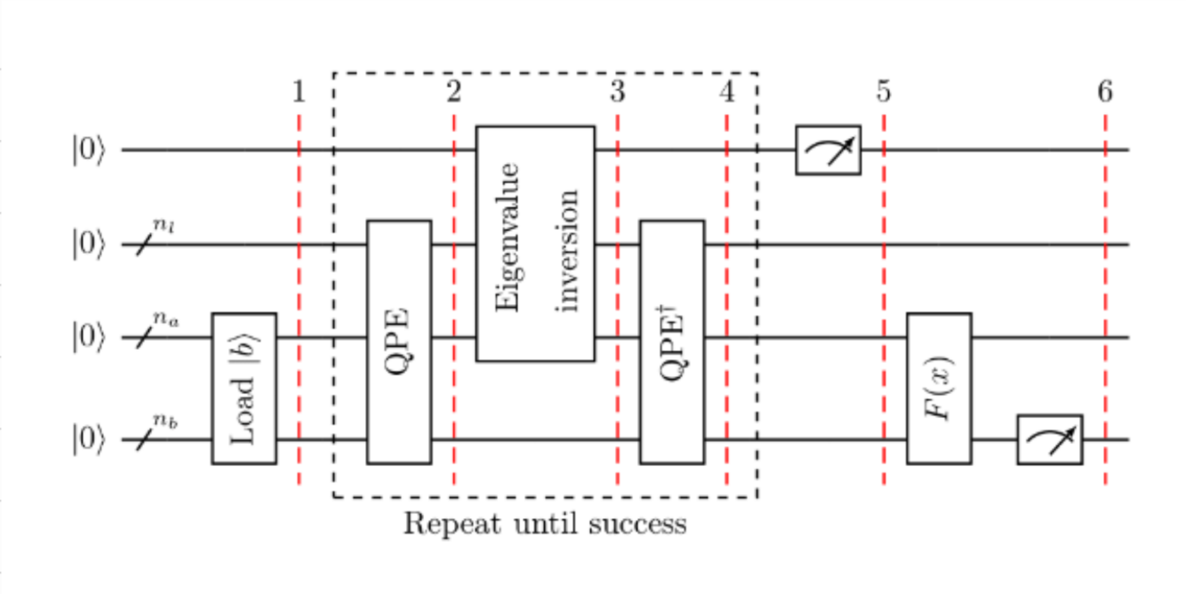
$$1 + w + w^2 + \cdots + w^{n'} = \frac{1-w^{n'}}{1-w} = 0$$

$$\text{if } n' = 2^n \to \frac{1-w^{2^n}}{1-w} = 0$$

$\therefore e^{\frac{2\pi i}{2^n}k(x-2^n\theta)}$에서 $x \neq 2^n\theta$인 경우, ⊛에 의해서 0이 되고,
$x = 2^n\theta$인 경우, 1이 된다.

$$\therefore |\psi_4\rangle = |2^n\theta\rangle \otimes |\psi\rangle$$

<1. Representing statevector of HHL Circuit>



① $|0\rangle_{n_b} \longmapsto |b\rangle_{n_b}$

② $\boxed{U = e^{iAt} = \sum_{j=0}^{N-1} e^{i\lambda_j t} |u_j\rangle\langle u_j|}$를 가지고 QPE를 적용해보자. (Exact한 경우를 가정)

pf)

Recall, Taylor exp. ; $e^x = 1 + \frac{1}{2!}x + \frac{1}{3!}x^2 + \cdots\cdots$

$\therefore e^{iAt} = I + \frac{1}{2!} iAt + \frac{1}{3!}(iAt)^2 + \cdots$ where $A = \sum_{j=0}^{N-1} \lambda_j |u_j\rangle\langle u_j|$

$$= \sum_{j=0}^{N-1}\left[|u_j\rangle\langle u_j| + \frac{1}{2!} i\lambda_j t |u_j\rangle\langle u_j| + \frac{1}{3!}(-1)\lambda_j^2 |u_j\rangle \cancel{\langle u_j|u_j\rangle}^{1} \langle u_j| t^2 + \cdots \right]$$

$$= \sum_{j=0}^{N-1} |u_j\rangle\langle u_j| \left(1 + \frac{1}{2!} i\lambda_j t + \frac{1}{3!}(i)^2 \lambda_j^2 t^2 + \cdots\right)$$

$$= \sum_{j=0}^{N-1} e^{i\lambda_j t} |u_j\rangle\langle u_j|$$

이진법으로 표현한 경우.

