

1. Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c
1	2	2	3
→ 2	3	2	1,4
3	4	1	2
4	1	4	/

2. Consider the class REG_A of all regular languages over the fixed one-letter alphabet $A=\{a\}$.

(a) Is REG_A countable?

(b) Is the class $NOTREG_A$ of all languages over A that are not regular countable?
(Note that $NOTREG_A = 2^{A^*} - REG_A$.)

(c) Is the class $REG_A \cap NOTREG_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, always using the constructions given in class:

(a) $(a \cup a^2)(a^3 \cup a^2)^*$ over the alphabet $\{a\}$

(b) $(00 \cup 11)^*((11)^* \cup (00)^*)$ over the alphabet $\{0,1\}$

4. Construct a regular expression over the alphabet $\{a,b\}$ for the language accepted by the following automaton:

	a	b
→ A	/	C
B	A	/
C	/	B,C

Points:

1: 12

2: 22

3: 44

4: 22

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DFA	a	b	c		
→ 2	3	2	1,4	1	
3	4	1	2	1	1 12
1,4	1,2	2,4	3	1	2 22
4	1	4	—	1	3 44
1	2	2	3	1	4 22
1,2	2,3	2	1,3,4	1	100 :)
2,4	1,3	2,4	1,4	1	
—	—	—	—	0	
2,3	3,4	1,2	1,2,4	1	
1,3,4	1,2,4	1,2,4	2,3	1	
1,3	2,4	1,2	2,3	1	
3,4	1,4	1,4	2	1	✓ 12
1,2,4	1,2,3	2,4	1,3,4	1	
1,2,3	2,3,4	1,2	1,2,3,4	1	
2,3,4	1,3,4	1,2,4	1,2,4	1	
1,2,3,4	1,2,3,4	1,2,4	1,2,3,4	1	

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(a) REGA is a regular language if it is accepted by a DFA, and we know ^{DFA} is finite automata.

If we have a DFA over fixed alphabet $A = \{a\}$ which are m in number and DFA has n states. Then, no. of combinations possible for DFA is $2^n \cdot n^{mn}$

which is countable

\therefore REGA is finite as regular language which is determined by using DFA.

\therefore REGA is countable.

(b) $\therefore A^*$ is countable infinite

$\therefore 2^{A^*}$ is also countable infinite

\therefore REGA is finite

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$\therefore 2^{A^*} - \text{REGA}$ is infinite

$\therefore \text{NOTREGA}$ is uncountable

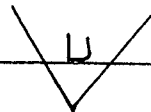
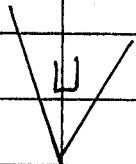
(c) $\therefore \text{REGA} \cap \text{NOTREGA} = \{\emptyset\}$

\therefore It is countable. ✓

3 (a) 1, 2, ..., 8 is renaming for q_1, q_2, \dots, q_8

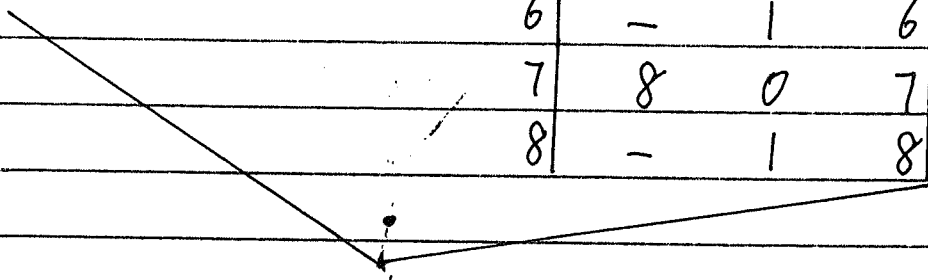
$$(a \cup a \cdot a) \cdot (a \cdot a \cdot a \cup a \cdot a)^*$$

a			a			a			a		
→ 0	1	0	→ 0	2	0	→ 0	4	0	→ 0	7	0
1	-	1	2	3	0	4	5	0	7	8	0
			3	-	1	5	6	0	8	-	1
						6	-	1			



a		
→ 0	1, 2	0
1	-	1
2	3	0
3	-	1

a			a		
→ 0	4, 7	0	→ 0	4, 7	1
4	5	0	4	5	0
5	6	0 *	5	6	0
6	-	1	6	4, 7	1
7	8	0	7	8	0
8	-	1	8	4, 7	1



a		
→ 0	1, 2	0
1	4, 7	1
2	3	0
3	4, 7	1
4	5	0
5	6	0
6	4, 7	1
7	8	0
8	4, 7	1

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the next page

3(a) continued

Corresponding DFA

a				a			
A	→ 0	1, 2	0	→ A	B	0	
B	1, 2	3, 4, 7	1	B	C	1	
C	3, 4, 7	4, 5, 7, 8	1	C	D	1	
D	4, 5, 7, 8	4, 5, 6, 7, 8	1	D	E	1	
E	4, 5, 6, 7, 8	4, 5, 6, 7, 8	1	E	E	1	

