CSC 665: Artificial Intelligence

Bayes Nets: Independence

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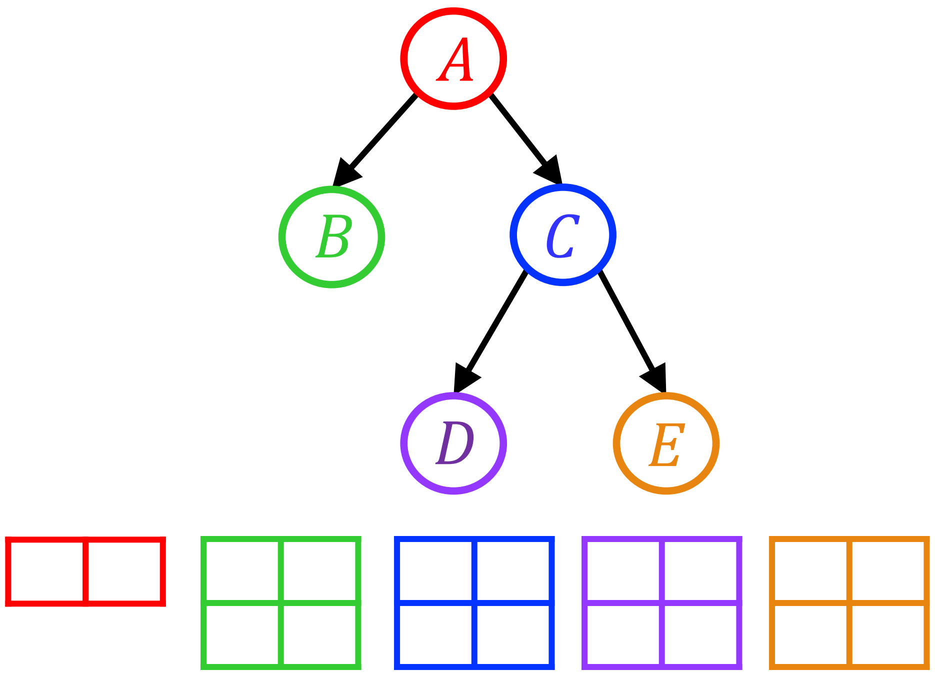
Bayes Nets: Syntax

- One Node per Random Variable
- Directed Acyclic Graph
- One Conditional Probability Table per Node: P(node | *Parents*(node))

