CSC 665: Artificial Intelligence

Informed Search

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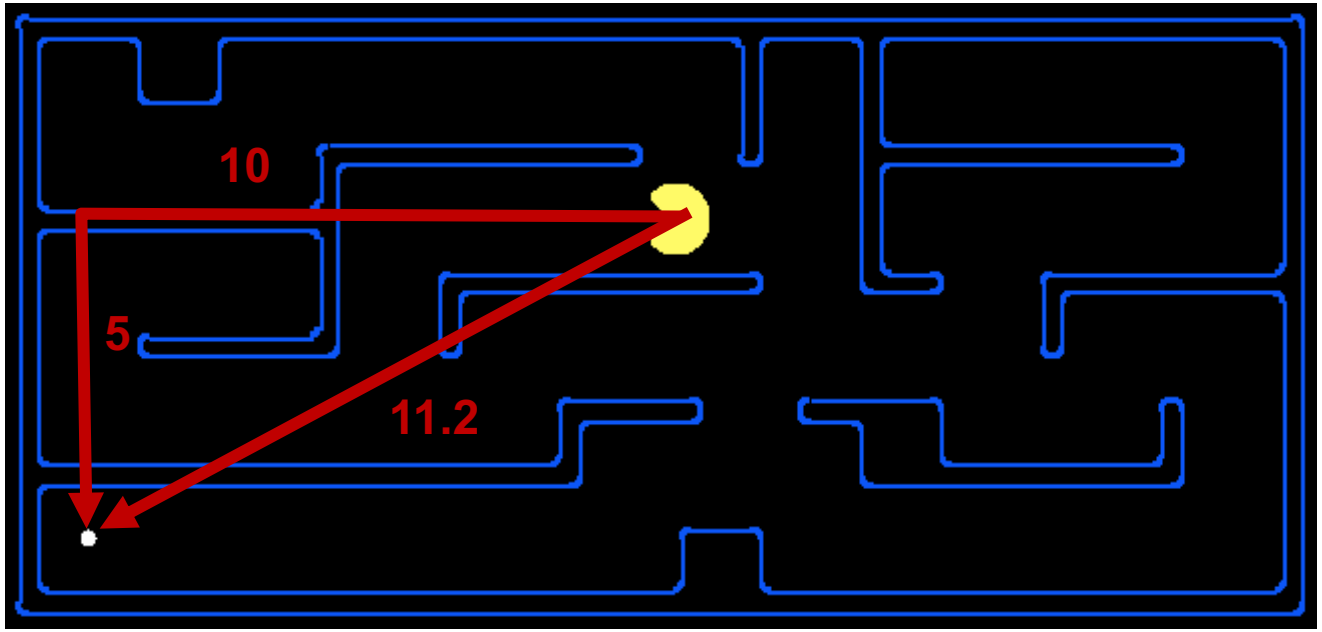
Today

- Informed Search
  - Heuristics
  - Best-First Search
  - A* Search

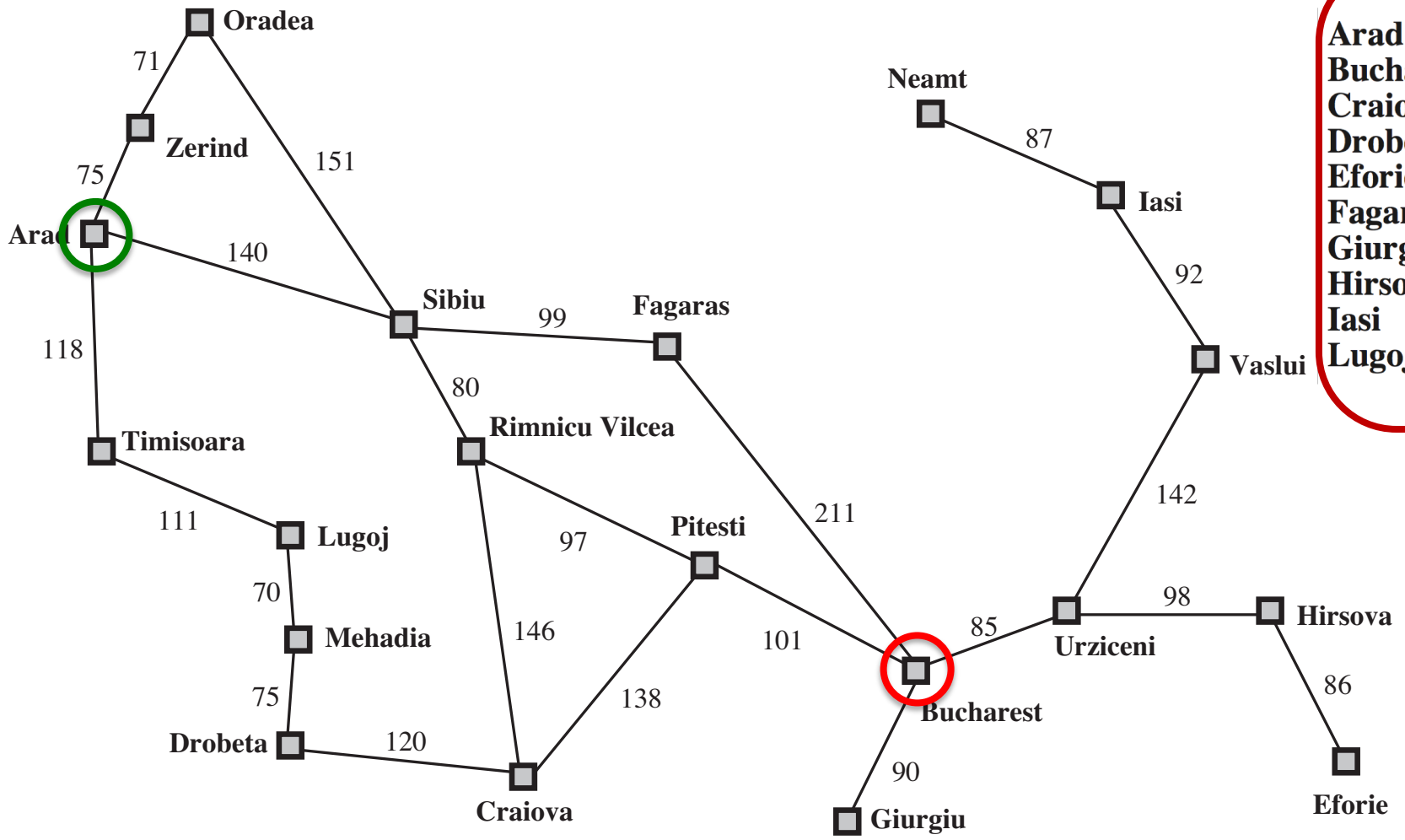
Heuristics

Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing



Example: Euclidean distance to Bucharest

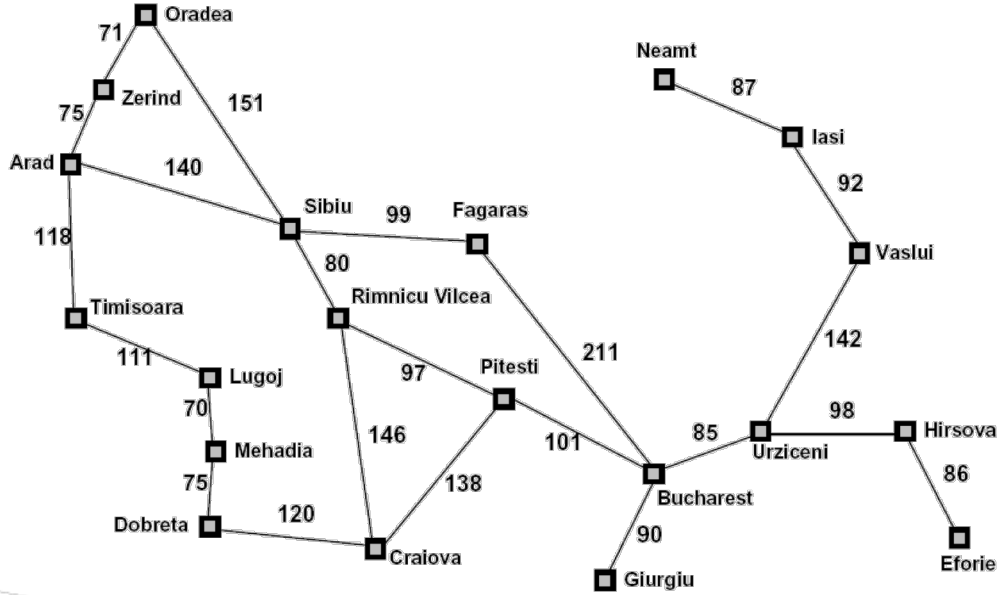


Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

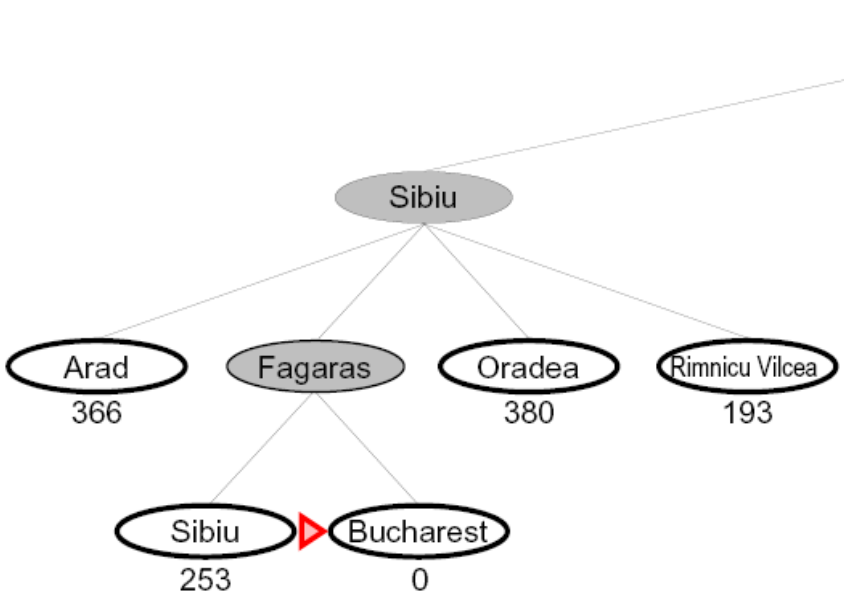
h(x)

