Supervised Learning

Linear Regression
**Trend*
**Market estimates*
**Forecasts*

Step 1. Hypothesis:

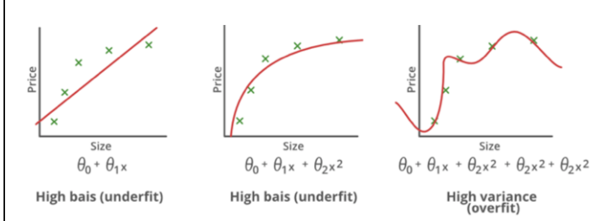
$$h_\theta(x)$$

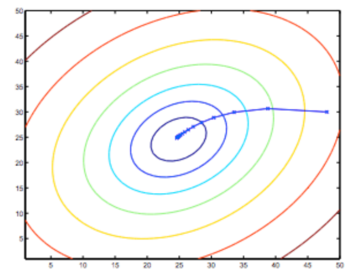
Step 2. Cost

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

Step 3: Gradients

$$\begin{cases} \theta_0 := \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_0^{(i)} \\ \theta_j := \theta_j\left(1 - \alpha\frac{\lambda}{m}\right) - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}, j = 1,2,3,\dots,n \end{cases}$$





Logistic Regression
**Binary classes*

Step 1. Hypothesis:

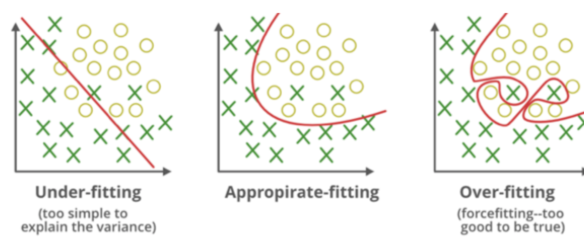
$$\begin{cases} h_\theta(z) = g(\theta^T x) \\ z = \theta^T x \\ g(z) = \frac{1}{1+e^{-z}} \end{cases}$$

Step 2. Cost

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1-y^{(i)}\right)\log\left(1-h_\theta\left(x^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

Step 3. Gradients:

$$\begin{cases} \theta_0 := \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_0^{(i)} \\ \theta_j := \theta_j\left(1 - \alpha\frac{\lambda}{m}\right) - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}, j = 1,2,3,\dots,n \end{cases}$$

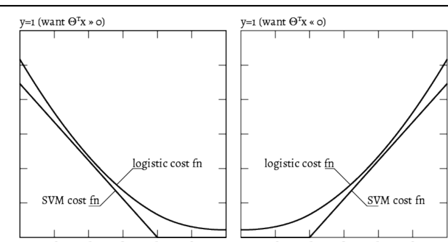


Support vector machines (SVM)

Cost

$$J(\theta) = C\sum_{i=1}^{m}\left[y^{(i)}\operatorname{cost}_1\left(\theta^T x^{(i)}\right) + \left(1-y^{(i)}\right)\operatorname{cost}_0\left(\theta^T x^{(i)}\right)\right] + \frac{1}{2}\sum_{j=1}^{m}\theta_j^2$$

$$y^{(i)} = \begin{cases} 1, & \theta^T x^{(i)} \geq 1 \\ 0, & \theta^T x^{(i)} \leq -1 \end{cases}$$



SVM with Gaussian Kernel

Step 1. Hypothesis

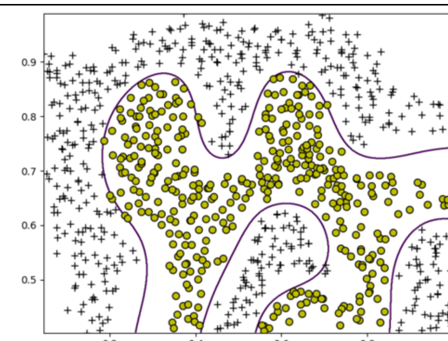
Given x, compute features $f \in \mathbb{R}^{m+1}$*, parameters* $\theta \in \mathbb{R}^{m+1}$
Predict "y=1" if $\theta^T f \geq 0,\ \theta_0 f_0 + \theta_1 f_1 + \cdots + \theta_m f_m \geq 0$

