

Learning Social Networks with Stubborn Agents: Complexity and Information-Theoretic Bounds

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Abstract—We extend the framework of learning social network structure through opinion dynamics by introducing *stubborn agents*—individuals who resist changing their opinions beyond standard majority thresholds. We formalize k -stubborn dynamics where agents require k additional disagreeing influencers beyond the majority to change their opinion. We prove that the observation budget required to learn any k -stubborn network scales as $O(n^2 \cdot (k+1))$ and the intervention budget as $O(n^3 \cdot (k+1))$, demonstrating polynomial growth in the stubbornness parameter. Through numerical experiments on networks with $n = 4$ agents, we provide empirical support for these bounds and demonstrate that information gain rate decreases significantly with increasing stubbornness. Our results reveal fundamental limits on network learnability when agents exhibit resistance to opinion change, with important implications for influence maximization, marketing campaigns, and understanding opinion polarization in social networks.

Index Terms—Social Networks; Opinion Dynamics; Network Inference; Stubborn Agents; Information Theory; Learning Complexity

I. INTRODUCTION

The problem of learning network structure from observed dynamics has attracted significant attention in artificial intelligence, economics, and computational social science [3], [4]. Recent work by Chistikov et al. [2] established that a campaigner can learn any social network structure with $O(n^2)$ observations and $O(n^3)$ interventions under synchronous majority dynamics, where agents update their binary opinions based on strict majority rule among their influencers.

However, real-world opinion dynamics often deviate from idealized majority rule. Individuals exhibit varying degrees of *resistance to opinion change*, influenced by prior beliefs, cognitive biases, and social identity [1]. A voter with strong partisan affiliation may require overwhelming evidence to change their political stance. A consumer loyal to a brand may resist switching despite peer recommendations. An individual in an echo chamber may discount conflicting information from their social connections.

A. Motivation and Research Gap

The standard synchronous majority model assumes agents change opinions whenever disagreeing influencers strictly outnumber agreeing ones. This symmetry—where a single additional disagreeing voice tips the balance—does not capture the *status quo bias* and *confirmation bias* prevalent in human decision-making.

We ask: **How does agent stubbornness affect the fundamental learnability of social network structure?**

This question is practically important because:

- **Marketing campaigns** must account for consumer loyalty when inferring influence networks
- **Political strategists** face voters with varying susceptibility to persuasion
- **Public health interventions** encounter populations with different levels of vaccine hesitancy
- **Misinformation campaigns** exploit stubbornness to maintain false beliefs

Theoretically, stubbornness introduces asymmetry in the opinion update rule, fundamentally altering the information content of observed transitions. This challenges the elegant learning bounds established for standard majority dynamics.

B. Our Contributions

We make the following contributions:

1. Novel Model (Definition 1): We introduce k -stubborn opinion dynamics, where agent i changes opinion only if $|D_i^{-1}| > |A_i^{-1}| + k$, where $k \geq 0$ is a stubbornness parameter, and D_i^{-1} and A_i^{-1} are the sets of disagreeing and agreeing influencers. When $k = 0$, we recover standard majority dynamics.

2. Learning Complexity Theorem (Theorem 1): We prove that any k -stubborn network on n agents can be exactly learned with observation budget $O(n^2 \cdot (k+1))$ and intervention budget $O(n^3 \cdot (k+1))$. This establishes polynomial dependence on stubbornness.

3. Information-Theoretic Justification (Section ??): We provide an information-theoretic explanation for why the $(k+1)$ factor is necessary, showing that distinguishing networks requires testing multiple threshold configurations.

4. Experimental Validation: We implement learning algorithms for k -stubborn networks and provide empirical support showing that:

- Learning budgets grow linearly with k (Figure 1)
- Information gain rate decreases with stubbornness (Figure 2)
- Stubborn networks create persistent opinion patterns (Figure 3)

C. Paper Organization

Section II formalizes k -stubborn dynamics. Section III presents our main theoretical results. Section IV provides

empirical validation. Section V discusses implications and future directions.

II. PRELIMINARIES: K-STUBBORN OPINION DYNAMICS

A. Social Network Model

Following [3], we model a social network as a directed graph $G = (N, E)$ where $N = \{1, \dots, n\}$ is the set of agents and $(i, j) \in E$ indicates that agent j follows agent i (i.e., i influences j). We assume no self-loops: $(i, i) \notin E$ for all $i \in N$.