Recall, $QPE(U, |\psi\rangle) = |\tilde{\theta}\rangle_n |\psi\rangle_m$.

$|\psi_1\rangle$을 eigen basis로써 표현해보면, $|\psi_1\rangle = \sum_{j=0}^{N-1} b_j |0\rangle_{n_l} |u_j\rangle_{n_b}$로 표현됨.

$$\therefore QPE\left(e^{iAt}, \sum_{j=0}^{N-1} b_j |0\rangle_{n_l} |u_j\rangle_{n_b}\right) = \sum_{j=0}^{N-1} b_j |\tilde{\lambda}_j\rangle_{n_l} |u_j\rangle_{n_b} = |\psi_2\rangle$$

Eigenvalue에 대한 n_l bit representation.

③ Eigenvalue Inversion.

$$|\psi_3\rangle = \sum_{j=0}^{N-1} b_j |\lambda_j\rangle_{n_l} |u_j\rangle_{n_b} \left(\sqrt{1-\frac{C^2}{\lambda_j^2}}\,|0\rangle + \frac{C}{\lambda_j}|1\rangle\right)$$

(where $C < \lambda_j$)

④ Inverse QPE ($QPE^\dagger$)

$$|\psi_4\rangle = \sum_{j=0}^{N-1} b_j |0\rangle_{n_l} |u_j\rangle_{n_b} \left(\sqrt{1-\frac{C^2}{\lambda_j^2}}\,|0\rangle + \frac{C}{\lambda_j}|1\rangle\right)$$

⑤ Outcomes 1의 경우 (auxiliary qubit) ; $|\psi_4\rangle$에서 $|0\rangle$은 무시!

$$\therefore |\psi_5\rangle = \sum_{j=0}^{N-1} b_j \cdot \frac{C}{\lambda_j} |0\rangle_{n_l} |u_j\rangle_{n_b} |1\rangle$$

normalize; $$\sum_{j=0}^{N-1} \left(\sqrt{\frac{\cancel{C^2}}{\lambda_j^2} b_j^2}\right)^{-1} \sum_{j=0}^{N-1} b_j \cdot \frac{\cancel{C}}{\lambda_j} |0\rangle_{n_l} |u_j\rangle_{n_b} |1\rangle$$

$$\therefore \text{(normalized) } |\psi_5\rangle = \frac{1}{\sum_{j=0}^{N-1} \sqrt{\frac{b_j^2}{\lambda_j^2}}} \sum_{j=0}^{N-1} b_j/\lambda_j\, |0\rangle_{n_l} |u_j\rangle_{n_b} |1\rangle$$

⑥ Observable M을 이용해서, $\langle x|M|x\rangle$를 얻어내어 $|x\rangle$를 추측.

$\because$ $|x\rangle$를 그냥 얻어내기 위해서는 알고리즘 실행횟수가 qubit 수에 따라 exponential하게 증가됨.

⊕ Non-Exact QPE의 경우를 고려한 HHL Alg.

QPE에서 $QFT^\dagger$를 거치고 난 직후의 상태를, 기존 노트에 표기된 순서에 맞추어 $|\psi_3\rangle$라고 해보자.

$$|\psi_3\rangle = \frac{1}{2^n} \sum_{x=0}^{2^n-1} \sum_{k=0}^{2^n-1} e^{-\frac{2\pi i}{2^n} k (x - 2^n \theta)} |x\rangle \otimes |\psi\rangle \quad - ⊛$$

[Qiskit - Solving Linear Systems of Equations using HHL]의 표기로 ⊛를 고치면,

$x \to l$, $n \to n_l$

$|\psi\rangle \to b_j |u_j\rangle_{n_b}$; $|\psi\rangle$에 HHL 상의 $|b\rangle$가 들어오기 때문.

$\theta = \frac{\lambda_j}{2\pi} t$

$$\therefore |\psi_3\rangle = \frac{1}{2^{n_l}} \sum_{l=0}^{2^{n_l}-1} \sum_{k=0}^{2^{n_l}-1} b_j\, e^{-\frac{2\pi i}{2^{n_l}} k (l - 2^{n_l}\theta)} |l\rangle_{n_l} \otimes |u_j\rangle_{n_b}$$

$$= \frac{1}{2^{n_l}} \sum_{l=0}^{2^{n_l}-1} \sum_{k=0}^{2^{n_l}-1} b_j \left(e^{2\pi i \left(\frac{\lambda_j}{2\pi} t - \frac{l}{2^{n_l}}\right)} |l\rangle_{n_l}\right)^k \otimes |u_j\rangle_{n_b}$$

