Supervised Learning

Linear regression

<u>Step 1. Hypothesis:</u>

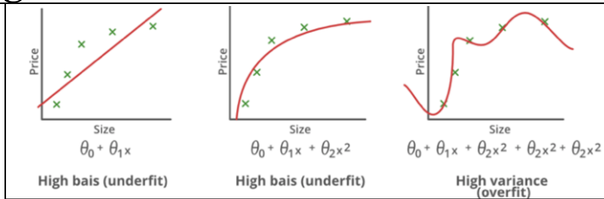
$$h_\theta(x)$$

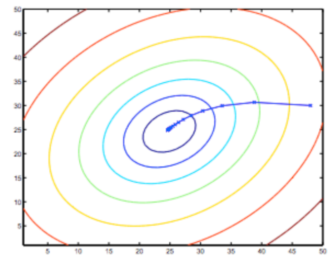
<u>Step 2. Cost</u>

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

<u>Step 3: Gradients</u>

$$\begin{cases} \theta_0 := \theta_0 - \alpha\dfrac{1}{m}\displaystyle\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_0^{(i)} \\ \theta_j := \theta_j\left(1 - \alpha\dfrac{\lambda}{m}\right) - \alpha\dfrac{1}{m}\displaystyle\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}, j = 1,2,3,\dots,n \end{cases}$$





Logistic regression

<u>Step 1. Hypothesis:</u>

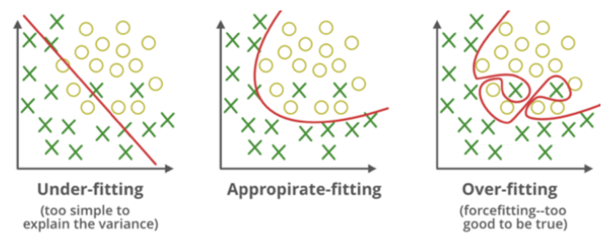
$$\begin{cases} h_\theta(z) = g(\theta^T x) \\ z = \theta^T x \\ g(z) = \dfrac{1}{1 + e^{-z}} \end{cases}$$

<u>Step 2. Cost</u>

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(h_\theta\left(x^{(i)}\right)\right) + (1 - y^{(i)})\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

<u>Step 3. Gradients:</u>

$$\begin{cases} \theta_0 := \theta_0 - \alpha\dfrac{1}{m}\displaystyle\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_0^{(i)} \\ \theta_j := \theta_j\left(1 - \alpha\dfrac{\lambda}{m}\right) - \alpha\dfrac{1}{m}\displaystyle\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}, j = 1,2,3,\dots,n \end{cases}$$

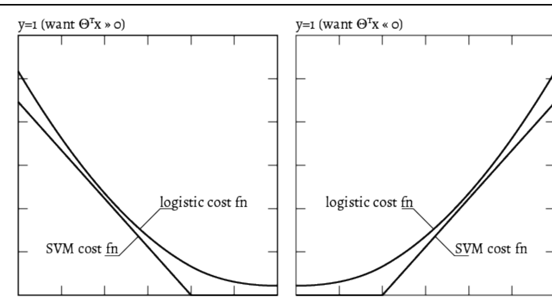


Support vector machines (SVM)

<u>Cost</u>

$$J(\theta) = C\sum_{i=1}^{m}\left[y^{(i)}\operatorname{cost}_1\left(\theta^T x^{(i)}\right) + (1 - y^{(i)})\operatorname{cost}_0\left(\theta^T x^{(i)}\right)\right] + \frac{1}{2}\sum_{j=1}^{m}\theta_j^2$$

$$y^{(i)} = \begin{cases} 1, & \theta^T x^{(i)} \geq 1 \\ 0, & \theta^T x^{(i)} \leq -1 \end{cases}$$



SVM with Gaussian Kernel

<u>Step 1. Hypothesis</u>

Given x, compute features $f \in \mathbb{R}^{m+1}$, *parameters* $\theta \in \mathbb{R}^{m+1}$