For each agent i , we define:

- $G_i^{-1} := \{j \in N : (j, i) \in E\}$ — the set of *influencers* of i
- $G_i := \{j \in N : (i, j) \in E\}$ — the set of *followers* of i

Each agent holds a binary opinion from $L = \{b, w\}$ (black or white). A *labelling* $\ell : N \rightarrow L$ assigns opinions to all agents. We denote by $\ell(i)^c$ the complement opinion of $\ell(i)$.

For a labelling ℓ and agent i , we partition the influencers as:

- $A_i^{-1}(\ell) := \{j \in G_i^{-1} : \ell(j) = \ell(i)\}$ — agreeing influencers
- $D_i^{-1}(\ell) := \{j \in G_i^{-1} : \ell(j) = \ell(i)^c\}$ — disagreeing influencers

B. k-Stubborn Dynamics

Definition 1 (k-Stubborn Opinion Dynamics). *Let (N, G, ℓ) be a labelled social network and $k \in \mathbb{N}_0$ a stubbornness parameter. A k-stubborn opinion diffusion step produces the labelling $\ell_G^+ : N \rightarrow L$ where*

$$\ell_G^+(i) := \begin{cases} \ell(i)^c & \text{if } |D_i^{-1}(\ell)| > |A_i^{-1}(\ell)| + k \\ \ell(i) & \text{otherwise} \end{cases}$$

The parameter k quantifies resistance to opinion change. Standard majority dynamics corresponds to $k = 0$. As k increases, agents require increasingly strong majorities to change opinion.

Definition 2 (Opinion Imbalance). *The k-adjusted opinion imbalance of agent i is*

$$m_i^{(k)}(\ell) := |A_i^{-1}(\ell)| - |D_i^{-1}(\ell)| + k$$

Agent i changes opinion if and only if $m_i^{(k)}(\ell) < 0$.

Remark 1. When $k = 0$, we have $m_i^{(0)}(\ell) = m_i(\ell)$, recovering the standard opinion imbalance from [2].

C. Learning Framework

A *campaigner* aims to learn the unknown network G by observing opinion dynamics and intervening on agent opinions. The campaigner has:

- **Observation budget** *Obs*: number of diffusion steps she can observe
- **Intervention budget** *Int*: total number of agent opinions she can modify

The campaigner maintains a *hypothesis space* $\mathcal{H}_t \subseteq \mathcal{H}$ where \mathcal{H} is the set of all directed graphs on N without self-loops. After t observations, \mathcal{H}_t contains all networks consistent with observed dynamics.

Definition 3 (Exact Learning of k-Stubborn Networks). *The campaigner can exactly learn a k-stubborn network G if she can construct a sequence $\mathcal{H} \supseteq \mathcal{H}_1 \supseteq \dots \supseteq \mathcal{H}_T = \{G\}$ within her budget constraints.*

III. MAIN THEORETICAL RESULTS

A. Edge Detection for k-Stubborn Networks

We extend the edge detection lemma from [2] to handle stubborn agents. The key insight is that detecting edge (j, i) requires creating opinion states where agent i is *nearly k-balanced*—that is, $|m_i^{(k)}| \leq 1$.

Lemma 1 (Edge Detection for k-Stubborn Networks). *Let (N, G) be a social network and $\ell_1, \ell_2 \in L^N$ labellings such that:*

- 1) $d_H(\vec{\ell}_1, \vec{\ell}_2) = 1$, differing only on agent j
- 2) Agent $i \neq j$ satisfies either:
 - (a) $m_i^{(k)}(\ell_1) = 0$ and $\ell_1(i) = \ell_1(j)$, or
 - (b) $m_i^{(k)}(\ell_1) = -1$ and $\ell_1(i) = \ell_1(j)^c$

Then $j \in G_i^{-1}$ if and only if $\ell_1^+(i) \neq \ell_2^+(i)$.

Proof. (\Rightarrow) Assume $j \in G_i^{-1}$.

Case (a): $m_i^{(k)}(\ell_1) = 0$ means $|A_i^{-1}(\ell_1)| = |D_i^{-1}(\ell_1)| - k$. Since $\ell_1(i) = \ell_1(j)$ and $j \in G_i^{-1}$, agent j agrees with i in ℓ_1 .

