Task

- Prove Max Flow == Min Cut Capacity
- Explain how you came to this
- Show work

Prove that the maximum flow is equal to the minimum cut capacity

Lemma 1 **There is no extra edge in the min-cut**

Given a graph, by definition the min-cut is a set of edges whose removal disconnects the start vertex from the target vertex with the least sum of flow (capacity) taken. Assuming that definition is true, should there exist an edge Q within the set of the min-cut whose removal from the set would reduce the capacity cut to separate S from T, this would contradict the min-cut definition that the original min-cut set of edges separate S from T with the minimum capacity. This proves that there is no extra edge that can be cut from the set of edges that make up the min-cut set.

Lemma 2 **All edges in the min-cut must be parts of augmenting paths**

Given a graph and the algorithm on slide 12 on Network Flow: min-max Theorem from class, to find a min-cut you start by finding a path from S to T and removing the smallest original weight edge from that path from all the edges in that path saving that smallest weight edge in the set min-cut. Repeating until there are no paths. Assuming that algorithm is true, suppose there exists an edge in min-cut that isn't part of an augmented path from S to T. This would contradict that the algorithm is adding only the smallest weight edge on each iteration of finding an available path from S to T. This proves that each edge in the min-cut must be part of augmenting paths which are primarily defined by bottleneck edges.

Lemma 3 **Each edge in the min-cut is the minimum weighted edge in each augmenting path**

Building off of leema 2, if there existed an edge in the min-cut that was part of an augmented path but not the minimum weighted edge of that augmented path, then that would contradict the method of finding a min-cut set of edges as the min-cut algorithm takes the smallest original edge in a path found and adds it to the min-cut set.

With these three leema proved we can say that the maximum flow is the equal to the min-cut capacity as the max. flow is the sum of minimum weighted edges of possible augmenting paths.