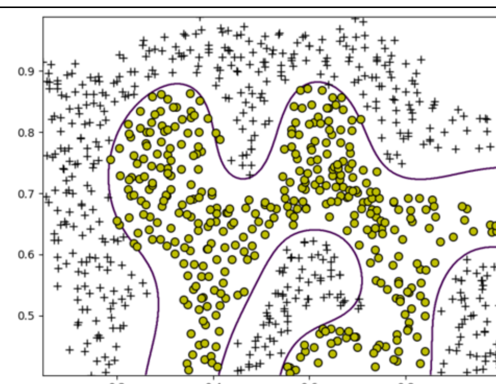
Predict "y=1" if $\theta^T f \geq 0,\ \theta_0 f_0 + \theta_1 f_1 + \cdots + \theta_m f_m \geq 0$

<u>Step 2. Training</u>

$$\min J(\theta) = C\sum_{i=1}^{m}\left[y^{(i)}\operatorname{cost}_1(\theta^T f_i) + (1 - y^{(i)})\operatorname{cost}_0(\theta^T f_i)\right] + \frac{1}{2}\sum_{j=1}^{m}\theta_j^2$$

$$f_i = similarity\left(x, l^{(i)}\right) = \exp\left(-\frac{\left\|x - l^{(i)}\right\|^2}{2\sigma^2}\right), or = \left(-\frac{\left\|x_1 - x_2\right\|^2}{2\sigma^2}\right)$$

$$Predict\ \text{“}y = 1\text{”}\ if\ \ \theta^T f_i \geq 0$$



Neural network
Classification

Step 1. Randomly initialize weights

Initialize parameters $\Theta^{(1)}, \Theta^{(2)}, \ldots, \Theta^{(L-1)}$ $[-\epsilon, \epsilon]$(i.e. $-\epsilon \le \Theta_{ji}^{(l)} \le \epsilon$)

Step 2. Forward propagation

$$h_\Theta\left(x^{(i)}\right) \in \mathbb{R}^K \quad \left(h_\Theta(X)\right)_i = i^{th}\ output$$

$a^{(1)} = x$

$z^{(2)} = \Theta^{(1)} a^{(1)}$

$a^{(2)} = g\left(z^{(2)}\right)\ \left(add\ a_0^{(2)}\right)$

$z^{(3)} = \Theta^{(2)} a^{(2)}$

$a^{(3)} = g\left(z^{(3)}\right)\ \left(add\ a_0^{(3)}\right)$

$z^{(4)} = \Theta^{(3)} a^{(3)}$

$a^{(4)} = h_\Theta(x) = g\left(z^{(4)}\right)$

Step 3. Cost function $J(\Theta)$

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^{(i)} \log\left(h_\Theta\left(x^{(i)}\right)\right)_k + \left(1 - y_k^{(i)}\right)\log\left(1 - \left(h_\Theta\left(x^{(i)}\right)\right)_k\right)\right]$$
$$+ \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(\Theta_{ji}^{(l)}\right)^2$$

Step 4. Backpropagation to compute partial derivatives $\frac{\partial}{\partial\Theta_{jk}^{(l)}}J(\Theta)$

$\delta_j^{(l)}$ = "error" of node j in layer l.

$$\delta^{(2)} = \left(\Theta^{(2)}\right)^T \delta^{(3)} * g'\left(z^{(2)}\right)$$
$$g'\left(z^{(2)}\right) = a^{(2)} * \left(1 - a^{(2)}\right)$$
$$\delta^{(3)} = \left(\Theta^{(3)}\right)^T \delta^{(4)} * g'\left(z^{(3)}\right)$$
$$g'\left(z^{(3)}\right) = a^{(3)} * \left(1 - a^{(3)}\right)$$
$$\delta^{(4)} = a^{(4)} - y$$

$\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)}\left(a^{(l)}\right)^T$

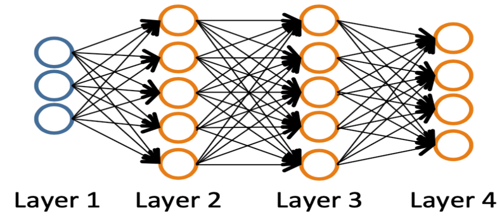
$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} + \lambda\Theta_{ij}^{(l)}, if\ j \ne 0$$
$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} \qquad , if\ j = 0$$

$$D_{ij}^{(l)} = \frac{\partial}{\partial\Theta_{ij}^{(l)}}J(\Theta) = \frac{\partial J(\Theta)}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial \Theta}$$

Step 5. Use gradient checking to compare $\frac{\partial}{\partial\Theta_{jk}^{(l)}}J(\Theta)$ computed using backpropagation vs. using numerical estimate of gradient of $J(\Theta)$.

