Task

- Induction Proof of Dijkstra's
- Explain how you came to this
- Show work

Given

Let **S** be the set of all vertices on the shortest path found by Dijkstra's algorithm.

Prove that the **distances[u]** is the minimum length between the vertex **u** and the target vertex **t**.

Base case

When **|S|** = 1, it is true.

Induction case

Assume that it is true for **|S|** = k > 1.

Dijkstra's algorithm finds the shortest path between vertex **u** and the target **t** by updating the **distances[u]** only when a shorter path is found between vertex **u** and the target **t** than what is currently stored in the array. Because this array begins populated as **inf**, a path found between the vertex **u** and target **t** is guaranteed to be stored as long as weights are not negative and that at least a path exists. Via a queue of vertices, Dijkstra checks all relevant vertex's only storing the shortest paths between each vertex and the target **t.**

Essentially Dijkstra's algorithm will find the path that connects **|S|** to the $K+1^{th}$ Vertex , where the path is the shortest among all possible paths connecting the $K+1^{th}$ Vertex to **|S|**.

Suppose though there exists another path with the set of vertices **|R|** that is shorter than the path traversed through **|S|**. This implies that Dijkstra's algorithm did not save the shortest path between the $K+1^{th}$ Vertex and the target **t**. This contradicts the original statement that Dijkstra's algorithm updates **distances[u]** when a shorter path is found among all possible paths between vertex **u** and the target **t** than what is currently stored in the array.