Best-First Search

Best-First Search

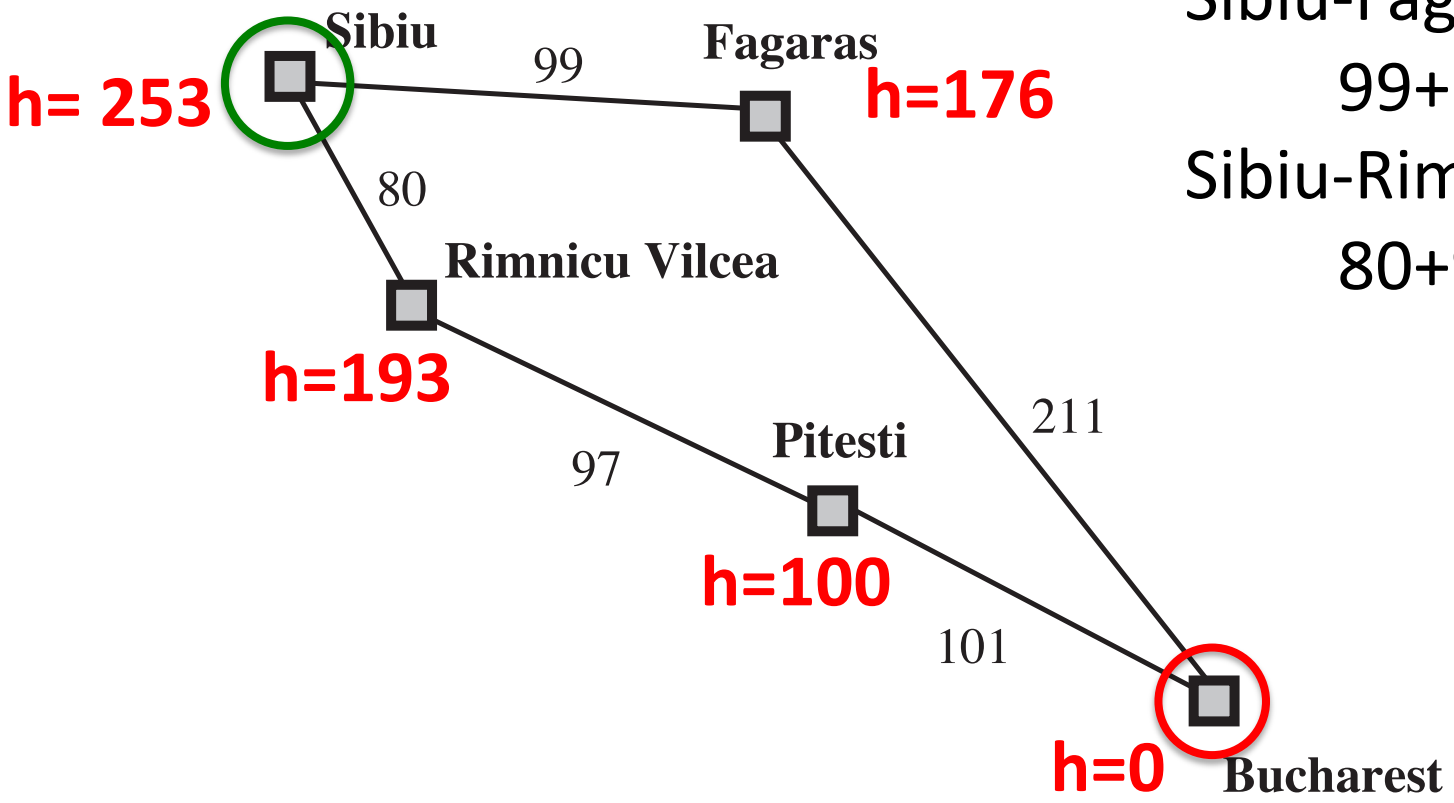


- Expand the node that seems closest…



- What can go wrong?

Best-First Search

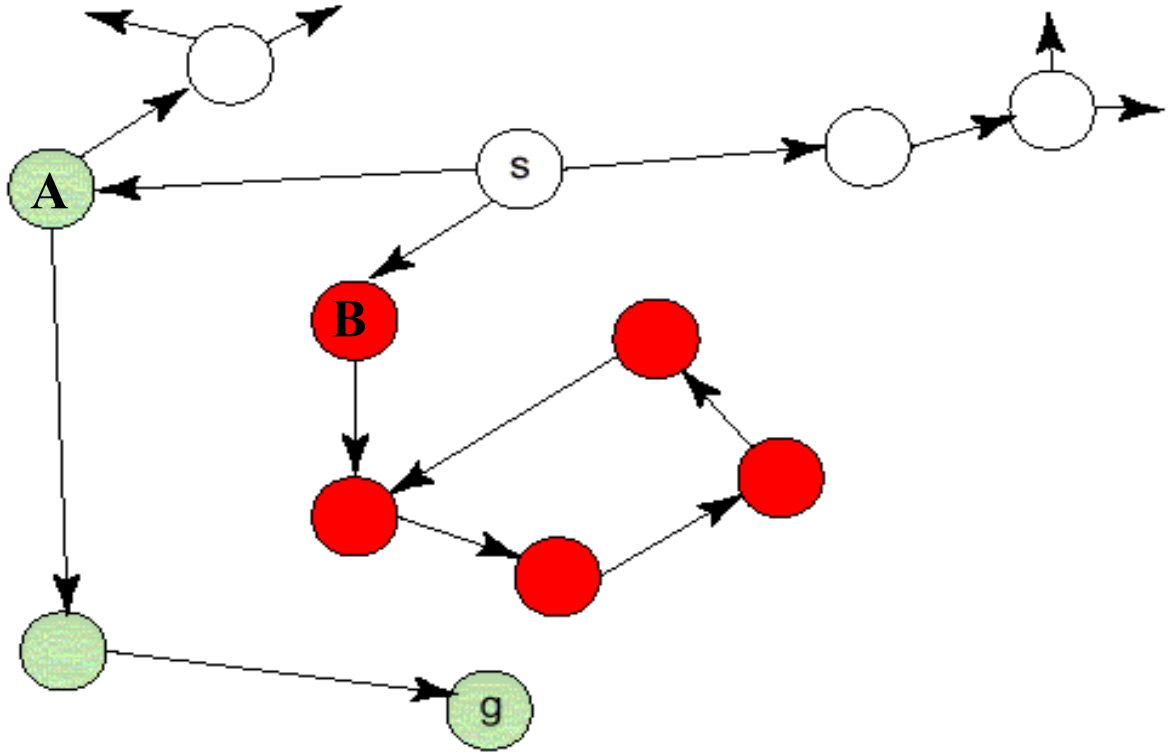


Sibiu-Fagaras-Bucharest =
99+211 = **310**
Sibiu-Rimnicu Vilcea-Pitesti-Bucharest =
80+97+101 = **278**

Best-First Search

- Idea: always choose the node on the frontier with the smallest h value.
- Best-First Search treats the frontier as a priority queue ordered by h.

Best-First Search



- A low heuristic value can mean that a cycle gets followed forever -> not complete

Best-First Search Properties

- Complete? No, see the example in the last slide
- Optimal? No, see the Romanian map example

The worst case is all the heuristic is the same

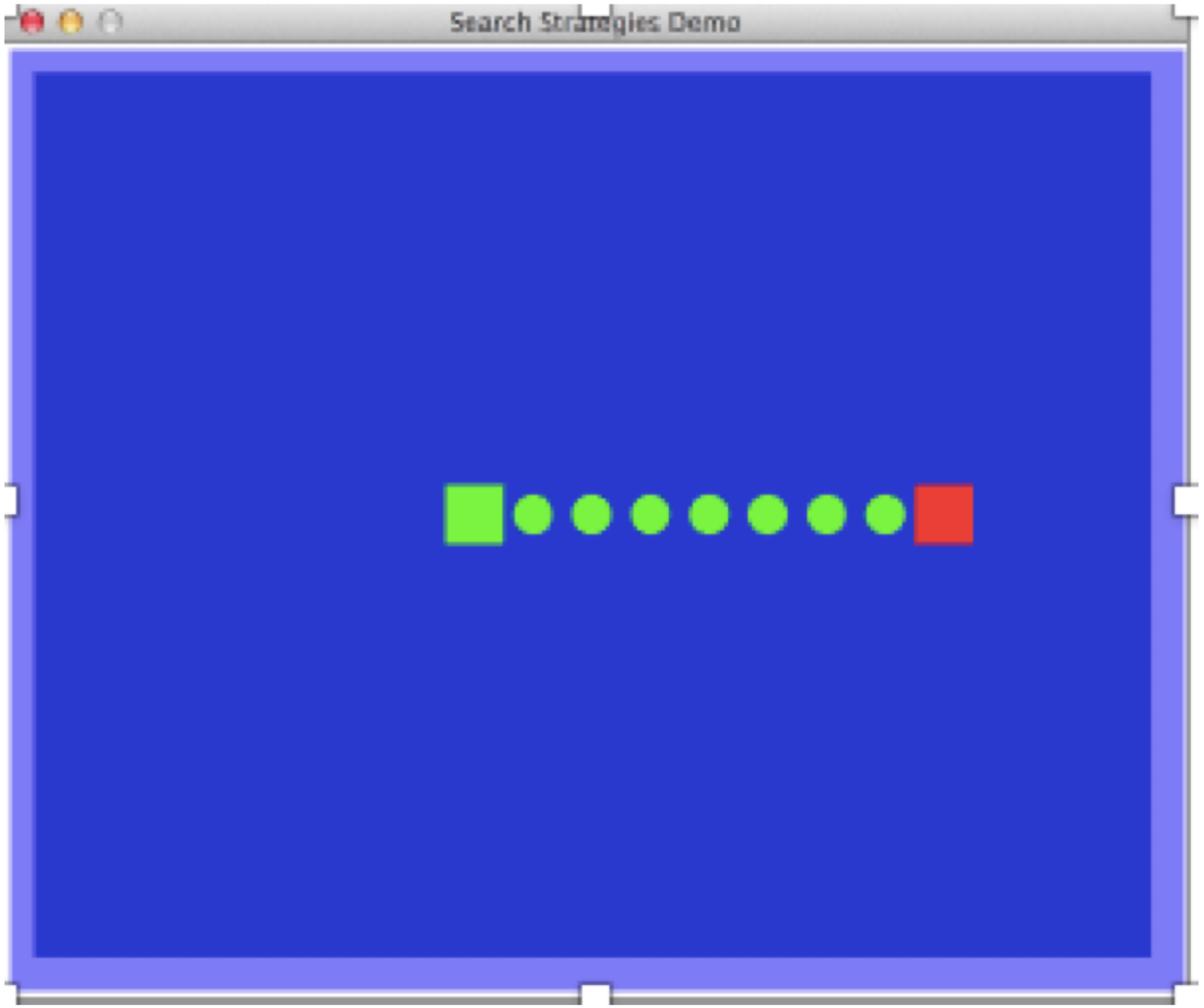
- Time Complexity $O(b^m)$ ✓ $O(m^b)$ $O(bm)$ $O(b+m)$