Step 2. Training

$$\min J(\theta) = C\sum_{i=1}^{m}\left[y^{(i)}\, cost_1(\theta^T f_i) + (1-y^{(i)})\, cost_0(\theta^T f_i)\right] + \frac{1}{2}\sum_{j=1}^{m}\theta_j^2$$

$$f_i = similarity\left(x, l^{(i)}\right) = \exp\left(-\frac{\left\|x - l^{(i)}\right\|^2}{2\sigma^2}\right), or = \left(-\frac{\left\|x_1 - x_2\right\|^2}{2\sigma^2}\right)$$

$$Predict\ \text{"}y = 1\text{"}\ if\ \ \theta^T f_i \geq 0$$



Neural network

**Pattern recognition*
**Fraud detection*
**Deep learning.*

Step 1. Randomly initialize weights

Initialize parameters $\Theta^{(1)}, \Theta^{(2)}, \ldots, \Theta^{(L-1)}$ $[-\epsilon, \epsilon]$(i.e. $-\epsilon \le \Theta_{ji}^{(l)} \le \epsilon$)

Step 2. Forward propagation

$$h_\Theta\left(x^{(i)}\right) \in \mathbb{R}^K \quad \left(h_\Theta(X)\right)_i = i^{th}\ output$$

$a^{(1)} = x$
$z^{(2)} = \Theta^{(1)}a^{(1)}$
$a^{(2)} = g\left(z^{(2)}\right)\ \left(add\ a_0^{(2)}\right)$
$z^{(3)} = \Theta^{(2)}a^{(2)}$
$a^{(3)} = g\left(z^{(3)}\right)\ \left(add\ a_0^{(3)}\right)$
$z^{(4)} = \Theta^{(3)}a^{(3)}$
$a^{(4)} = h_\Theta(x) = g\left(z^{(4)}\right)$

Step 3. Cost function $J(\Theta)$

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^{(i)}\log\left(h_\Theta\left(x^{(i)}\right)\right)_k + \left(1-y_k^{(i)}\right)\log\left(1-\left(h_\Theta\left(x^{(i)}\right)\right)_k\right)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(\Theta_{ji}^{(l)}\right)^2$$

Step 4. Backpropagation to compute partial derivatives

$$\frac{\partial}{\partial\Theta_{jk}^{(l)}}J(\Theta)$$

$\delta_j^{(l)}$ = "error" of node j in layer l.

$$\delta^{(2)} = \left(\Theta^{(2)}\right)^T\delta^{(3)} * g'\left(z^{(2)}\right)$$
$$g'\left(z^{(2)}\right) = a^{(2)} * \left(1-a^{(2)}\right)$$
$$\delta^{(3)} = \left(\Theta^{(3)}\right)^T\delta^{(4)} * g'\left(z^{(3)}\right)$$
$$g'\left(z^{(3)}\right) = a^{(3)} * \left(1-a^{(3)}\right)$$
$$\delta^{(4)} = a^{(4)} - y$$

$\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)}\left(a^{(l)}\right)^T$

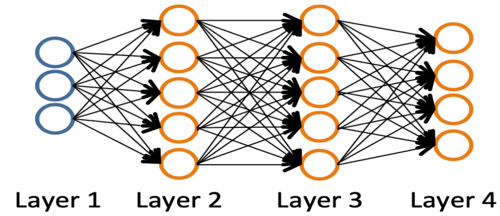
$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} + \lambda\Theta_{ij}^{(l)}, if\ j \ne 0$$
$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} \qquad , if\ j = 0$$

$$D_{ij}^{(l)} = \frac{\partial}{\partial\Theta_{ij}^{(l)}}J(\Theta) = \frac{\partial J(\Theta)}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial\Theta}$$

Step 5. Use gradient checking to compare $\frac{\partial}{\partial\Theta_{jk}^{(l)}}J(\Theta)$ computed using backpropagation vs. using numerical estimate of gradient of $J(\Theta)$.