Bayes Net

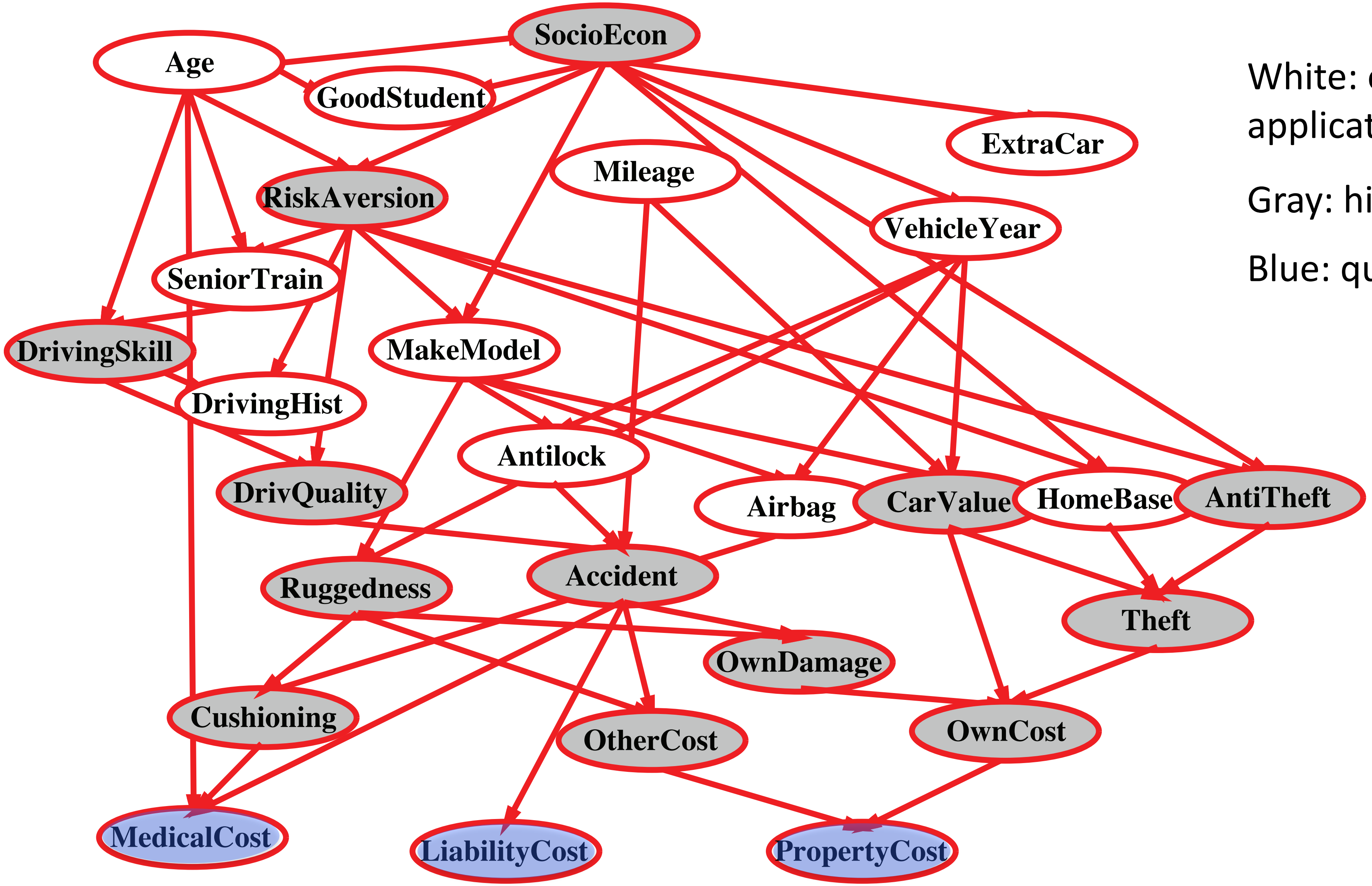


Bayes Net: Semantics

- Encode joint distributions as product of conditional distributions on each variable:

$$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | parents(X_i))$$

Example Bayes Net: Driver Insurance



White: observe on application form

Gray: hidden variables

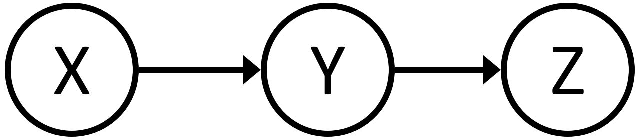
Blue: query variables

Bayes Nets

- ✓ Representation
- Conditional Independences
- Probabilistic Inference
- Learning Bayes' Nets from Data

Independence in a BN

- Important question about a BN:
  - Are two nodes independent given certain evidence?
  - Example:



  - Question: are X and Z necessarily independent?
    - Answer: no. Example: low pressure causes rain, which causes traffic.
    - X can influence Z, Z can influence X (via Y)
    - Addendum: they *could* be independent: how?