  - Worst case: has to explore all nodes

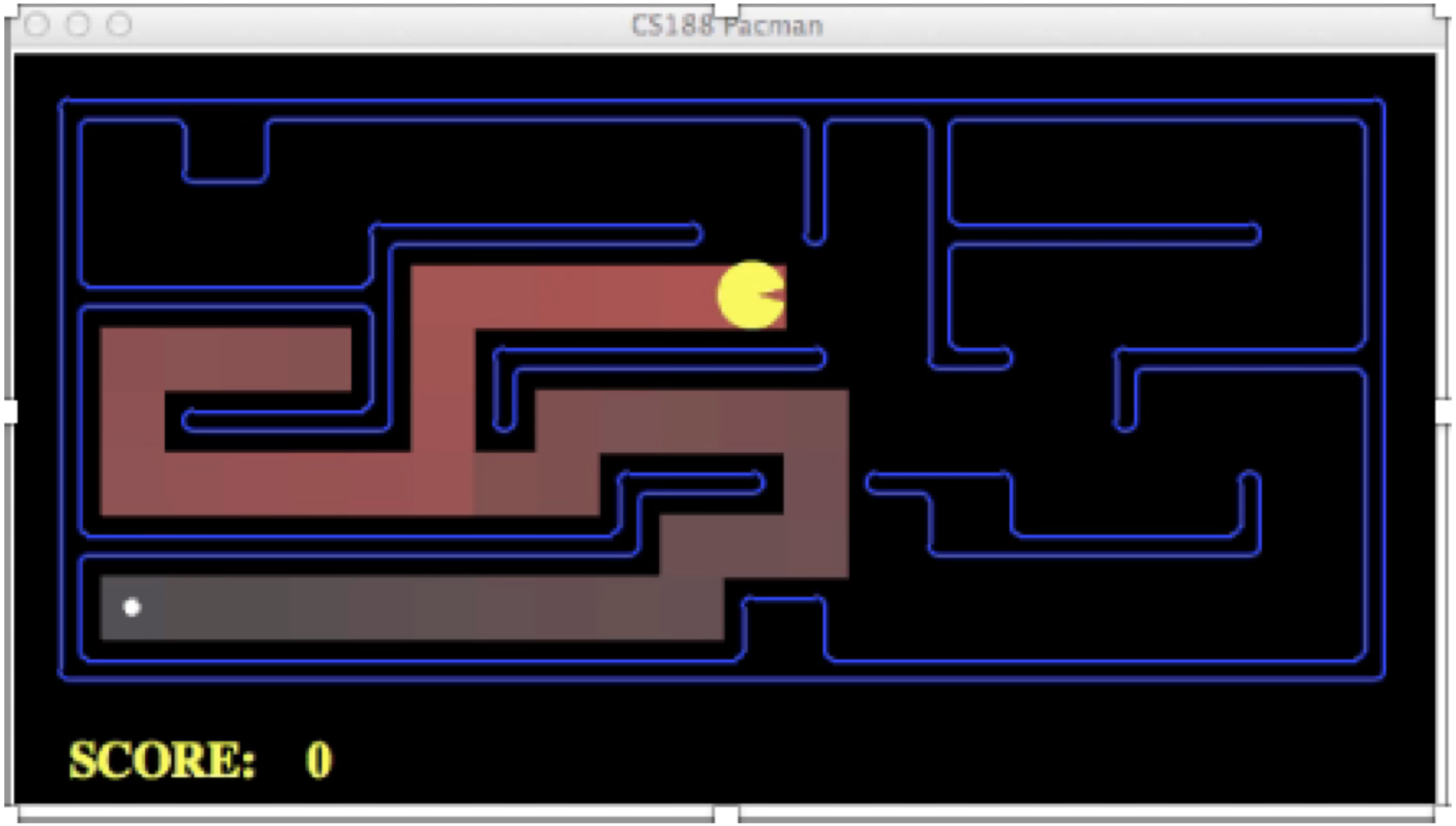
- Space Complexity $O(b^m)$ ✓ $O(m^b)$ $O(bm)$ $O(b+m)$

  - Heuristic could be chosen to emulate BFS:
    E.g. $h(n) = 0$

Video of Demo Contours BestFS (Empty)



Video of Demo Contours BestFS (Pacman Small Maze)



A* Search

A* Search

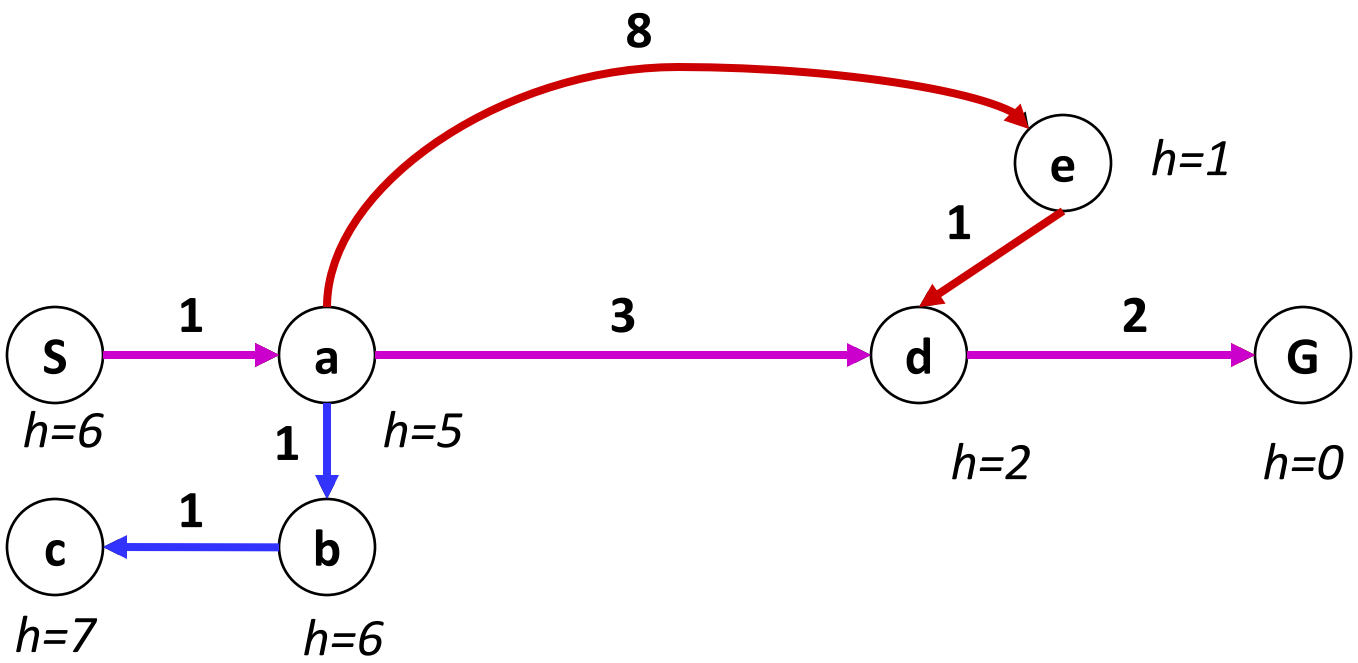
Uniform Cost Search

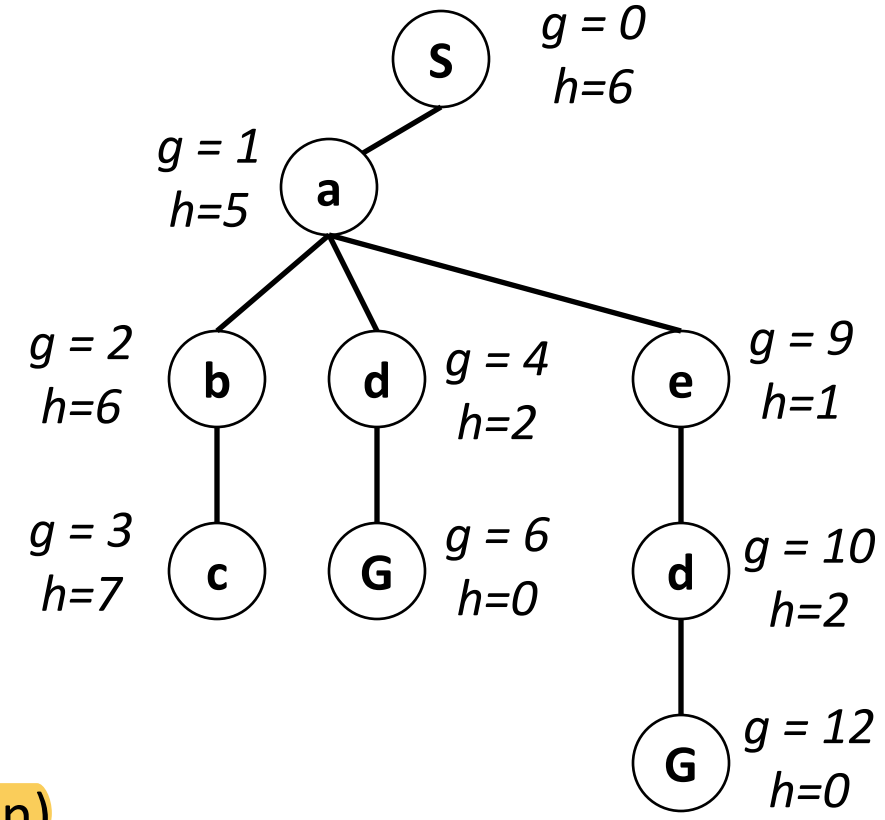
Best-First Search

A*

Combining UCS and BestFS

- Uniform-Cost-Search orders by path cost, or *backward cost* g(n)
- Best-First orders by goal proximity, or *forward cost* h(n)



