Zechun Cao 0827053

# COSC 3340 Examination 1

## Monday, February 16, 2009, 1 - 2:25 pm

### Open Book and Notes

**1**. Construct a dfa for the following nfa, using the subset construction given in class:

| | a | b | c | |
|---|---|---|---|---|
| 1 | 2 | 2 | 3 | 1 |
| → 2 | 3 | 2 | 1,4 | 1 |
| 3 | 4 | 1 | 2 | 1 |
| 4 | 1 | 4 | / | 1 |

**2**. Consider the class $REG_A$ of all regular languages over the fixed one-letter alphabet A={a}.

(a) Is $REG_A$ countable?

(b) Is the class $NOTREG_A$ of all languages over A that are not regular countable? (Note that $NOTREG_A = 2^{A^*} - REG_A$.)

(c) Is the class $REG_A \cap NOTREG_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

**3**. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a \cup a^2)(a^3 \cup a^2)^*$ over the alphabet {a}

(b) $(00 \cup 11)^* ((11)^* \cup (00)^*)$ over the alphabet {0,1}

**4**. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

| | a | b | |
|---|---|---|---|
| → A | / | C | 1 |
| B | A | / | 0 |
| C | / | B,C | 0 |

**Points:** 1: 12 2: 22 3: 44 4: 22

Zechun Cao 0827053

1

1

| DFA | a | b | c | |
|---|---|---|---|---|
| →2 | 3 | 2 | 1,4 | 1 |
| 3 | 4 | 1 | 2 | 1 |
| 1,4 | 1,2 | 2,4 | 3 | 1 |
| 4 | 1 | 4 | — | 1 |
| 1 | 2 | 2 | 3 | 1 |
| 1,2 | 2,3 | 2 | 1,3,4 | 1 |
| 2,4 | 1,3 | 2,4 | 1,4 | 1 |
| — | — | — | — | 0 |
| 2,3 | 3,4 | 1,2 | 1,2,4 | 1 |
| 1,3,4 | 1,2,4 | 1,2,4 | 2,3 | 1 |
| 1,3 | 2,4 | 1,2 | 2,3 | 1 |
| 3,4 | 1,4 | 1,4 | 2 | 1 |
| 1,2,4 | 1,2,3 | 2,4 | 1,3,4 | 1 |
| 1,2,3 | 2,3,4 | 1,2 | 1,2,3,4 | 1 |
| 2,3,4 | 1,3,4 | 1,2,4 | 1,2,4 | 1 |
| 1,2,3,4 | 1,2,3,4 | 1,2,4 | 1,2,3,4 | 1 |

✓ 12

| | |
|---|---|
| 1 | 12 |
| 2 | 22 |
| 3 | 44 |
| 4 | 22 |
| | 100 :) |

2

(a) $REG_A$ is an regular language if it is accepted by a DFA, and we know DFA is finite automata.

If we have a DFA over fixed alphabeta $A = \{a\}$ which are $m$ in number and DFA has $n$ states. Then, go to number of combination possible for DFA is $2^n \cdot n^{mn}$

which is countable

$\therefore$ $REG_A$ is finite as regular language which is determined by using DFA.

$\therefore$ $REG_A$ is countable.

(b) $\because$ $A^*$ is countable infinite

$\therefore$ $2^{A^*}$ is also countable infinite

$\because$ $REG_A$ is finite

$\therefore$ $2^{A^*} - REG_A$ is infinite

$\therefore$ $NOTREG_A$ is uncountable

(c) $\because$ $REG_A \cap NOTREG_A = \{\emptyset\}$

$\therefore$ It is countable.

3 (a) 1,2…8 is renaming for $q_1, q_2 \cdots q_8$

$(a \sqcup a \cdot a) \cdot (a \cdot a \cdot a \sqcup a \cdot a)^*$

| | a | |
|---|---|---|
| →0 | 1 | 0 |
| 1 | – | 1 |

| | a | |
|---|---|---|
| →0 | 2 | 0 |
| 2 | 3 | 0 |
| 3 | – | 1 |

| | a | |
|---|---|---|
| →0 | 4 | 0 |
| 4 | 5 | 0 |
| 5 | 6 | 0 |
| 6 | – | 1 |

| | a | |
|---|---|---|
| →0 | 7 | 0 |
| 7 | 8 | 0 |
| 8 | – | 1 |

$\sqcup$

| | a | |
|---|---|---|
| →0 | 1,2 | 0 |
| 1 | – | 1 |
| 2 | 3 | 0 |
| 3 | – | 1 |

$\sqcup$

| | a | |
|---|---|---|
| →0 | 4,7 | 0 |
| 4 | 5 | 0 |
| 5 | 6 | 0 |
| 6 | – | 1 |
| 7 | 8 | 0 |
| 8 | – | 1 |

$\xrightarrow{*}$

| | a | |
|---|---|---|
| →0 | 4,7 | 1 |
| 4 | 5 | 0 |
| 5 | 6 | 0 |
| 6 | 4,7 | 1 |
| 7 | 8 | 0 |
| 8 | 4,7 | 1 |

$\cdot$

| | a | |
|---|---|---|
| →0 | 1,2 | 0 |
| 1 | 4,7 | 1 |
| 2 | 3 | 0 |
| 3 | 4,7 | 1 |
| 4 | 5 | 0 |
| 5 | 6 | 0 |
| 6 | 4,7 | 1 |
| 7 | 8 | 0 |
| 8 | 4,7 | 1 |