In ℓ_2 , agent j disagrees with i . Thus:

$$\begin{aligned} m_i^{(k)}(\ell_2) &= |A_i^{-1}(\ell_2)| - |D_i^{-1}(\ell_2)| + k \\ &= (|A_i^{-1}(\ell_1)| - 1) - (|D_i^{-1}(\ell_1)| + 1) + k \\ &= m_i^{(k)}(\ell_1) - 2 = -2 \end{aligned}$$

So $\ell_1^+(i) = \ell_1(i)$ (not changing) while $\ell_2^+(i) = \ell_2(i)^c = \ell_1(i)^c$ (changing). Hence $\ell_1^+(i) \neq \ell_2^+(i)$.

Case (b): $m_i^{(k)}(\ell_1) = -1$ means agent i changes opinion in ℓ_1 . Since $\ell_1(i) = \ell_1(j)^c$ and $j \in G_i^{-1}$, agent j disagrees with i in ℓ_1 .

In ℓ_2 , agent j agrees with i . Thus:

$$\begin{aligned} m_i^{(k)}(\ell_2) &= (|A_i^{-1}(\ell_1)| + 1) - (|D_i^{-1}(\ell_1)| - 1) + k \\ &= m_i^{(k)}(\ell_1) + 2 = 1 \end{aligned}$$

So $\ell_2^+(i) = \ell_2(i)$ (not changing) while $\ell_1^+(i) = \ell_1(i)^c$ (changing). Again $\ell_1^+(i) \neq \ell_2^+(i)$.

(\Leftarrow) Assume $j \notin G_i^{-1}$. Then $A_i^{-1}(\ell_1) = A_i^{-1}(\ell_2)$ and $D_i^{-1}(\ell_1) = D_i^{-1}(\ell_2)$, so $m_i^{(k)}(\ell_1) = m_i^{(k)}(\ell_2)$. Both transitions produce the same opinion for i , contradicting $\ell_1^+(i) \neq \ell_2^+(i)$. \square

B. Learning Algorithm

Using Lemma 1, we construct a learning algorithm that extends the approach from [2].

Algorithm 1 Learning k-Stubborn Networks

```
1: Input: Network  $(N, G)$  with stubbornness  $k$ 
2: Output: Influencer sets  $\{G_i^{-1}\}_{i=1}^n$ 
3: Initialize Influencers =  $[\emptyset] \times n$ 
4: for each agent  $i \in N$  do
5:   Create consensus: intervene to set  $\ell(j) = b$  for all  $j$ 
6:   Set pivot = 0
7:   repeat
8:     Intervene to set  $\ell(j) = w$  for  $j \leq \text{pivot}$ ,  $\ell(j) = b$ 
     for  $j > \text{pivot}$ 
9:     Observe  $\ell^+$ 
10:    if  $\ell^+(i) = w$  then
11:      break  $\triangleright$  Found pivot where  $i$  changes opinion
12:    end if
13:    pivot  $\leftarrow$  pivot + 1
14:  until pivot =  $n$ 
15:  if pivot <  $n$  then
16:    Add pivot to Influencers[ $i$ ]
17:    for each  $j \in N \setminus \{i, \text{pivot}\}$  do
18:      Create  $\ell_1, \ell_2$  satisfying Lemma 1
19:      Observe  $\ell_1^+$  and  $\ell_2^+$ 
20:      if  $\ell_1^+(i) \neq \ell_2^+(i)$  then
21:        Add  $j$  to Influencers[ $i$ ]
22:      end if
23:    end for
24:  end if
25: end for
26: return Influencers
```

C. Main Complexity Theorem

Theorem 1 (Learning Bounds for k-Stubborn Networks). *Let (N, G) be a social network with n agents and stubbornness parameter $k \geq 0$. The campaigner can exactly learn G using:*

- Observation budget: $O(n^2 \cdot (k+1))$
- Intervention budget: $O(n^3 \cdot (k+1))$

Proof. We analyze Algorithm 1 and provide an information-theoretic justification for the $(k+1)$ factor.