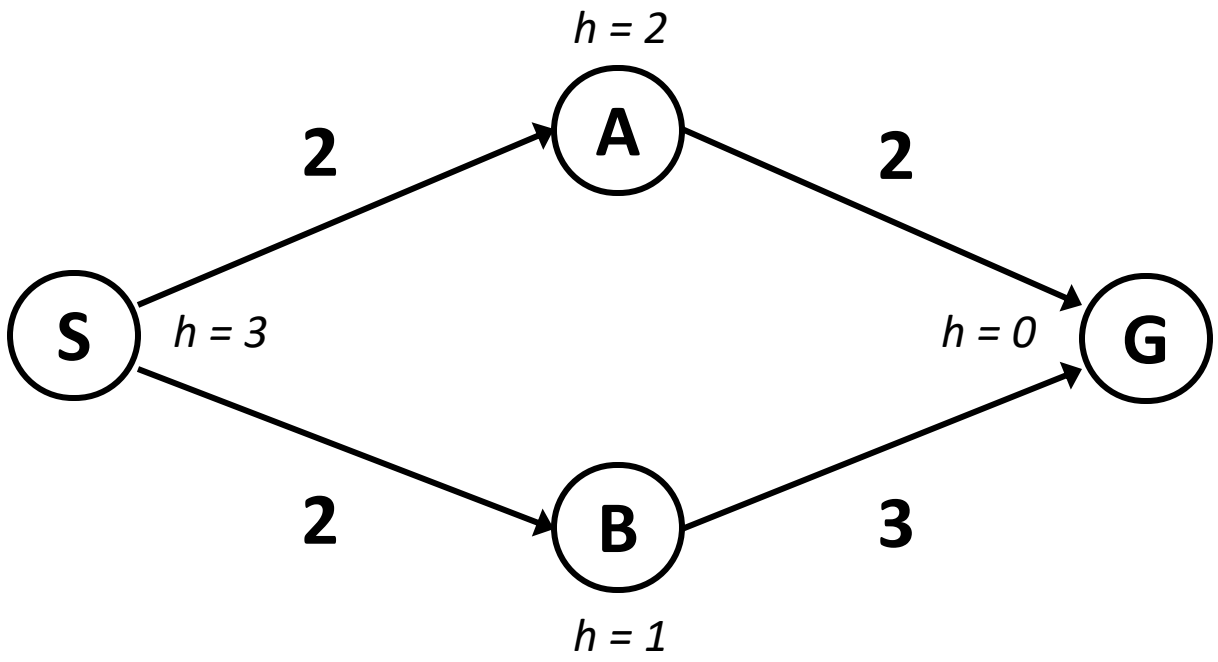


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

When should A* terminate?

- Should we stop when we enqueue (frontier) a goal?



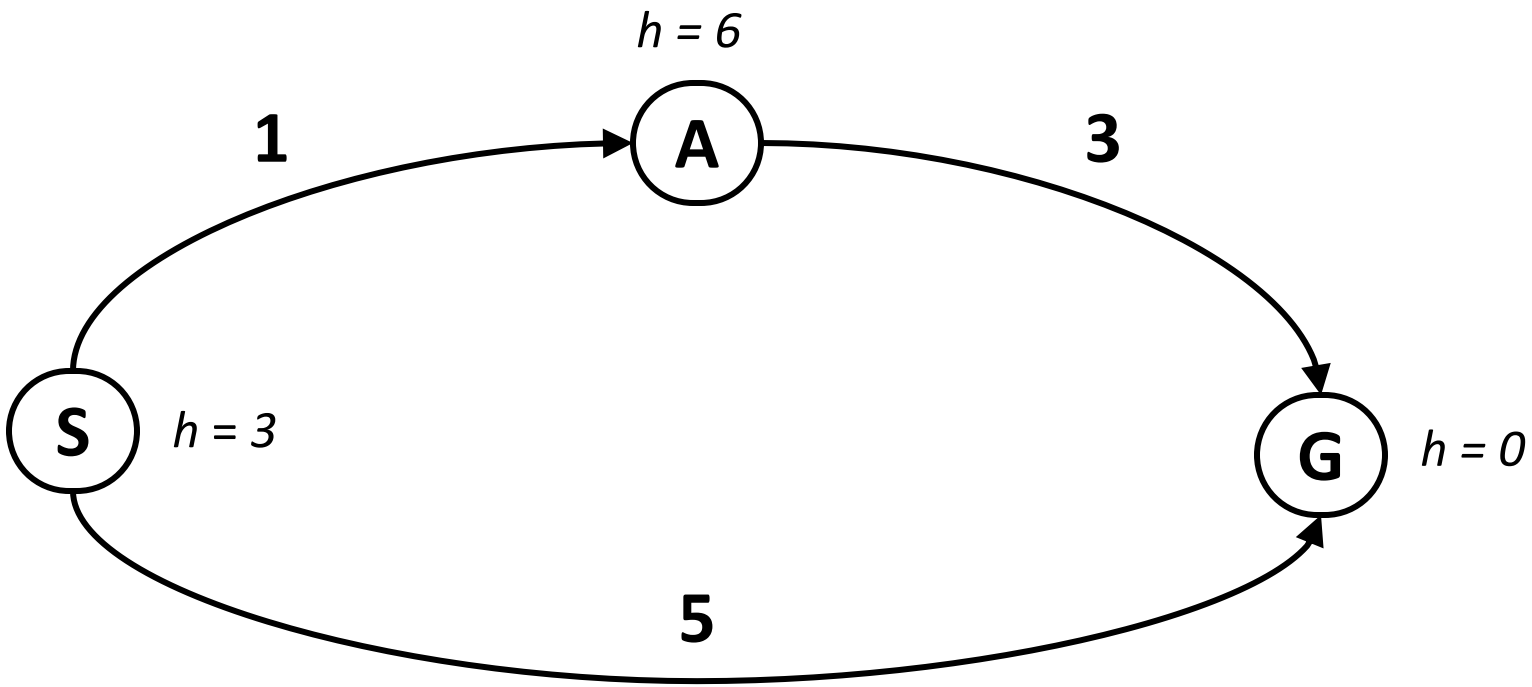
- No: only stop when we dequeue (frontier) a goal

Time / Space Complexity of A^*

- Time complexity is $O(b^m)$ the heuristic could be completely uninformative and the edge costs could all be the same, meaning that A^* does the same thing as BFS.

- Space complexity is $O(b^m)$ like BFS, A^* maintains a frontier which grows with the size of the tree.

Is A* Optimal?



- What went wrong?
  - Actual goal cost < estimated goal cost
- We need estimates to be less than equal to the actual costs!

Admissible Heuristics

Admissible Heuristics

- A heuristic *h* is *admissible* if:

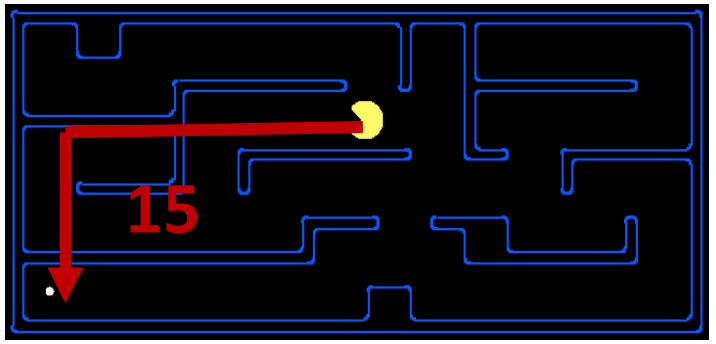
$$0 \leq h(n) \leq h^*(n) \qquad \textit{For all nodes n}$$

, where $h^*(n)$ is the true cost to a nearest goal,

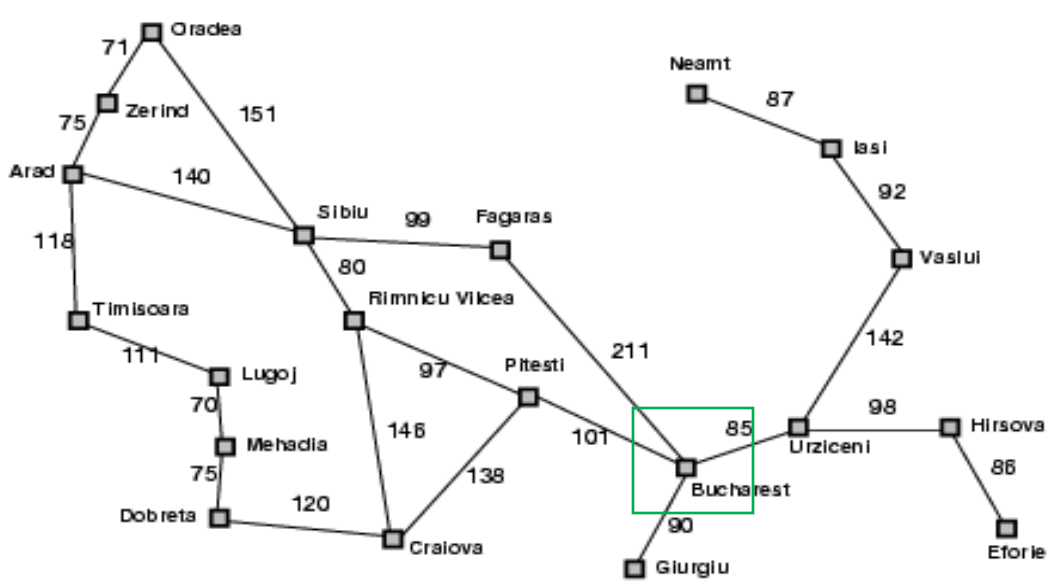
- For all nodes, it is an *underestimate* of the cost to any goal.
- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Admissible Heuristics

- Example 1: is the Manhattan distance admissible?
  - Yes!



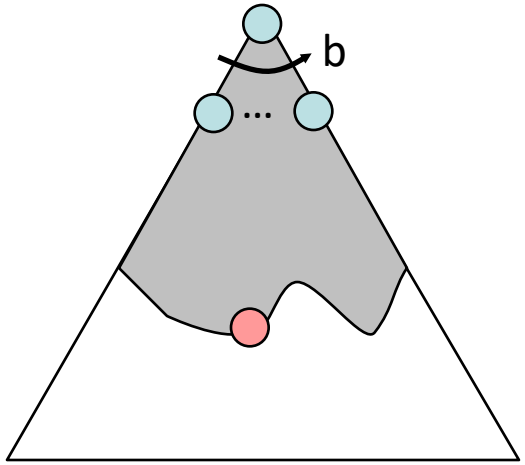
- Example 2: is the straight-line distance admissible?
  - Yes! The shortest distance between two points is a line.