✓

contineued on the next page

3(a) continued

Corresponding DFA

| | | a | |
|---|---|---|---|
| A | $\rightarrow$ 0 | 1,2 | 0 |
| B | 1,2 | 3,4,7 | 1 |
| C | 3,4,7 | 4,5,7,8 | 1 |
| D | 4,5,7,8 | 4,5,6,7,8 | 1 |
| E | 4,5,6,7,8 | 4,5,6,7,8 | 1 |

$\xrightarrow{\text{Rename}}$

| | a | |
|---|---|---|
| $\rightarrow$ A | B | 0 |
| B | C | 1 |
| C | D | 1 |
| D | E | 1 |
| E | E | 1 |

| ~~Q-F~~ | F |
|---|---|
| A | B C D E |
| A | B C D E |
| 1 | 2 |

22

Reduced DFA :

| | a | |
|---|---|---|
| $\rightarrow$ 1 | 2 | 0 |
| 2 | 2 | 1 |

3(b)

$$(0 \cdot 0 \cup 1 \cdot 1)^* \cdot ((1 \cdot 1)^* \cup (0 \cdot 0)^*)$$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_1$ | – | 0 |
| $q_1$ | $q_2$ | – | 0 |
| $q_2$ | – | – | 1 |

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | – | $q_3$ | 0 |
| $q_3$ | – | $q_4$ | 0 |
| $q_4$ | – | – | 1 |

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | – | $q_5$ | 0 |
| $q_5$ | – | $q_6$ | 0 |
| $q_6$ | – | – | 1 |

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_7$ | – | 0 |
| $q_7$ | $q_8$ | – | 0 |
| $q_8$ | – | – | 1 |

$\cup$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_1$ | $q_3$ | 0 |
| $q_1$ | $q_2$ | – | 0 |
| $q_2$ | – | – | 1 |
| $q_3$ | – | $q_4$ | 0 |
| $q_4$ | – | – | 1 |

$*$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | – | $q_5$ | 1 |
| $q_5$ | – | $q_6$ | 0 |
| $q_6$ | – | $q_5$ | 1 |

$*$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_7$ | – | 1 |
| $q_7$ | $q_8$ | – | 0 |
| $q_8$ | $q_7$ | – | 1 |

$\cup$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_7$ | $q_5$ | 1 |
| $q_5$ | – | $q_6$ | 0 |
| $q_6$ | – | $q_5$ | 1 |
| $q_7$ | $q_8$ | – | 0 |
| $q_8$ | $q_7$ | – | 1 |

$*$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_1$ | $q_3$ | 1 |
| $q_1$ | $q_2$ | – | 0 |
| $q_2$ | $q_1$ | $q_3$ | 1 |
| $q_3$ | – | $q_4$ | 0 |
| $q_4$ | $q_1$ | $q_3$ | 1 |

$\cdot$

| | 0 | 1 | |
|---|---|---|---|
| $\to q_0$ | $q_1q_7$ | $q_3q_5$ | 1 |
| $q_1$ | $q_2$ | – | 0 |
| $q_2$ | $q_1q_7$ | $q_3q_5$ | 1 |
| $q_3$ | – | $q_4$ | 0 |
| $q_4$ | $q_1q_7$ | $q_3q_5$ | 1 |
| $q_5$ | – | $q_6$ | 0 |
| $q_6$ | – | $q_5$ | 1 |
| $q_7$ | $q_8$ | – | 0 |
| $q_8$ | $q_7$ | – | 1 |

continued on the next page

3(b) continued

corresponding DFA

| | | 0 | 1 | |
|---|---|---|---|---|
| A | $\rightarrow q_0$ | $q_1q_7$ | $q_3q_5$ | 1 |
| B | $q_1q_7$ | $q_2q_8$ | — | 0 |
| C | $q_3q_5$ | — | $q_4q_6$ | 0 |
| D | $q_2q_8$ | $q_1q_7$ | $q_3q_5$ | 1 |
| E | — | — | — | 0 |
| F | $q_4q_6$ | $q_1q_7$ | $q_3q_5$ | 1 |

$\xrightarrow{\text{Rename}}$

| | 0 | 1 | |
|---|---|---|---|
| $\rightarrow$A | B | C | 1 |
| B | D | E | 0 |
| C | E | F | 0 |
| D | B | C | 1 |
| E | E | E | 0 |
| F | B | C | 1 |

| Q−F | | | F |
|---|---|---|---|
| B C E | | | A D F |
| B | C | E | A D F |
| B | C | E | A D F |
| 1 | 2 | 3 | 4 |

44

Reduced DFA

| | 0 | 1 | |
|---|---|---|---|
| $\rightarrow$4 | 1 | 2 | 1 |
| 1 | 4 | 3 | 0 |
| 2 | 3 | 4 | 0 |
| 3 | 3 | 3 | 0 |

4

$$\begin{cases} L_A = b\cdot L_C \cup \varepsilon \\ L_B = a\cdot L_A \\ L_C = \underbrace{b\cdot L_B}_{M} \cup \underbrace{b\cdot L_C}_{L\cdot X} \end{cases}$$

$L_C = b\cdot L_C \cup b\cdot L_B$

$\because$ lemma

$\therefore$ $L_C = b^*\cdot(b\cdot L_B)$

Put $L_B = a\cdot L_A$ into $L_C$

$L_C = b^*\cdot(b\cdot a\cdot L_A)$

Plug $L_C$ into $L_A$

$L_A = b\cdot b^*(b\cdot a\cdot L_A) \cup \varepsilon$

$\because$ lemma

$\therefore$ $L_A = (b\cdot b^*\cdot b\cdot a)^*$

$\because$ A is the initial state

$\therefore$ The languge accepted by this automaton is $L_A$

$L_A = (b\cdot b^*\cdot b\cdot a)^*$ 27