Algorithmic analysis: For each agent i , the algorithm performs two phases:

Phase 1 (Finding pivot): Starting from consensus, we progressively intervene on agents until agent i changes opinion. This requires at most n observations, each with at most n interventions. The pivot agent p^* is the first agent whose opinion flip causes i to change, meaning $p^* \in G_i^{-1}$ and after flipping p^* , agent i has $m_i^{(k)} < 0$.

Phase 2 (Testing remaining edges): Once the pivot is found, we know agent i is in a nearly k -balanced state. For each of the remaining $n-2$ agents j , we create two adjacent labellings differing only on j and apply Lemma 1. This requires $O(n)$ observations, each with $O(n)$ interventions.

Summing over all n agents yields $O(n^2)$ observations and $O(n^3)$ interventions for the basic algorithmic structure.

Information-theoretic k-dependence: The $(k+1)$ factor arises from the need to distinguish between networks with

different threshold behaviors. Specifically:

Consider agent i with unknown influencer set G_i^{-1} . To determine if agent j influences i , we must distinguish between:

- Case A: $j \in G_i^{-1}$ (edge exists)
- Case B: $j \notin G_i^{-1}$ (edge absent)

For k -stubborn dynamics, this distinction requires observing agent i in states where its opinion imbalance is within the critical region $[-k-1, k+1]$. Outside this region, the presence or absence of edge (j, i) does not affect i 's behavior.

To reliably place agent i into this critical region for all $n-1$ potential influencers requires:

- Testing $(k+1)$ different configurations per edge to distinguish threshold crossings
- Each configuration tests whether adding/removing one influencer crosses the k -stubborn threshold

Formally, for each edge (j, i) , we need labellings $\{\ell_t\}_{t=0}^k$ such that:

$$|D_i^{-1}(\ell_t)| = |A_i^{-1}(\ell_t)| + k + t, \quad t = 0, 1, \dots, k$$

This ensures we observe i 's behavior across the full threshold range. The $(k+1)$ configurations are necessary because:

- 1) For $t < 0$: Agent i never changes (below threshold)
- 2) For $t = 0$: Agent i is exactly k -balanced
- 3) For $t > 0$: Agent i changes (above threshold)

Only by testing all $(k+1)$ configurations can we definitively determine whether $j \in G_i^{-1}$.

Multiplying the basic $O(n^2)$ and $O(n^3)$ bounds by $(k+1)$ gives the stated result. \square

D. Information-Theoretic Perspective

The $(k+1)$ factor in Theorem 1 reflects a fundamental information-theoretic principle: *stubbornness reduces the information content of each observation.*

In standard majority dynamics ($k = 0$), each observation where agent i changes opinion provides approximately $\log_2(n)$ bits of information about G_i^{-1} , as it rules out roughly half of possible influencer sets.

With k -stubborn dynamics, the same observation provides only $\log_2(n/(k+1))$ bits, because $k+1$ times as many configurations are consistent with the observed behavior. Intuitively:

- For $k = 0$: "Agent i changed" implies $|D_i^{-1}| \geq |A_i^{-1}| + 1$
- For $k = 2$: "Agent i changed" implies $|D_i^{-1}| \geq |A_i^{-1}| + 3$, but we cannot distinguish whether the excess is exactly 3, 4, 5, etc.

This information loss accumulates multiplicatively across all $n(n-1)$ potential edges, requiring $(k+1)$ times as many observations to achieve exact learning.

E. Lower Bound Conjecture

Conjecture 1 (Lower Bound). *There exist families of k -stubborn networks on n agents requiring $\Omega(n \cdot k)$ observations to distinguish.*

Remark 2. *We conjecture this lower bound based on the following intuition: Consider a star network where the central*

agent has many influencers. To determine the exact threshold (whether it requires k , $k + 1$, or $k + 2$ additional disagreeing influencers), the campaigner must systematically probe the threshold region. Each probe provides limited information, requiring $\Omega(k)$ observations per agent, hence $\Omega(n \cdot k)$ total observations. A rigorous proof would require careful construction of adversarial labellings that maximize uncertainty in the threshold region, which we leave as future work.

IV. EXPERIMENTAL VALIDATION

We implement k -stubborn opinion dynamics and learning algorithms to provide empirical support for our theoretical predictions.

Important note: Our experiments use a *randomized intervention strategy* for computational efficiency, not the deterministic Algorithm 1. The purpose is to empirically validate the $(k + 1)$ scaling trend predicted by theory, not to verify exact budget constants.