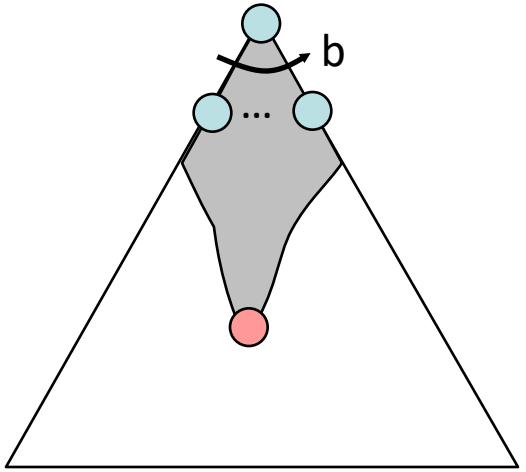
Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

UCS vs A*

Uniform-Cost-Search

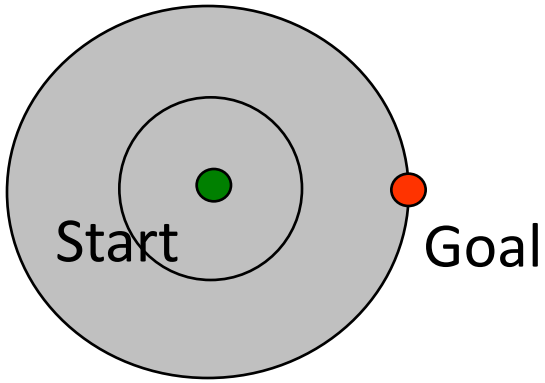


A*

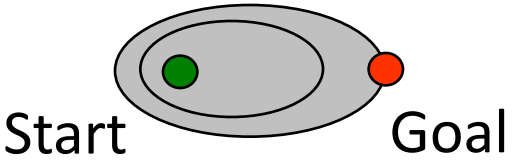


UCS vs A* Contours

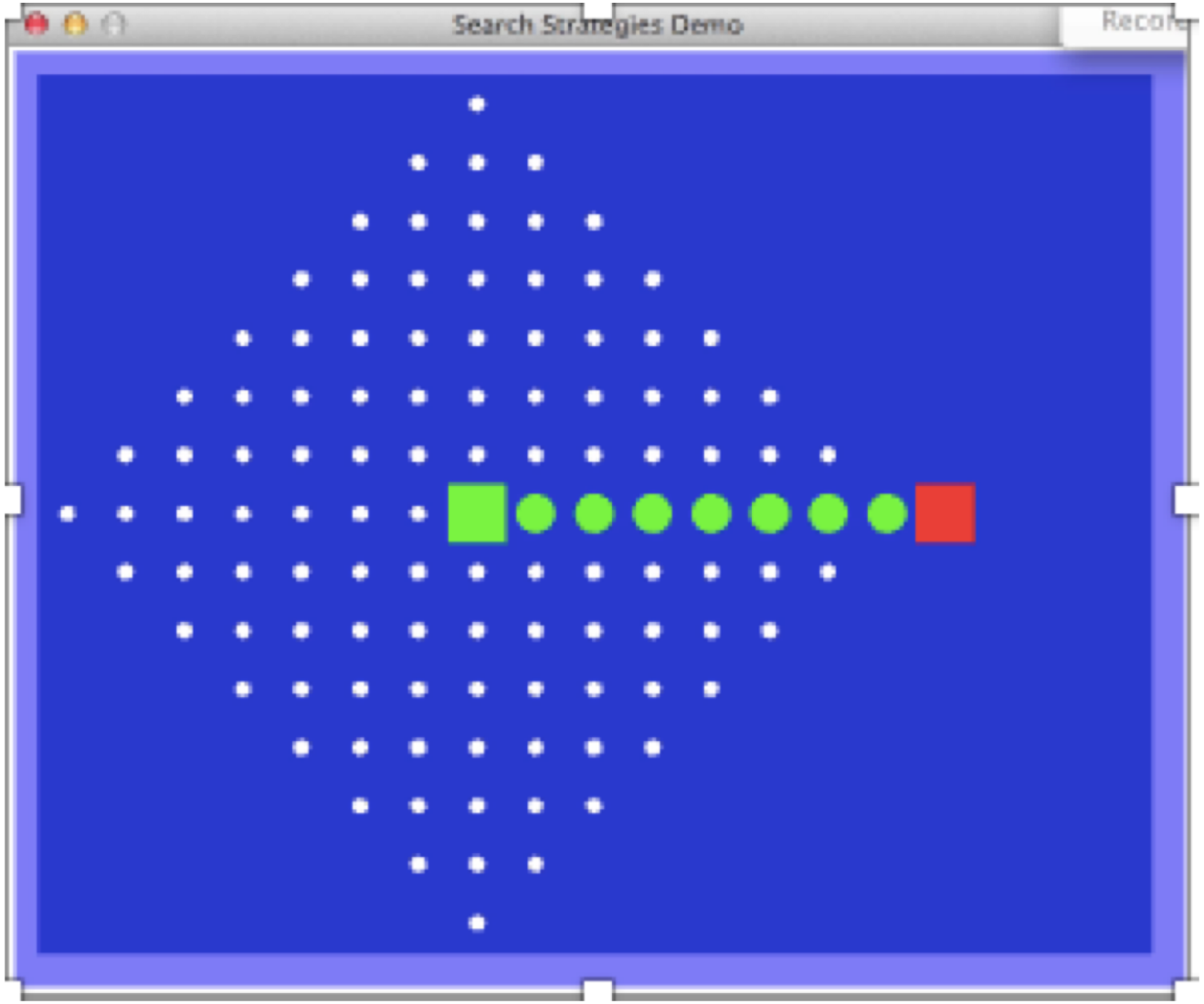
- Uniform-Cost Search expands equally in all “directions”



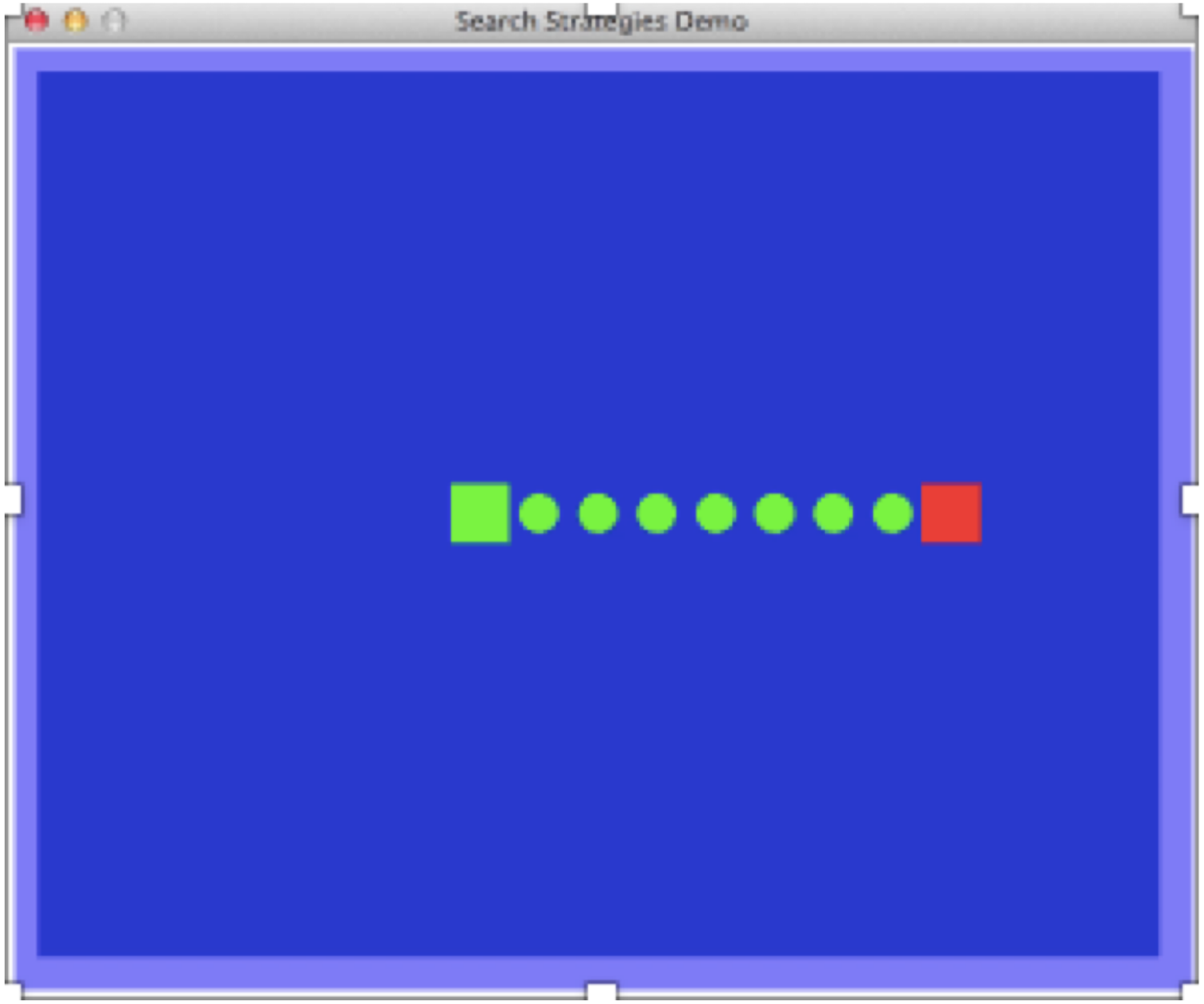
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