Step 6. Use gradient descent or advanced optimization method with backpropagation to try to minimize $J(\Theta)$ as a function of parameters Θ .

```
result = minimize(cost_func, initial_nn_params, method='CG',
jac=grad_func,
        options={'disp': True, 'maxiter': 50.0})
nn_params = result.x
Jcost = result.fun
```



<table>
<tr><th colspan="3">Unsupervised Learning</th></tr>
<tr>
<td>K-means</td>
<td>
<u>Step 1. Centroids</u>
$$c^{(i)} = index\ of\ min\left\|x^{(i)} - \mu_j\right\|^2$$
$c^{(i)} \in \mathbb{R}^K$, i $= 1,2,\dots,$ m denotes the index of cluster centroids closet to $x^{(i)}$

<u>Step 2. Means</u>
$$\mu_k = \frac{\sum_{i=1,\ \{c^{(i)}=k\}}^{m} x^{(i)}}{\sum_{i=1,\ \{c^{(i)}=k\}}^{m} 1}$$
$\mu_k \in \mathbb{R}^K, k = 1,2,\dots,K$ denotes the average(mean) of points assigned to cluster k

<u>Step 3. Cost function</u>
$$\boldsymbol{J_{(c,\mu)}} = \sum_{i}^{m}\left\|\boldsymbol{x^{(i)}} - \boldsymbol{\mu_{c^{(i)}}}\right\|^2$$
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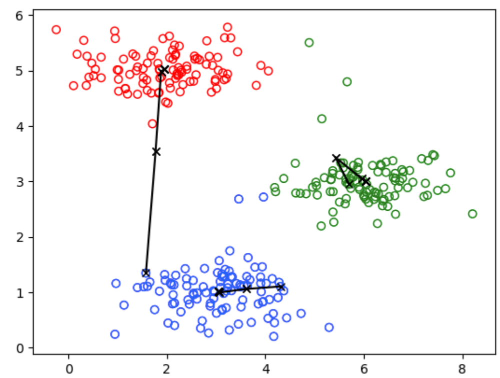
<td>
K=3


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<tr>
<td>Principal Component Analysis
(PCA)
*Dimensionality Reduction,
*Facial recognition,
*Data compression,
*Computer vision and image compression</td>
<td>
<u>Step1. Feature scaling (Mean normalization)</u>
Mean: $\bar{X} = \mu_j = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$
Standard deviation: $s = \sigma = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(x^{(i)} - \mu)^2}$
Mean normalize: $x^{(i)} = \frac{x^{(i)} - \mu}{\sigma}$

<u>Step 2. Calculate U, S, V.</u>
$\boldsymbol{sigma} = \frac{1}{m}\sum_{i=1}^{m}\left(\boldsymbol{x^{(i)}}\right)\left(\boldsymbol{x^{(i)}}\right)^{T} = \frac{1}{m}\boldsymbol{X^T X}$
U, S, V = numpy.linalg.svd(sigma)
$$\boldsymbol{U} = \begin{pmatrix} | & | & | & | & | & | \\ \boldsymbol{u^{(1)}} & \boldsymbol{u^{(2)}} & \cdots & \boldsymbol{u^{(k)}} & \cdots & \boldsymbol{u^{(n)}} \\ | & | & | & | & | & | \end{pmatrix}$$
Ureduce = U[:, 0:K].T
Z = Ureduce*X = X_norm * U[:, 0:K]
X_approximate = X_recovered = Z * U[:, 0:K].T

<u>Step 3. Pick the smallest value of k,</u>
$$\frac{\frac{1}{m}\sum_{i=1}^{m}\left\|x^{(i)} - x_{approx}^{(i)}\right\|^2}{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2} \le 0.01?$$
$$S = \begin{pmatrix} S_{11} & & \cdots & & 0 \\ & S_{22} & \dots & 0 & \\ \vdots & \vdots & S_{33} & \vdots & \vdots \\ & 0 & \dots & \ddots & \\ 0 & & \cdots & & S_{nn} \end{pmatrix}$$
$$1 - \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \le 0.01 \ \rightarrow\ \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \ge 0.99$$
99% of variance retained
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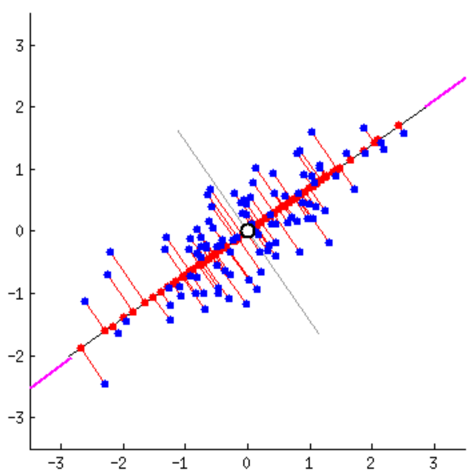
<td>
$2D \rightarrow 1D$