이동!

$$\Rightarrow |\psi_3\rangle = \sum_{j=0}^{N-1} b_j \left(\sum_{l=0}^{2^{n_l}-1} a_{l|j} |l\rangle_{n_l} \right) |u_j\rangle_{n_b}$$

In here, $a_{l|j} = \frac{1}{2^{n_l}} \sum_{k=0}^{2^{n_l}-1} \left(e^{2\pi i \left(\frac{\lambda_j t}{2\pi} - \frac{l}{2^{n_l}} \right)} \right)^k$

<논문 속 HHL 알고리즘의 예시>

논문 ; 비-허미시안 행렬에 대한 저-복잡도 양자 역행렬 알고리즘.

(given) $\boxed{\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

비-허미시안.

∵ $\dagger$를 취한 것이 원래 행렬과 동일하지 × ⇒ Hermitian으로 바꿔야 함.

cf Qiskit: Matrix A should be Hermitian.

$$\underbrace{\begin{pmatrix} 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 4 \\ 2 & 1 & 0 & 0 \\ -1 & 4 & 0 & 0 \end{pmatrix}}_{\equiv A} \underbrace{\begin{pmatrix} 0 \\ 0 \\ x_1 \\ x_2 \end{pmatrix}}_{= x} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{\equiv b}$$

Eigenvalue & Eigenvector of A:

(not normalized!)

$\lambda_1 = \sqrt{10}-1$

$\lambda_2 = -\sqrt{10}+1$

$\lambda_3 = \sqrt{10}+1$

$\lambda_4 = -\sqrt{10}-1$

$$|e_1\rangle = \begin{pmatrix} -\sqrt{10}-3 \\ -1 \\ -\sqrt{10}-3 \\ 1 \end{pmatrix} \quad |e_2\rangle = \begin{pmatrix} \sqrt{10}+3 \\ 1 \\ -\sqrt{10}-3 \\ 1 \end{pmatrix} \quad |e_3\rangle = \begin{pmatrix} -\sqrt{10}+3 \\ 1 \\ \sqrt{10}-3 \\ 1 \end{pmatrix} \quad |e_4\rangle = \begin{pmatrix} \sqrt{10}-3 \\ -1 \\ \sqrt{10}-3 \\ 1 \end{pmatrix}$$

(normalize eigenvectors)

e_1, e_4 : $\sqrt{(-\sqrt{10}-3)^2 + (-1)^2 + (-\sqrt{10}-3)^2 + 1^2} = \sqrt{40+12\sqrt{10}}$

e_2, e_3 : $\sqrt{40-12\sqrt{10}}$

Let $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ 라두고, $|b\rangle$를 A의 Eigenvector들을 기저로 갖도록 A의 고유 벡터공간에 표현해보자.

(computational basis)

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \boxed{\begin{bmatrix} \begin{pmatrix} e_1 \end{pmatrix} & \begin{pmatrix} e_2 \end{pmatrix} & \begin{pmatrix} e_3 \end{pmatrix} & \begin{pmatrix} e_4 \end{pmatrix} \end{bmatrix}} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

A의 Eigenvector로 표현된 기저들

⇒ QLSA에서 $|u_j\rangle$에 해당되고, 이는 A의 벡터공간을 구성하는 기저벡터들.

즉, $b_j|u_j\rangle$로 A의 상태공간에 표현한 과정을 예제에서 보인 것.

$$\equiv A_{eigen}$$

$$\therefore |b\rangle = A^{+}_{eigen} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

각각 기저들이 orthonormal하므로, transpose가 곧 역행렬이다.

pf) Let $U = [\, u_1, u_2 \cdots u_n \,]$, 각각의 column들은 orthonormal함.

$$\therefore U^T U = \begin{pmatrix} u_1^T u_1 & \cdots & \cdots \\ \vdots & u_2^T u_2 & \\ \vdots & & \ddots \\ & & & u_n^T u_n \end{pmatrix} = I$$

$\Rightarrow U^{+} U = I = U^T U$ 이므로, $U^T = U^{+}$이다.