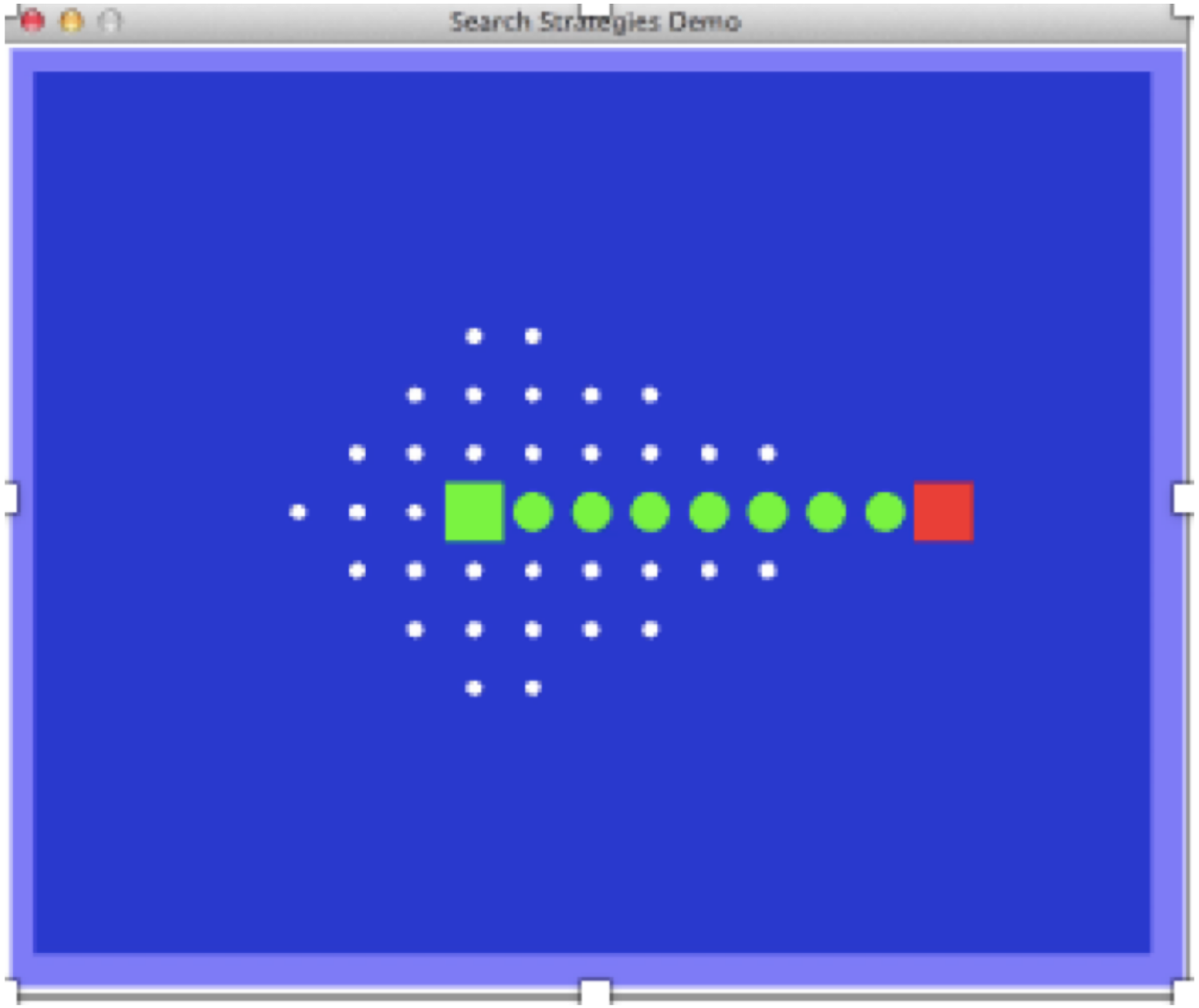
Video of Demo Contours (Empty) -- UCS



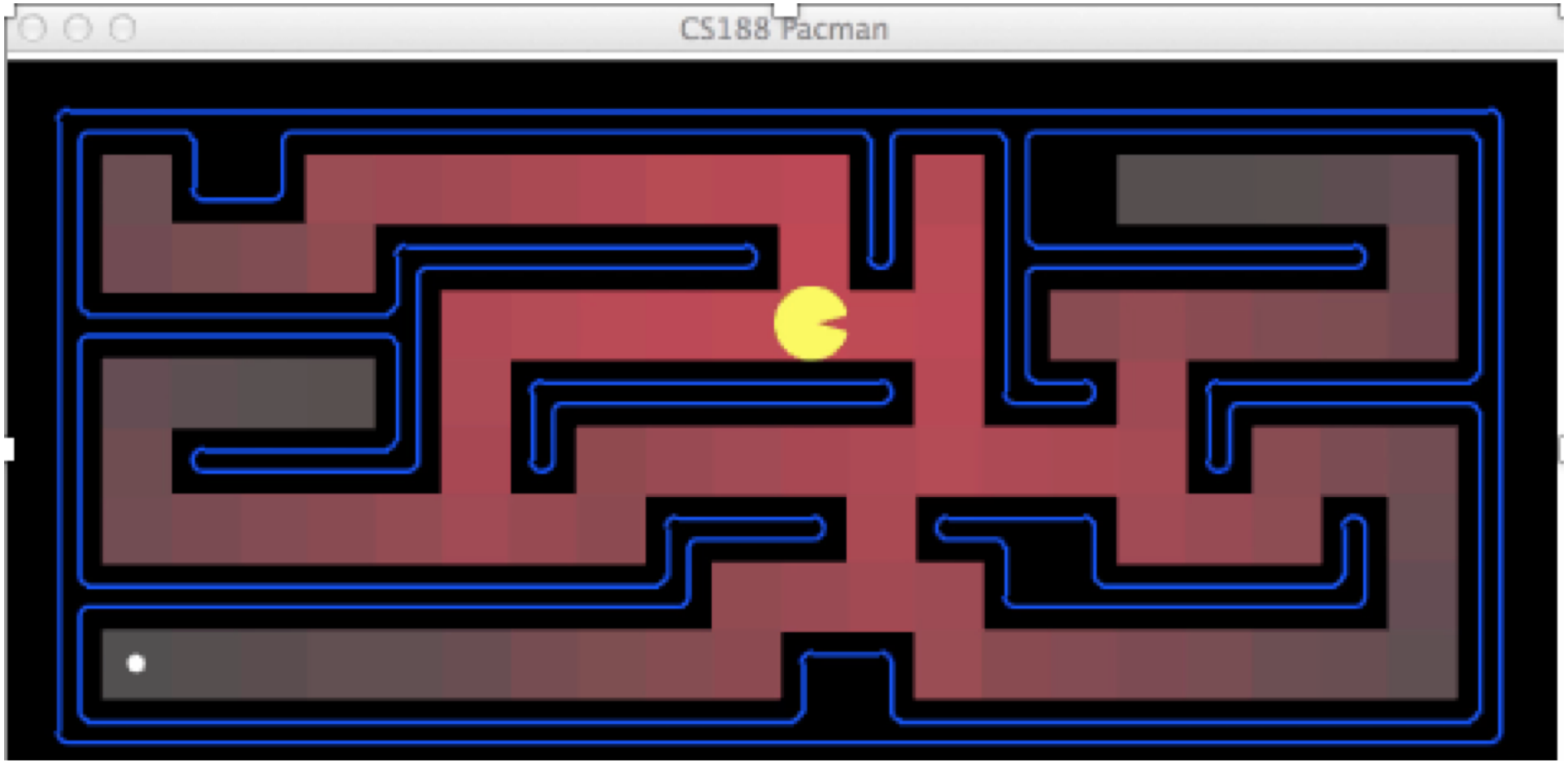
Video of Demo Contours (Empty) -- BestFS



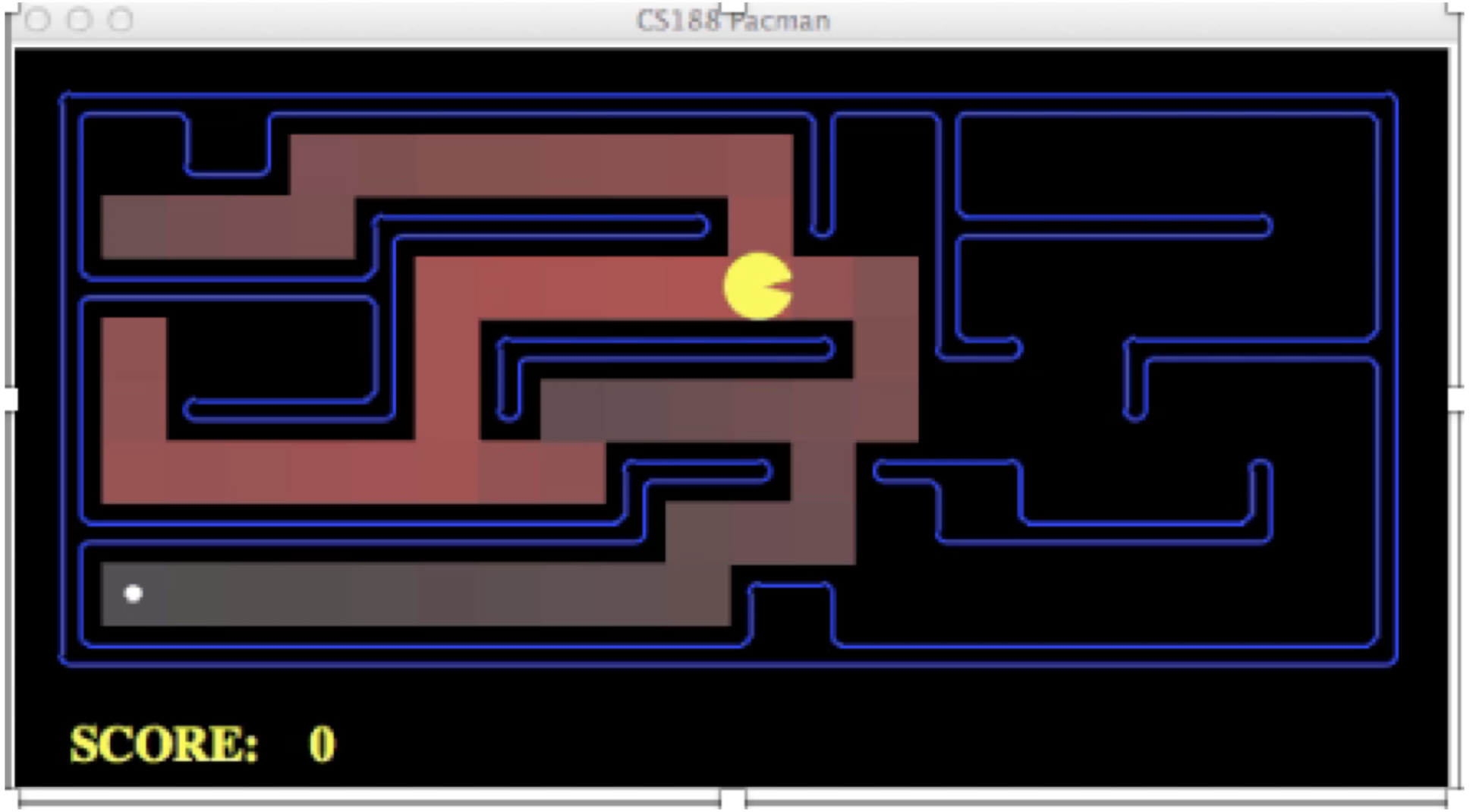
Video of Demo Contours (Empty) – A*



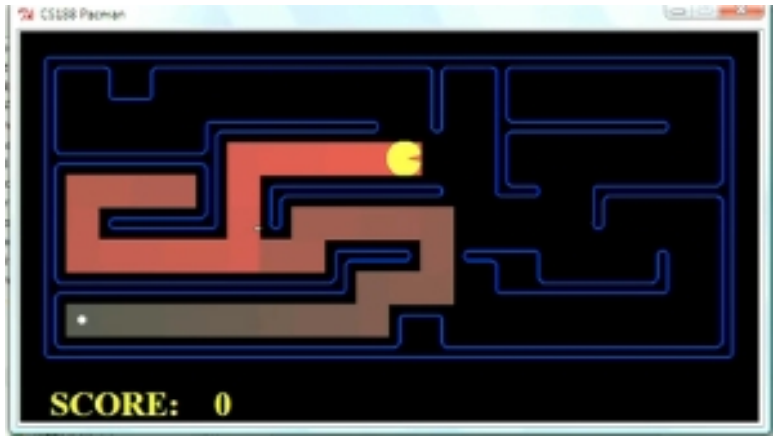
Video of Demo Contours Pacman Small Maze -UCS