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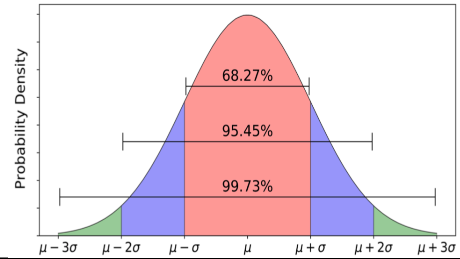
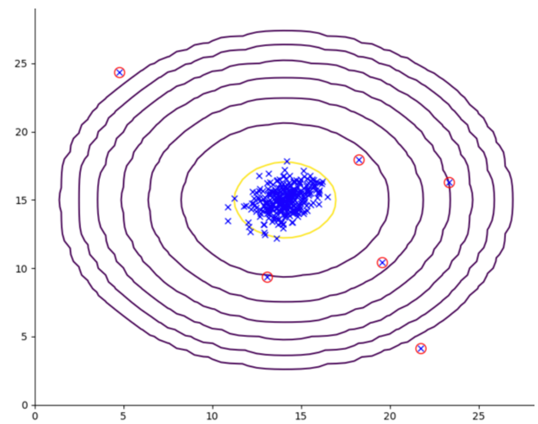
Anomaly Detection *Fraud detection *Intrusion detection *system health *monitoring	**Gaussian (Normal) distribution** $\boldsymbol{X \sim N(\mu, \sigma^2)}$ **Mean:** $\boldsymbol{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}}$ **Variance:** $\boldsymbol{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2}$ **Probability:** $\boldsymbol{p(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}}$	
Original model	<u>**Step 1. Choose feature**</u> Training set: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}, x^{(i)} \in \mathbb{R}^n$ Density estimation: $\boldsymbol{x_j \sim N(\mu_j, \sigma_j^2), j = 1, 2, \ldots, n}$ Choose features x_i that might be indicative of anomalous examples. <u>**Step 2. Fit parameters**</u> $$\boldsymbol{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}}$$ $$\boldsymbol{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2}$$ <u>**Step 3. Given new example $\boldsymbol{x \in \mathbb{R}^n}$, compute $\boldsymbol{p(x)}$**</u> <u>**Probability**</u> $$\boldsymbol{p(x) = \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} e^{\left(-\frac{(x_j-\mu_j)^2}{2\sigma_j^2}\right)}}$$ $$\boldsymbol{y = \begin{cases} 1, & if\ p(x) < \epsilon (anomaly) \\ 0, & if\ p(x) \geq \epsilon (normal) \end{cases}}$$	
Multivariate Gaussian	<u>**Step 1. Choose feature**</u> Training set: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}, x^{(i)} \in \mathbb{R}^n$ Density estimation: $x_j \sim N(\mu_j, \sigma_j^2), j = 1, 2, \ldots, n$ <u>**Step 2. Fit parameters**</u> Parameters: $\mu \in \mathbb{R}^n, \Sigma \in \mathbb{R}^{n \times n}$ (covariance matrix) Mean: $\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$ Variance: $\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$ <u>**Step 3. Given new example $\boldsymbol{x}$, compute $\boldsymbol{p(x)}$**</u> Diagonal Sigma: $\Sigma = \begin{pmatrix} \Sigma_1 & & \cdots & & 0 \\ & \Sigma_2 & \cdots & 0 & \\ \vdots & \vdots & \Sigma_3 & \vdots & \vdots \\ & 0 & \cdots & \ddots & \\ 0 & & \cdots & & \Sigma_n \end{pmatrix}$ Probability: $$p(x_j; \mu_j, \Sigma) = \frac{1}{\sqrt{2\pi}\lvert\Sigma\rvert^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-u)\right)}$$ $$y = \begin{cases} 1, & if\ p(x) < \epsilon (anomaly) \\ 0, & if\ p(x) \geq \epsilon (normal) \end{cases}$$