Step 6. Use gradient descent or advanced optimization method with backpropagation to try to minimize $J(\Theta)$ as a function of parameters Θ .

```
result = minimize(cost_func, initial_nn_params, method='CG',
jac=grad_func,
          options={'disp': True, 'maxiter': 50.0})
nn_params = result.x
Jcost = result.fun
```



Unsupervised Learning

K-means

<u>Step 1. Centroids</u>

$$c^{(i)} = index\ of\ min\left\|x^{(i)} - \mu_j\right\|^2$$

$c^{(i)} \in \mathbb{R}^K$, i = 1,2, …, m denotes the index of cluster centroids closet to $x^{(i)}$

<u>Step 2. Means</u>

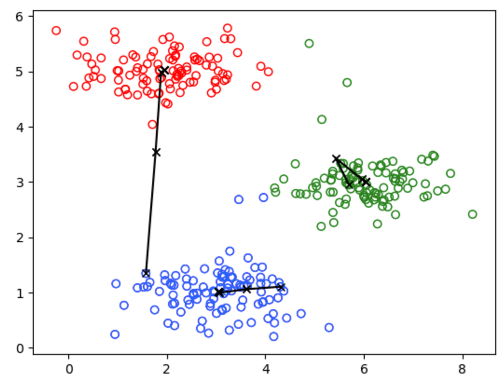
$$\mu_k = \frac{\sum_{i=1,\ \{c^{(i)}=k\}}^{m} x^{(i)}}{\sum_{i=1,\ \{c^{(i)}=k\}}^{m} 1}$$

$\mu_k \in \mathbb{R}^K, k = 1,2, \dots, K$ denotes the average(mean) of points assigned to cluster k

<u>Step 3. Cost function</u>

$$\boldsymbol{J}_{(\boldsymbol{c},\boldsymbol{\mu})} = \sum_{\boldsymbol{i}}^{\boldsymbol{m}} \left\|\boldsymbol{x}^{(i)} - \boldsymbol{\mu}_{\boldsymbol{c}^{(i)}}\right\|^2$$

K=3



Principal Component Analysis (PCA)

Dimensionality Reduction

Data compression

<u>Step1. Feature scaling (Mean normalization)</u>

Mean: $\bar{X} = \mu_j = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$

Standard deviation: $s = \sigma = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(x^{(i)} - \mu)^2}$

Mean normalize: $x^{(i)} = \frac{x^{(i)}-\mu}{\sigma}$

<u>Step 2. Calculate U, S, V.</u>

$$\boldsymbol{sigma} = \frac{\boldsymbol{1}}{\boldsymbol{m}}\sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{m}}(\boldsymbol{x}^{(i)})(\boldsymbol{x}^{(i)})^{\boldsymbol{T}} = \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}$$

U, S, V = numpy.linalg.svd(sigma)

$$\boldsymbol{U} = \begin{pmatrix} | & | & | & | & | & | \\ \boldsymbol{u}^{(1)} & \boldsymbol{u}^{(2)} & \cdots & \boldsymbol{u}^{(k)} & \cdots & \boldsymbol{u}^{(n)} \\ | & | & | & | & | & | \end{pmatrix}$$

Ureduce = U[:, 0:K].T

Z = Ureduce*X = X_norm * U[:, 0:K]

X_approximate = X_recovered = Z * U[:, 0:K].T

<u>Step 3. Pick the smallest value of k,</u>

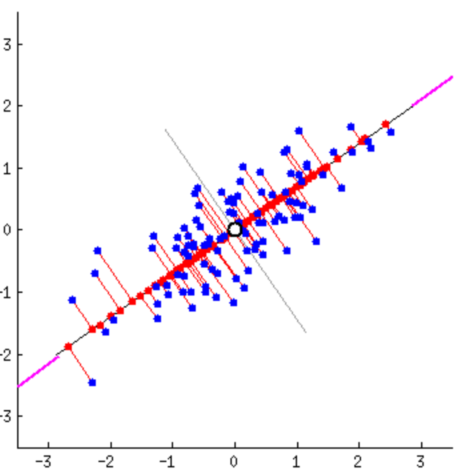
$$\frac{\frac{1}{m}\sum_{i=1}^{m}\left\|x^{(i)} - x_{approx}^{(i)}\right\|^2}{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2} \leq 0.01?$$