Rename

D-F

F

A
A
①

B C D E
B C D E
②

Reduced DFA :

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a		
→ 1	2	0
2	2	1

3(b)

$$(0 \cdot 0 \cup 1 \cdot 1)^* \cdot ((1 \cdot 1)^* \cup (0 \cdot 0)^*)$$

	0	1		0	1		0	1		0	1
$\rightarrow q_0$	q_1	- 0	$\rightarrow q_0$	- q_3	0	$\rightarrow q_0$	- q_5	0	$\rightarrow q_0$	q_7	- 0
q_1	q_2	- 0	q_3	- q_4	0	q_5	- q_6	0	q_7	q_8	- 0
q_2	-	- 1	q_4	-	- 1	q_6	-	- 1	q_8	-	- 1



	0				1				0				1			
	0				1				0				1			
					$\rightarrow q_0$	-				q_5	1	$\rightarrow q_0$	q_7 - 1			
$\rightarrow q_0$	q_1	q_3	0		q_5	-				q_6	0	q_7	q_8	- 0		
q_1	q_2	-		0	q_6	-				q_5	1	q_8	q_7	- 1		
q_2	-		-	1												
q_3	-		q_4	0												
q_4	-		-	1	$\rightarrow q_0$											



	0	1			0	1		
$\rightarrow q_0$	q_1	q_3	1		$\rightarrow q_0$	$q_1 q_7$	$q_3 q_5$	1
q_1	q_2	—	0		q_1	q_2	—	0
q_2	q_1	q_3	1	✓	q_2	$q_1 q_7$	$q_3 q_5$	1
q_3	—	q_4	0		q_3	—	q_4	0
q_4	q_1	q_3	1		q_4	$q_1 q_7$	$q_3 q_5$	1

	0	1	
$\rightarrow q_0$	q_7	q_5	1
q_5	-	q_6	0
q_6	-	q_5	1
q_7	q_8	-	0
q_8	q_7	-	1

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the next page

3(b) continued

corresponding DFA

	0	1			0	1
A $\rightarrow q_0$	$q_1 q_7$	$q_3 q_5$	1		$\rightarrow A$	B C 1
B $q_1 q_7$	$q_2 q_8$	—	0		B	D E 0
C $q_3 q_5$	—	$q_4 q_6$	0	\rightarrow Rename	C	E F 0
D $q_2 q_8$	$q_1 q_7$	$q_3 q_5$	1		D	B C 1
E —	—	—	0		E	E E 0
F $q_4 q_6$	$q_1 q_7$	$q_3 q_5$	1		F	B C 1

	A-F			F		
	B	C	E	A	D	F
B	C	E		A	D	F
C	B	C	E	A	D	F
E	①	②	③	④		

Reduced DFA

	0	1	
$\rightarrow 4$	1	2	1
1	4	3	0
2	3	4	0
3	3	3	0

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$$\begin{cases} L_A = b \cdot L_C \cup \epsilon \\ L_B = a \cdot L_A \\ L_C = \underbrace{b \cdot L_B}_M \cup \underbrace{b \cdot L_C}_{L \cdot X} \end{cases}$$

$$L_C = b \cdot L_C \cup b \cdot L_B$$

\therefore lemma

$$\therefore L_C = b^* \cdot (b \cdot L_B)$$

Put $L_B = a \cdot L_A$ into L_C

$$L_C = b^* \cdot (b \cdot a \cdot L_A)$$

Plug L_C into L_A

$$L_A = b \cdot b^* (b \cdot a \cdot L_A) \cup \epsilon$$

\therefore lemma

$$\therefore L_A = (b \cdot b^* \cdot b \cdot a)^*$$

\therefore A is the initial state

\therefore The language accepted by this automaton is L_A

$$L_A = (b \cdot b^* \cdot b \cdot a)^*$$