Observing Y

D-separation: Outline

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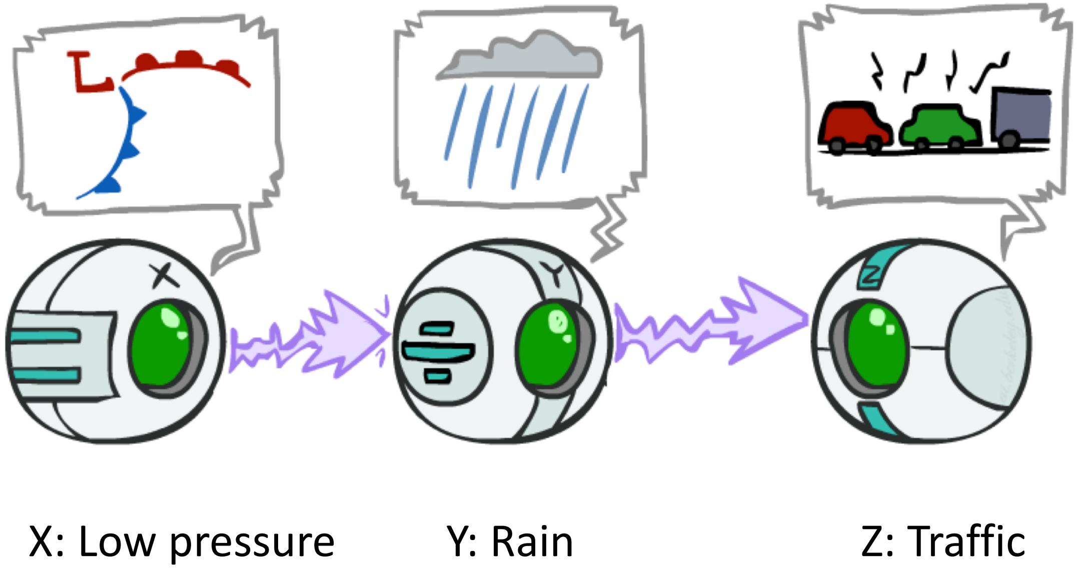
- Study independence properties for triples
- Analyze complex cases in terms of member sub triples

Causal Chains

- This configuration is a “causal chain”



$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

- Guaranteed X independent of Z ? *No!*

Causal Chains

- This configuration is a "causal chain"

X: Low pressure Y: Rain Z: Traffic

$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

- Guaranteed X independent of Z given Y?

$$P(z|x, y) = \frac{P(x, y, z)}{P(x, y)}$$

$$= \frac{P(x)P(y|x)P(z|y)}{P(x)P(y|x)}$$

$$= P(z|y)$$

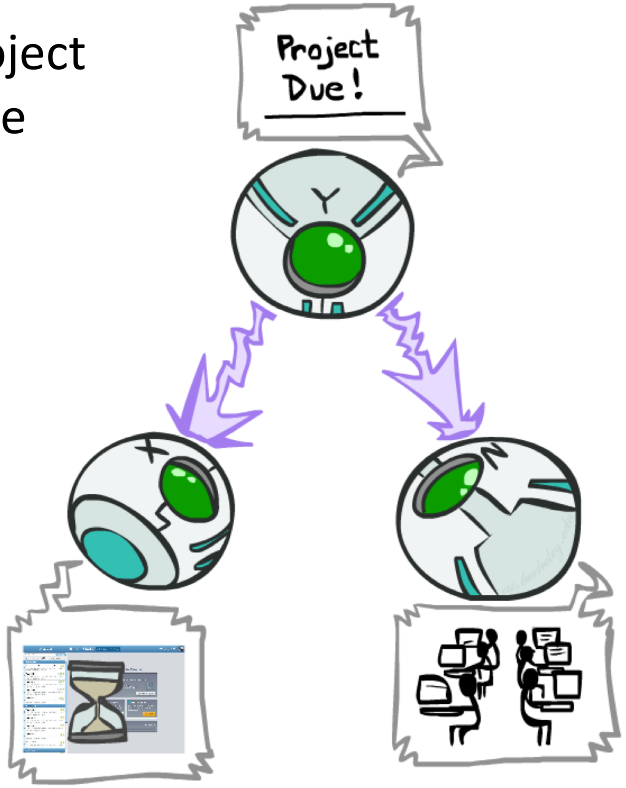
Yes!

- Evidence along the chain "blocks" the influence

Common Cause

- This configuration is a "common cause"