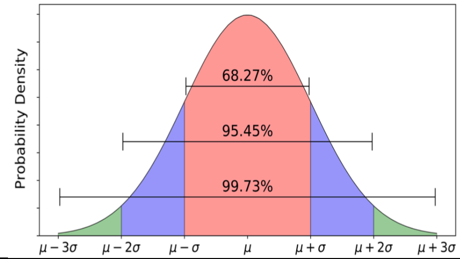
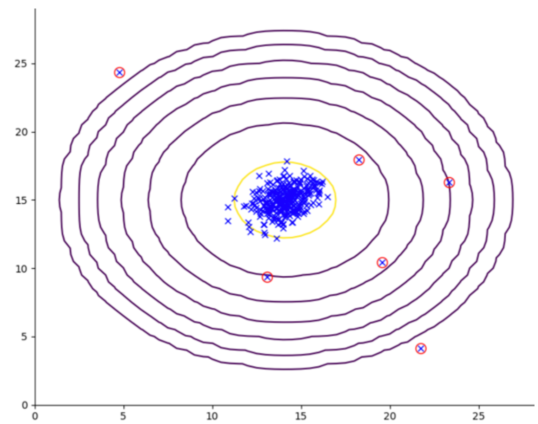
$$S = \begin{pmatrix} S_{11} & & \cdots & & 0 \\ & S_{22} & \cdots & 0 & \\ \vdots & \vdots & S_{33} & \vdots & \vdots \\ & 0 & \cdots & \ddots & \\ 0 & & \cdots & & S_{nn} \end{pmatrix}$$

$$1 - \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \leq 0.01 \ \rightarrow \ \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \geq 0.99$$

99% of variance retained

$2D \rightarrow 1D$



Anomaly Detection *Fraud detection, Manufacturing, Monitoring computers in a data center*	**Gaussian (Normal) distribution** $\boldsymbol{X \sim N(\mu, \sigma^2)}$ **Mean:** $\boldsymbol{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}}$ **Variance:** $\boldsymbol{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2}$ **Probability:** $\boldsymbol{p(x,\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}}$	
Original model	<u>Step 1. Choose feature</u> Training set: $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}, x^{(i)} \in \mathbb{R}^n$ Density estimation: $\boldsymbol{x_j \sim N(\mu_j, \sigma_j^2), j = 1, 2, \dots, n}$ Choose features x_i that might be indicative of anomalous examples. <u>Step 2. Fit parameters</u> $$\boldsymbol{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}}$$ $$\boldsymbol{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2}$$ <u>Step 3. Given new example $x \in \mathbb{R}^n$, compute $\boldsymbol{p(x)}$</u> <u>Probability</u> $$\boldsymbol{p(x) = \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} e^{\left(-\frac{(x_j-\mu_j)^2}{2\sigma_j^2}\right)}}$$ $$\boldsymbol{y = \begin{cases} 1, & if\ p(x) < \epsilon (anomaly) \\ 0, & if\ p(x) \geq \epsilon (normal) \end{cases}}$$	
Multivariate Gaussian	<u>Step 1. Choose feature</u> Training set: $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}, x^{(i)} \in \mathbb{R}^n$ Density estimation: $x_j \sim N(\mu_j, \sigma_j^2), j = 1, 2, \dots,$ n <u>Step 2. Fit parameters</u> Parameters: $\mu \in \mathbb{R}^n, \Sigma \in \mathbb{R}^{n \times n}$ (covariance matrix) Mean: $\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$ Variance: $\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$ <u>Step 3. Given new example x, compute $p(x)$</u> Diagonal Sigma: $\Sigma = \begin{pmatrix} \Sigma_1 & & \cdots & & 0 \\ & \Sigma_2 & \dots & 0 & \\ \vdots & \vdots & \Sigma_3 & \vdots & \vdots \\ & 0 & \dots & \ddots & \\ 0 & & \cdots & & \Sigma_n \end{pmatrix}$ Probability: $$p(x_j; \mu_j, \Sigma) = \frac{1}{\sqrt{2\pi}\lvert\Sigma\rvert^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-u)\right)}$$ $$y = \begin{cases} 1, & if\ p(x) < \epsilon (anomaly) \\ 0, & if\ p(x) \geq \epsilon (normal) \end{cases}$$