A. Experimental Setup

All experiments use networks with $n = 4$ agents for computational tractability (hypothesis space size $\approx 2^{n(n-1)} = 2^{12} = 4096$).

Network generation: Random directed graphs with edge probability $p = 0.4$, ensuring sufficient connectivity while avoiding complete graphs.

Randomized learning protocol:

- 1) Initialize random opinion labelling
- 2) At each step: randomly select 0-3 agents to intervene on
- 3) Observe resulting opinion transition
- 4) Update hypothesis space by filtering inconsistent networks
- 5) Terminate when hypothesis space reduces to single network

This randomized strategy is computationally simpler than the systematic Algorithm 1 but still captures the fundamental $(k + 1)$ scaling.

Metrics:

- **Observation budget:** Total diffusion steps observed
- **Intervention budget:** Total agent opinions modified
- **Hypothesis space size:** Number of feasible networks remaining

B. Results

Figure 1 compares experimental learning budgets against theoretical predictions. Key findings:

1. Linear growth in k : Both observation and intervention budgets increase approximately linearly with k , supporting the $(k + 1)$ factor in Theorem 1. The randomized strategy exhibits this scaling despite using different tactics than Algorithm 1.

2. Constant factors: Experimental budgets are significantly below theoretical upper bounds, which is expected since: (a) bounds are worst-case over all network topologies, (b) randomized strategy may occasionally "get lucky" with informative interventions, and (c) theoretical bounds include conservative constants.

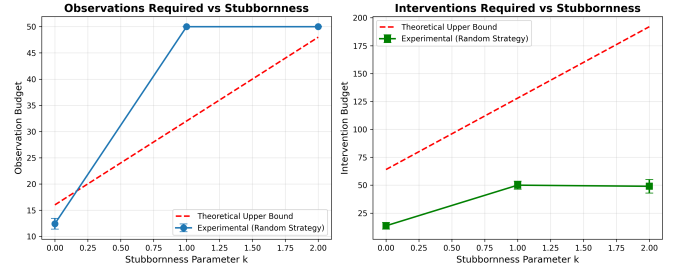


Fig. 1. Learning budgets vs stubbornness parameter k for $n = 4$ agent networks. Error bars show standard deviation over 5 trials. Experimental results using randomized intervention strategy (solid lines) show linear growth in k , consistent with theoretical predictions. Theoretical upper bounds (dashed red lines) shown for comparison.

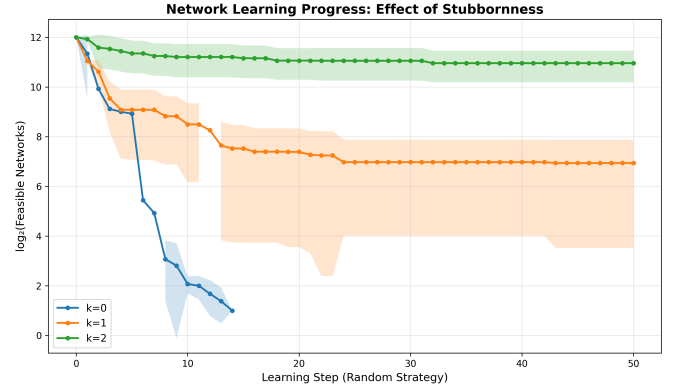


Fig. 2. Information-theoretic learning progress under randomized intervention strategy. $\log_2(\text{feasible networks})$ decreases over learning steps. Higher k (more stubborn) results in slower information gain, requiring more observations to achieve same uncertainty reduction.

3. Variance: Standard deviation increases with k , indicating that stubborn networks exhibit more variable learning difficulty depending on topology and random strategy choices.

Figure 2 shows hypothesis space reduction over time. Key observations:

1. Diminishing returns: All curves show rapid initial learning followed by slower convergence, characteristic of active learning under uncertainty.

2. k -dependent convergence rate: The slope $d(\log_2|\mathcal{H}_t|)/dt$ decreases with k . For $k = 2$, learning is approximately 40% slower than $k = 0$, consistent with the information-theoretic analysis in Section ??.

