**MATH 32A Spring 2018, Midterm Exam 2 Practice Problems**

1. Show that, for an arc-length parameterisation and some constant $k$,
$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = k\mathbf{N}.$$
Recall $\mathbf{B}(s) = \mathbf{T}(s) \times \mathbf{N}(s)$, and/or try differentiating $\mathbf{B} \cdot \mathbf{B}$.

2. Let $\kappa(t)$ be the curvature of a path $\mathbf{r}(t) = (x(t), y(t), z(t))$. Show that the scaled path $\mathbf{r}_1(t) = (\lambda x(t), \lambda y(t), \lambda z(t))$ has curvature $\kappa(t)/\lambda$. Use
$$\kappa(t) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{||r'(t)||^3}.$$

3. A particle follows the path $\mathbf{r}(t)$ for $0 \leq t \leq T$, starting from the origin, $\mathbf{r}(0) = 0$. The quantity
$$\bar{\mathbf{v}} = \frac{1}{T}\int_0^T \mathbf{r}'(t)\,\mathrm{d}t$$
is called the average velocity.
Suppose the $\bar{\mathbf{v}} = \mathbf{0}$. Where is the particle at time $T$? Is the particles average speed ($\frac{1}{T}\int_0^T v\,\mathrm{d}t$) also equal to zero?

4. Find the path of an object starting at position $\mathbf{r}_0 = (0, 0)$ with initial velocity $\mathbf{v}_0 = (1, 3)$, with constant acceleration $\mathbf{a} = (-1, -1)$. Find the position of the object at time $t = 2$.

5. Decompose the acceleration $\mathbf{a}(t)$ at $t = 0$ into components parallel and perpendicular to $\mathbf{r}'(0)$ for an object following the path $\mathbf{r}(t) = (t, e^t, te^t)$. Hence find $v'$, $v$, and $\kappa$ at $t = 0$.

6. Evaluate $\lim\limits_{(x,y)\to(1,2)} (x^2 + y)$.

7. Evaluate $\lim\limits_{(x,y)\to(1/9,2/9)} \dfrac{x}{y}$.

8. If $\lim\limits_{(x,y)\to(2,5)} f(x, y) = 3$ and $\lim\limits_{(x,y)\to(2,5)} g(x, y) = 7$, evaluate $\lim\limits_{(x,y)\to(2,5)} \dfrac{f(x, y)}{f(x, y) + g(x, y)}$.

9. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 + y^3}{xy^2}$ does not exist.

10. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4y + xy^4}{x^4 + 2x^2y^2 + y^4}$ or show the limit doesn't exist.

11. Is the function $f(x, y) = \begin{cases} x^2 + y^2, & \text{if } x^2 + y^2 < 1, \\ 1, & \text{if } x^2 + y^2 \geq 1, \end{cases}$ continuous?

12. Calculate $f_x$, $f_y$, $f_{xx}$, $f_{yy}$ and $f_{xy}$ for $f(x, y) = \dfrac{xy}{y + 1}$.

13. Calculate $U_r$ and $U_t$ for $U(r, t) = \dfrac{e^{-rt}}{r}$.

*Answers:* 3: $\mathbf{0}$, no, 4: $(0, 4)$, 5: $(0, 1, 2) = (1, 1, 1) + (-1, 0, 1)$, $v' = \sqrt{3}$, $v = \sqrt{3}$, $\kappa = \sqrt{2}/3$, 6: 3, 7: 1/2, 8: 3/10, 10: 0, 11: yes, 12: $f_x = y/(y + 1)$, $f_y = x/(y + 1)^2$, $f_{xx} = 0$, $f_{yy} = -2x/(y + 1)^3$, $f_{xy} = 1/(y + 1)^2$, 13: $U_r = -te^{-rt}/r - e^{-rt}/r^2$, $U_t = -e^{-rt}$,