$$\therefore \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} \frac{-\sqrt{10}+3}{\beta} & \frac{1}{\beta} & \frac{\sqrt{10}-3}{\beta} & \frac{1}{\beta} \\ \frac{\sqrt{10}-3}{\alpha} & -\frac{1}{\alpha} & \frac{-\sqrt{10}-3}{\alpha} & \frac{1}{\alpha} \\ \frac{\sqrt{10}+3}{\alpha} & \frac{1}{\alpha} & \frac{-\sqrt{10}-3}{\alpha} & \frac{1}{\alpha} \\ \frac{\sqrt{10}-3}{\beta} & \frac{1}{\beta} & \frac{\sqrt{10}-3}{\beta} & \frac{1}{\beta} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\beta\sqrt{5}(\sqrt{10}+2)}{10} \\ \frac{-\alpha\sqrt{5}(\sqrt{10}-2)}{10} \\ \frac{\alpha\sqrt{5}(\sqrt{10}-2)}{10} \\ \frac{-\beta\sqrt{5}(\sqrt{10}+2)}{10} \end{pmatrix}$$

Recall, QPE를 거치고나면 $\sum_{j}^{N-1} b_j |\tilde{\lambda}_j\rangle_{n_q} |e_j\rangle_{n_b}$

$$\boxed{\tilde{\lambda}_j = \frac{2^{n_q}\lambda_j t}{2\pi}}$$

$$\therefore \frac{\tilde{\lambda}_j}{2^{n_q}} = \frac{\lambda_j t}{2\pi} \quad \text{(Exact한 경우)}$$

* 여기서 t의 역할은 eigenvalue를 2π 안에 가두기 위함.

Let $t = \frac{2\pi}{16}$, $n_q = 6$으로 해보자. $\Rightarrow$ $\boxed{\tilde{\lambda}_j = 2^2 \lambda_j}$

Recall, HHL Alg을 다 거치고 나면, $\frac{1}{\sqrt{\sum_{j=1}^{N-1}\left(b_j/\tilde{\lambda}_j\right)^2}} \sum_{j}^{N-1} \frac{b_j}{\tilde{\lambda}_j} |e_j\rangle_{n_b} |0\rangle_{n_q} |1\rangle$ 이고,

이는 $x = A^{-1}b = \sum_{j=0}^{N-1} \frac{b_j}{\lambda_j} |e_j\rangle$의 상수배(normalize constant)만 다름.

$\therefore \frac{b_1}{\tilde{\lambda}_1} = \frac{\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9}$, $\frac{b_2}{\tilde{\lambda}_2} = \frac{-\alpha}{8\sqrt{5}} \cdot \left(\frac{8-\sqrt{10}}{9}\right)$, $\frac{b_3}{\tilde{\lambda}_3} = \frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9}$, $\frac{b_4}{\tilde{\lambda}_4} = \frac{\beta}{8\sqrt{5}} \cdot \frac{8+\sqrt{10}}{9}$

$$\therefore \frac{1}{N} \sum_{j}^{N-1} \frac{b_j}{\tilde{\lambda}_j} |e_j\rangle_{n_b} |0\rangle_{n_q} |1\rangle = \frac{1}{N}\left[\frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9}\left(|e_2\rangle + |e_3\rangle\right) + \frac{\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9}\left(|e_1\rangle + |e_4\rangle\right)\right] |0\rangle_{n_q} |1\rangle$$

$$N = \sqrt{\sum_{j=1}^{N-1}\left(\frac{b_j}{\tilde{\lambda}_j}\right)^2} = \sqrt{\frac{4160}{25920}} = \frac{\sqrt{13}}{9}$$

A의 eigenstate를 기저로하는 경우, $x = \frac{1}{N}\begin{pmatrix} \frac{\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9} \\ \frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9} \\ \frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9} \\ \frac{+\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9} \end{pmatrix}$

A의 eigenstate에서 computational basis로 바꾸자! (위에서 언급한 A eigen 이용)

$$x = A_{eigen} \cdot \frac{1}{N}\begin{pmatrix} \frac{\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9} \\ \frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9} \\ \frac{-\alpha}{8\sqrt{5}} \cdot \frac{(8-\sqrt{10})}{9} \\ \frac{+\beta}{8\sqrt{5}} \cdot \frac{(8+\sqrt{10})}{9} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5/\sqrt{26} \\ 1/\sqrt{26} \end{pmatrix}$$

$$\therefore x = \frac{5}{\sqrt{26}}|10\rangle + \frac{1}{\sqrt{26}}|11\rangle$$

$$\therefore \begin{pmatrix} 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 4 \\ 2 & 1 & 0 & 0 \\ -1 & 4 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \frac{5}{\sqrt{26}}C \\ \frac{1}{\sqrt{26}}C \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{9}{\sqrt{26}}C \\ \frac{9}{\sqrt{26}}C \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$\therefore C = \frac{\sqrt{26}}{9}$ 이므로,

$$\boxed{x = \frac{5}{9}|10\rangle + \frac{1}{9}|11\rangle}$$