Video of Demo Contours Pacman Small Maze – A*

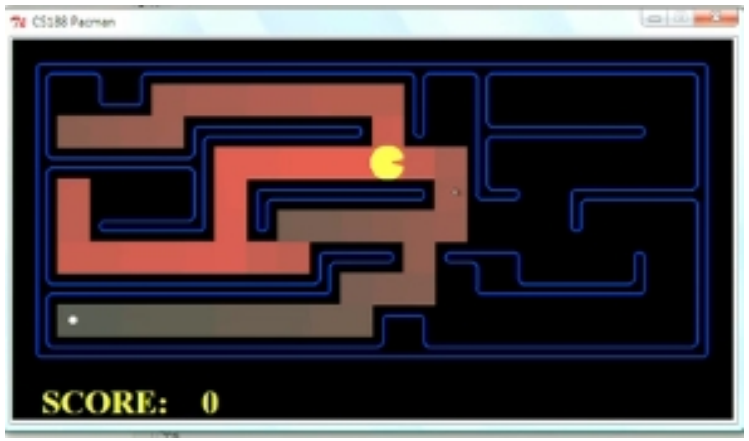


Comparison



Best-First Search

Uniform-Cost Search

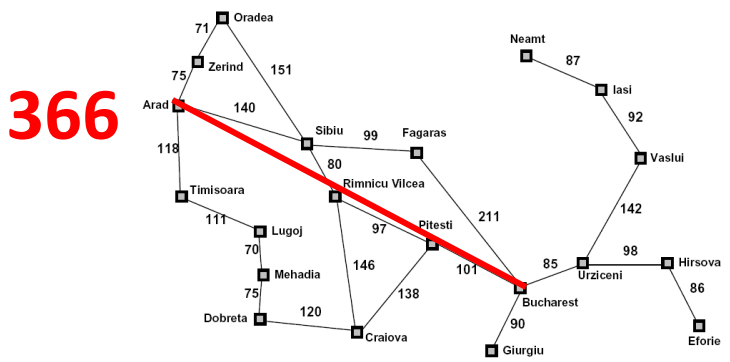


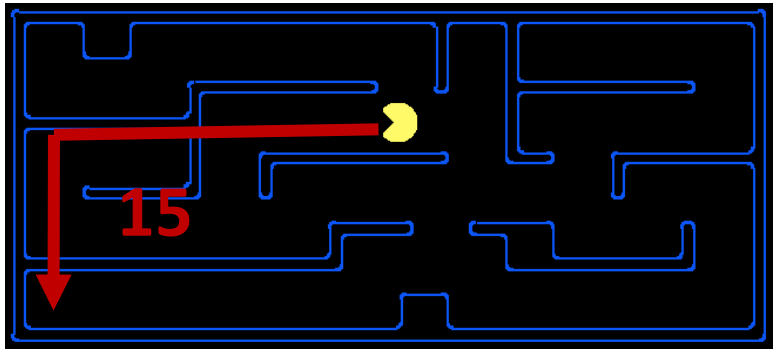
A*

Creating Heuristics

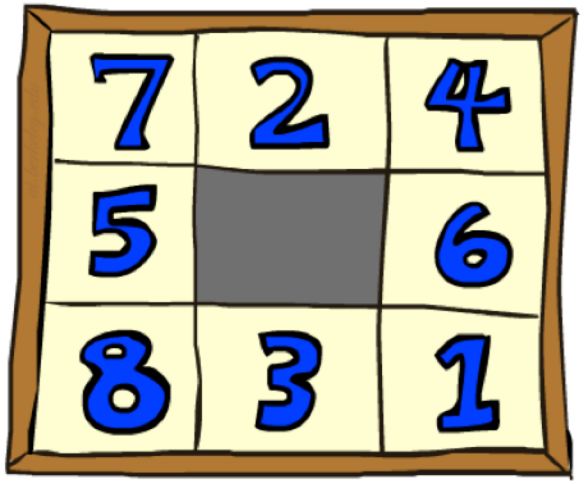
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available

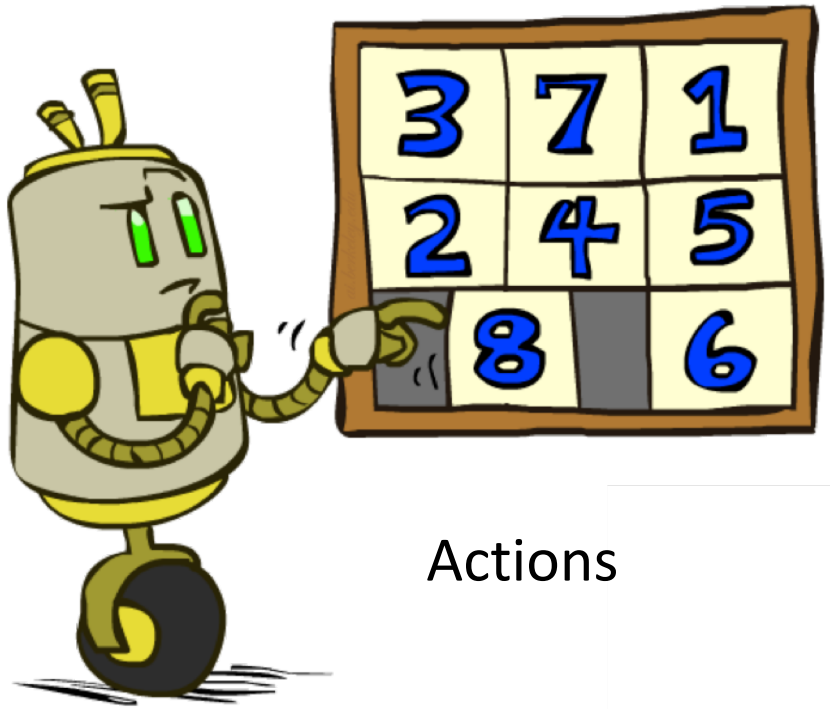




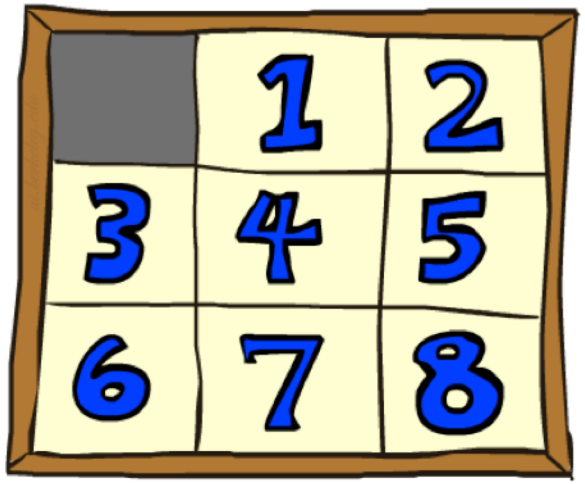
Example: 8 Puzzle



Start State



Actions

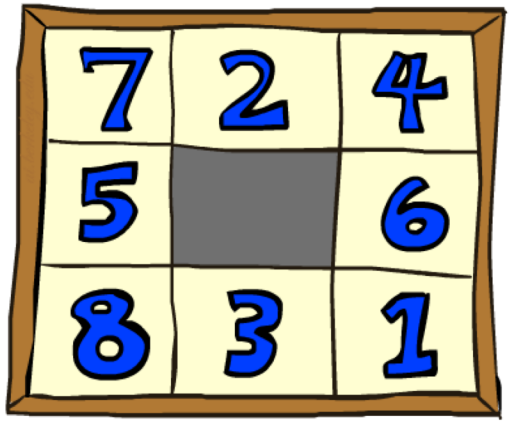


Goal State

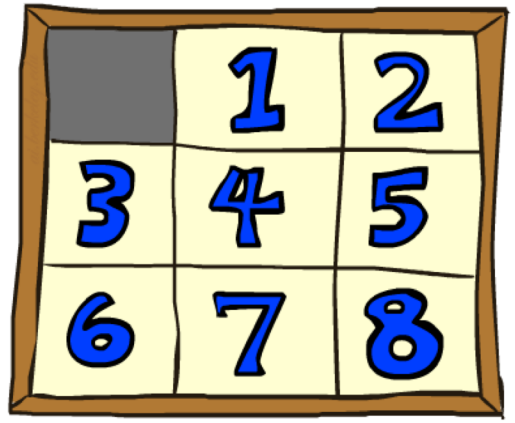
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8
- This is a *relaxed-problem* heuristic



Start State



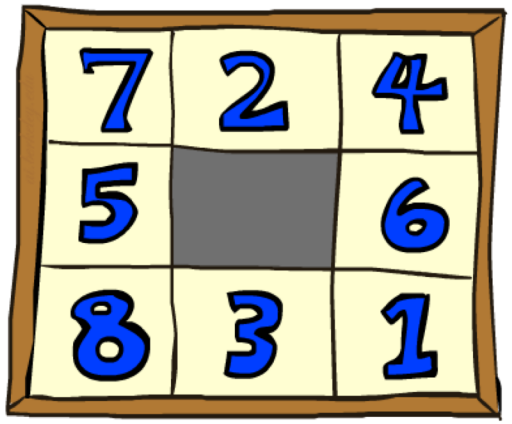
Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
A*, TILES	13	39	227

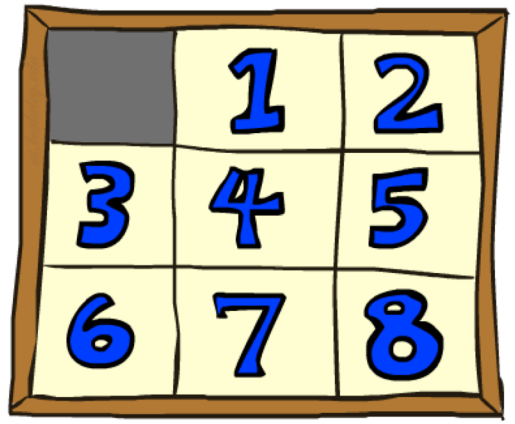
Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State



Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
A*, TILES	13	39	227
A*, MANHATTAN	12	25	73

Cost more computing in Heuristics

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?

- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

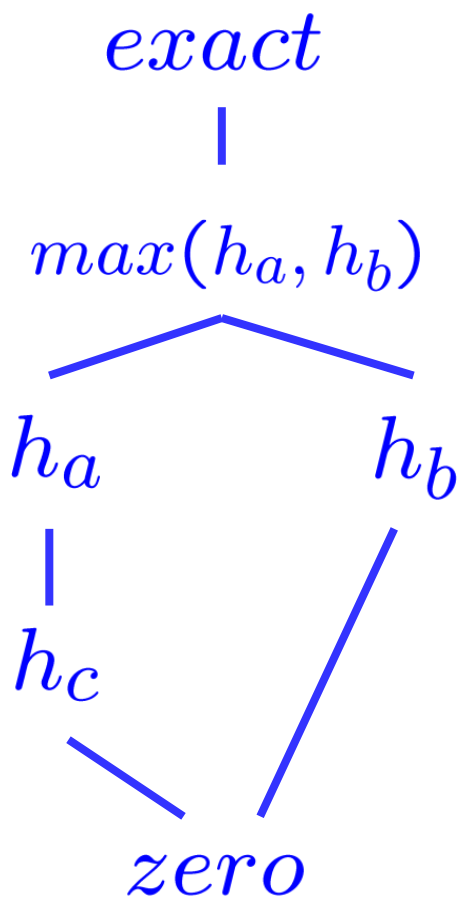
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

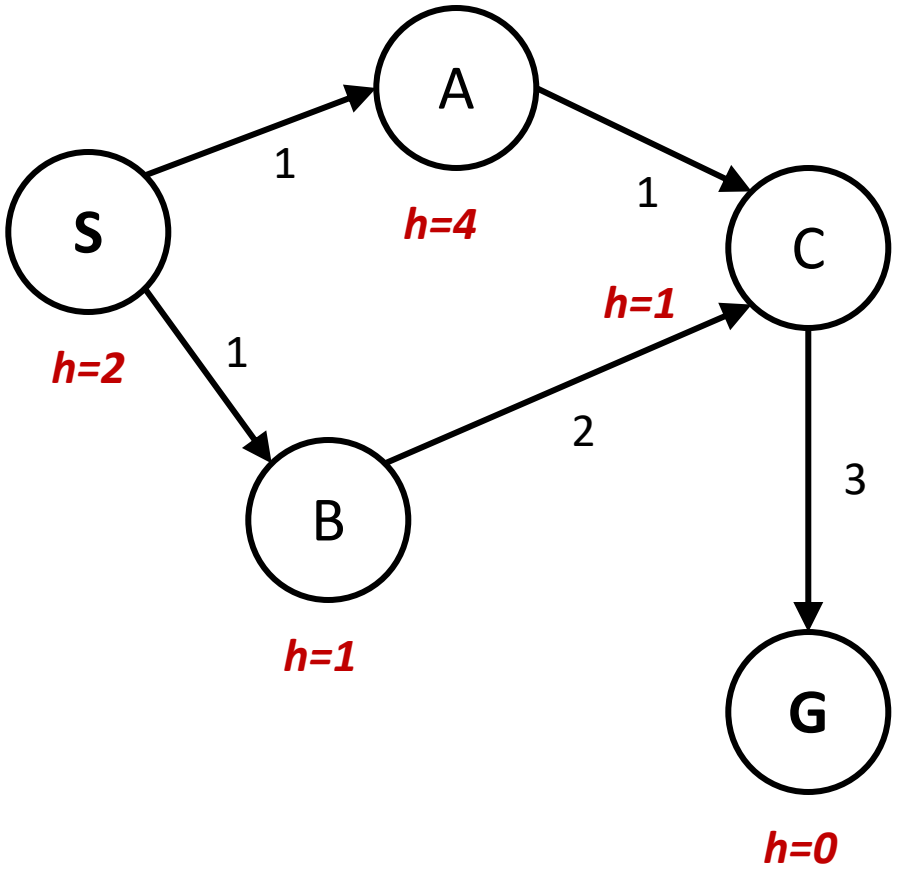
- Trivial heuristics
  - Bottom of lattice is the zero heuristic
  - Top of lattice is the exact heuristic



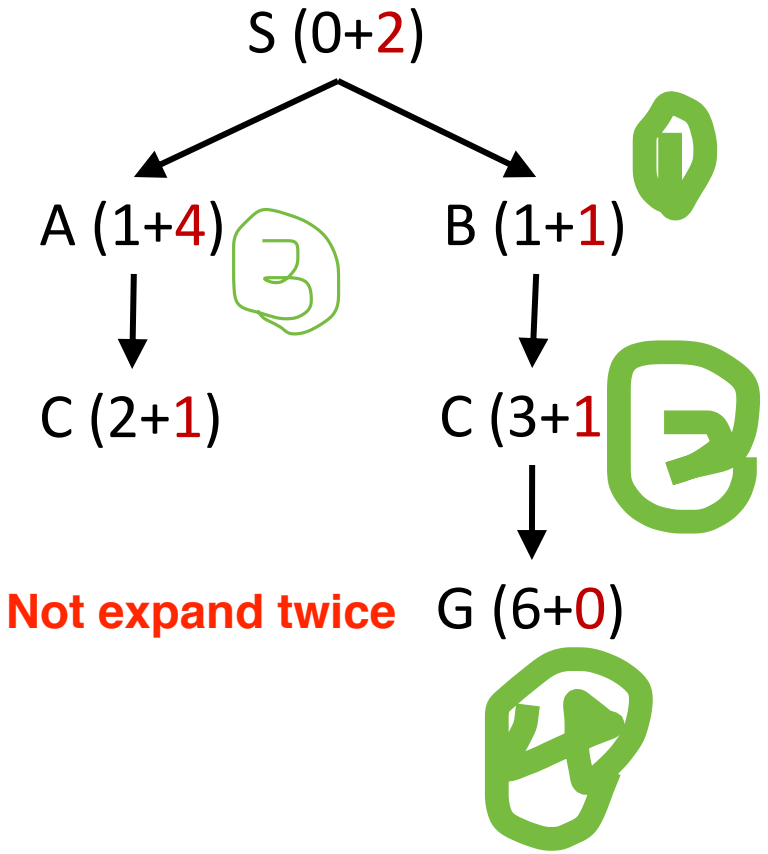
Consistency of Heuristics

A* Graph Search Gone Wrong?

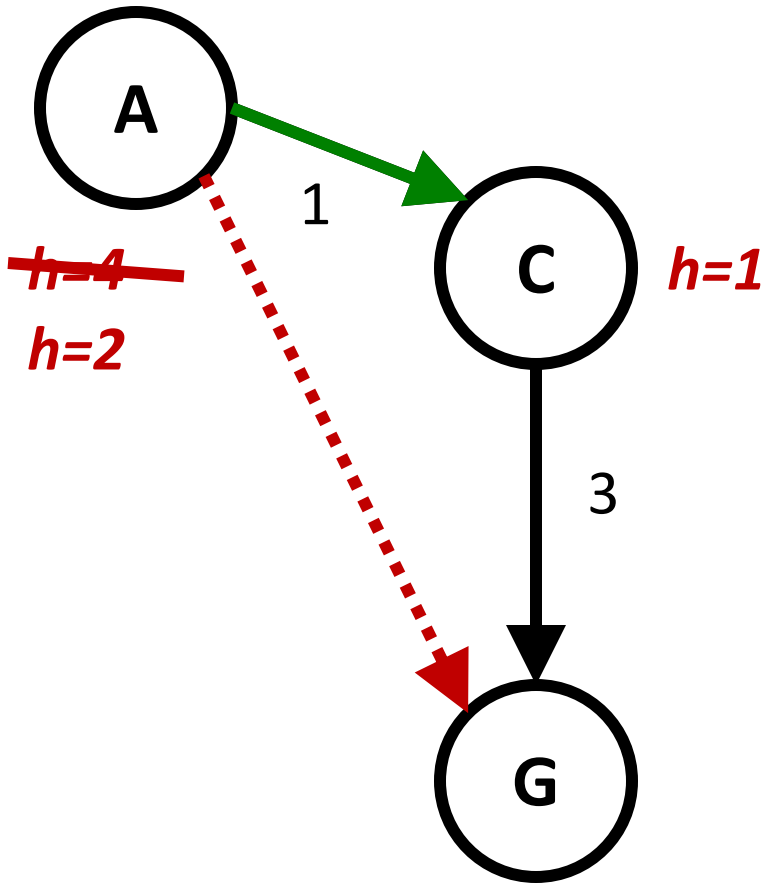
State space graph



Search tree



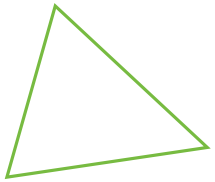
Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)



- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

A* Optimality

- Trees:
  - A* is optimal if heuristic is admissible
- Graphs:
  - A* optimal if heuristic is consistent
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems

Search Methods Summary

	Complete	Optimal	Time	Space
DFS	N (Y if no cycles)	N	$O(b^m)$	$O(mb)$
BFS	Y	Y	$O(b^m)$	$O(b^m)$
IDS	Y	Y	$O(b^m)$	$O(mb)$
UCS	Y Arc Costs > 0	Y Arc Costs > 0	$O(b^m)$	$O(b^m)$
Best First	N	N	$O(b^m)$	$O(b^m)$
A*	Y Arc Costs > 0 *h* admissible	Y Arc Costs > 0 *h* admissible	$O(b^m)$	$O(b^m)$

Reading

- Chapters 3.5-6 in the AIMA textbook