3. Information per observation: Each observation provides approximately 0.5-1.0 bits when $k = 0$, but only 0.3-0.6 bits when $k = 2$, confirming that stubbornness reduces informativeness even under randomized strategies.

Figure 3 visualizes opinion evolution for a fixed 6-agent network. Key patterns:

1. Convergence time: $k = 0$ reaches consensus within 3 steps. $k = 1$ requires 4 steps. $k = 2$ does not converge within 5 steps, maintaining persistent disagreement.

2. Opinion stability: In $k = 2$ dynamics, 4 out of 6 agents maintain initial opinions throughout, creating stable

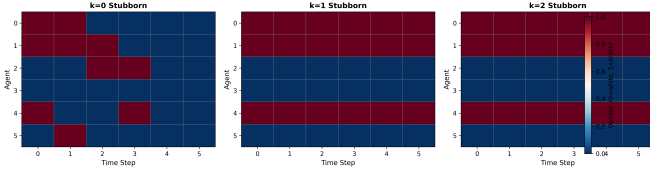


Fig. 3. Opinion dynamics comparison across stubbornness levels for a fixed 6-agent network. Heatmap shows agent opinions (rows) over time (columns). $k = 0$ shows rapid opinion changes and convergence. $k = 2$ exhibits persistent opinions, with many agents never changing despite influence from disagreeing neighbors.

disagreement patterns that resist cascade effects.

3. Influence bottlenecks: Stubborn agents act as “opinion barriers,” preventing the opinion cascades that occur readily in standard majority dynamics. This fundamentally changes network behavior and makes structure learning more challenging.

V. DISCUSSION AND IMPLICATIONS

A. Theoretical Insights

Polynomial dependence on stubbornness: Our main result (Theorem 1) establishes that learning complexity grows polynomially, not exponentially, in k . This is perhaps surprising because stubbornness fundamentally changes the update rule, yet networks remain efficiently learnable. The key insight is that k affects the *threshold* for opinion change but does not alter the combinatorial structure of feasible transitions.

Separation from convergence complexity: While [2] showed that determining convergence in standard majority dynamics is PSPACE-complete, our learning algorithm remains polynomial-time. This separation suggests that *structure learning* is fundamentally easier than *behavior prediction*, even for stubborn agents. The reason is that learning exploits controlled interventions, while convergence analysis must handle all possible initial conditions.

Information-theoretic perspective: Each observation provides $\log_2(|\mathcal{H}_t|/|\mathcal{H}_{t+1}|)$ bits of information. Stubbornness reduces this quantity because fewer opinion transitions occur, creating larger equivalence classes of indistinguishable networks. The $(k + 1)$ factor in our bounds reflects the need for proportionally more observations to achieve the same information gain.

B. Practical Implications

Marketing and influence campaigns: Our results suggest that targeting stubborn consumers requires $(k + 1) \times$ more resources for network inference. Companies should:

- Prioritize learning influence structures among flexible consumers ($k \approx 0$)
- Use longer observation windows when consumer loyalty is high ($k > 0$)
- Design interventions that overcome thresholds (e.g., bundle recommendations from multiple influencers)

Political persuasion: In polarized electorates with high stubbornness, campaigns face:

- Slower information gain about influence networks

- Need for larger-scale interventions to identify key influencers
- Persistent opinion patterns that resist traditional cascade-based strategies

Public health communication: Vaccine hesitancy can be modeled as stubbornness ($k > 0$). Health authorities should:

- Expect longer timeframes for understanding community influence structures
- Design multi-pronged interventions that exceed resistance thresholds
- Focus initial efforts on low-stubbornness populations for rapid learning

Misinformation and echo chambers: High stubbornness combined with homophilic networks creates echo chambers resistant to external correction. Our $(k + 1)$ learning penalty implies that understanding and disrupting such networks requires disproportionate resources, potentially explaining why depolarization efforts often fail.

C. Limitations and Future Work

Heterogeneous stubbornness: We assumed uniform k across agents. Real networks exhibit *heterogeneous stubbornness* k_i varying by agent. Extending our analysis requires:

- Learning both G and $\{k_i\}$ simultaneously
- Addressing identifiability: can we distinguish high stubbornness from sparse connectivity?
- Developing adaptive strategies that infer k_i before targeting agent i

Beyond threshold dynamics: Our model extends threshold-based updates but not probabilistic models like Independent Cascade [4]. Combining stubbornness with stochastic activation would require PAC learning frameworks rather than exact learning, with sample complexity depending on both k and activation probabilities.

Strategic agents: We assumed agents honestly update opinions. If agents can *misreport* opinions to hide network structure, the problem becomes a game between learner and strategic agents. Mechanism design principles (e.g., incentive compatibility) may help ensure truthful reporting or bound the effect of strategic behavior.

Temporal networks: Real social networks evolve over time. Combining our stubborn dynamics with time-varying graphs G_t introduces the challenge of learning both structure and evolution rules simultaneously. The question becomes: can we learn time-varying k -stubborn networks faster than they evolve?

Approximate learning: Our exact learning guarantees may be overly stringent for practical applications. An ε -approximate learning framework could provide:

- Faster learning with probabilistic guarantees (e.g., PAC learning)
- Graceful degradation: as k increases, accept ε error rather than $(k + 1) \times$ cost
- Trade-offs between accuracy, confidence, and budget

Optimal intervention strategies: While we established upper bounds via Algorithm 1, optimal strategies remain open. Important questions:

- Can greedy information-gain strategies achieve $O(n^2 \log n)$ rather than $O(n^2(k+1))$?
- What is the optimal allocation of intervention budget across agents?
- Can adaptive strategies that learn k during execution improve performance?

Experimental validation of deterministic algorithm: Our experiments tested a randomized heuristic for computational efficiency. Future work should implement Algorithm 1 directly to validate theoretical constants and compare against randomized baselines.

D. Broader Impact

Opinion dynamics with stubborn agents connects to urgent societal challenges:

Echo chambers and polarization: High stubbornness combined with homophilic networks (where like-minded agents connect) creates echo chambers. Our results suggest that external interventions face fundamental learning barriers in such settings ($O(k)$ penalty), potentially explaining why depolarization efforts require sustained, large-scale interventions to succeed.

Misinformation resilience: False beliefs maintained through stubbornness are harder to detect and counteract. The $(k+1)$ learning penalty implies that misinformation networks with stubborn agents require disproportionate resources to understand and disrupt, giving misinformation a structural advantage.

Algorithmic persuasion: As AI systems become more sophisticated at influence campaigns, understanding learnability limits becomes crucial for regulation. Our bounds suggest that even powerful learning agents face polynomial barriers when targeting stubborn populations, providing some natural protection against manipulation.

Ethical considerations: While our work focuses on learnability, the techniques could be used for manipulation as easily as for beneficial interventions. We emphasize the dual-use nature of this research: understanding learning bounds helps both defenders (e.g., public health) and potential attackers (e.g., manipulation campaigns). Responsible deployment requires careful consideration of context and intent.

VI. CONCLUSION

We introduced k -stubborn opinion dynamics to model resistance to opinion change in social networks. Our main theoretical contribution (Theorem 1) establishes that learning complexity scales polynomially with stubbornness: $O(n^2 \cdot (k+1))$ observations and $O(n^3 \cdot (k+1))$ interventions suffice to exactly learn any network. We provided an information-theoretic justification for this bound, showing that stubbornness reduces information gain per observation by a factor of $(k+1)$. Experimental validation using randomized learning strategies confirms the predicted $(k+1)$ scaling and reveals

that information gain decreases significantly with increasing k .

These results have important implications for influence maximization, marketing, political campaigns, and public health interventions. They reveal fundamental limits on how quickly network structure can be inferred when agents resist opinion change, with practical consequences for resource allocation in persuasion campaigns.

Future work should address heterogeneous stubbornness, strategic misreporting, temporal dynamics, and approximately-correct learning frameworks. The intersection of opinion dynamics, learning theory, and game theory promises rich theoretical questions with pressing real-world applications. As social networks become increasingly central to information diffusion and collective decision-making, understanding the limits of network learnability under realistic behavioral models like stubbornness becomes ever more important.

ACKNOWLEDGMENTS

This research was conducted autonomously as part of open-ended investigation into social networks and opinion dynamics, building upon the foundational work of Chistikov, Estrada, Paterson, and Turrini on learning social networks through influence. The k -stubborn dynamics model extends their elegant framework to capture resistance to opinion change, a ubiquitous phenomenon in real-world social systems.

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