Y: Project due



X: Forums busy

Z: Lab full

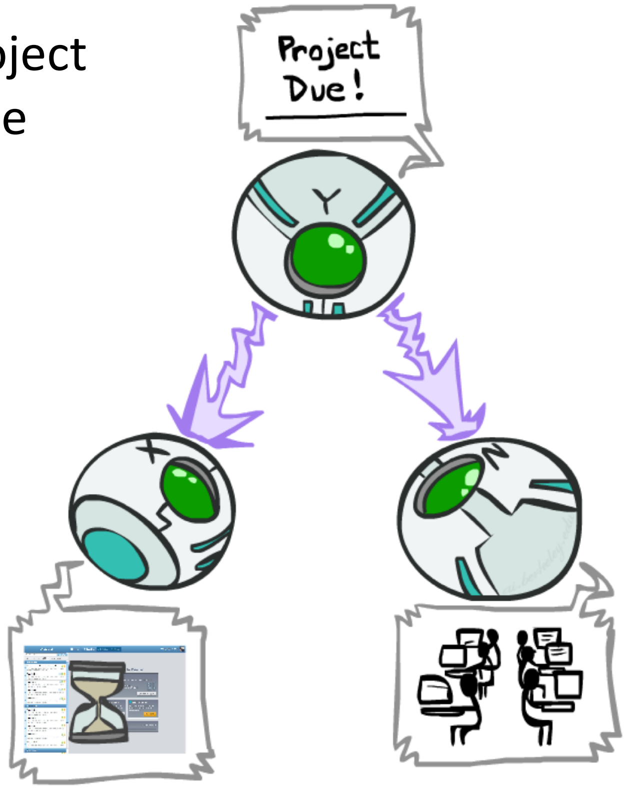
$$P(x, y, z) = P(y)P(x|y)P(z|y)$$

- Guaranteed X independent of Z ? *No!*

Common Cause

- This configuration is a "common cause"

Y: Project due



X: Forums busy

Z: Lab full

$$P(x, y, z) = P(y)P(x|y)P(z|y)$$

- Guaranteed X and Z independent given Y?

$$P(z|x, y) = \frac{P(x, y, z)}{P(x, y)}$$

$$= \frac{P(y)P(x|y)P(z|y)}{P(y)P(x|y)}$$

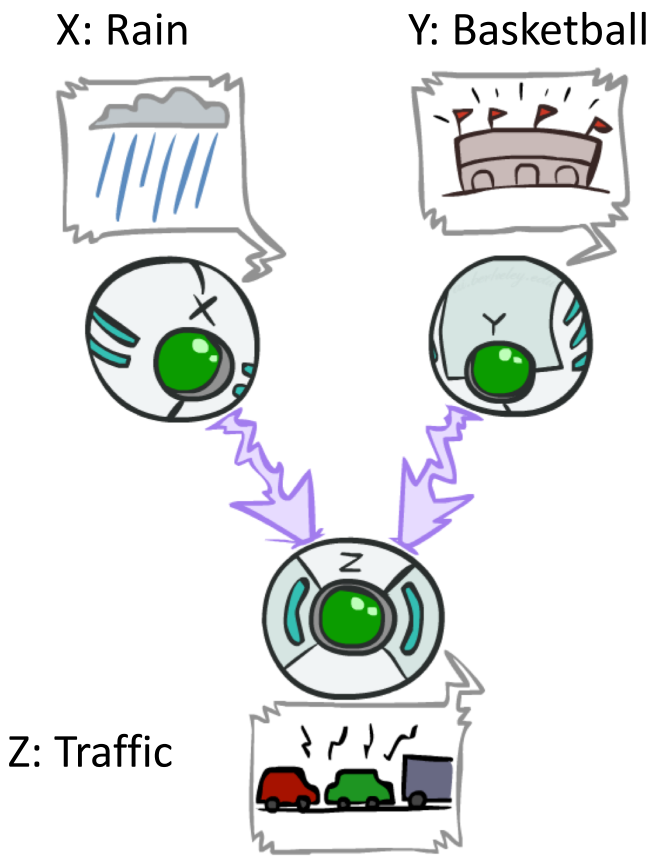
$$= P(z|y)$$

Yes!

- Observing the cause blocks influence between effects.

Common Effect

- Last configuration: two causes of one effect (v-structures)



- Are X and Y independent?
  - *Yes*: the basketball game and the rain cause traffic, but they are not correlated
- Are X and Y independent given Z?

  There is a basketball game, and traffic outside, does it change the belief of rain?
  - *No*: seeing traffic puts the rain and the basketball game in competition as explanation.
- This is backwards from the other cases
- Observing an effect activates influence between possible causes.

The General Case

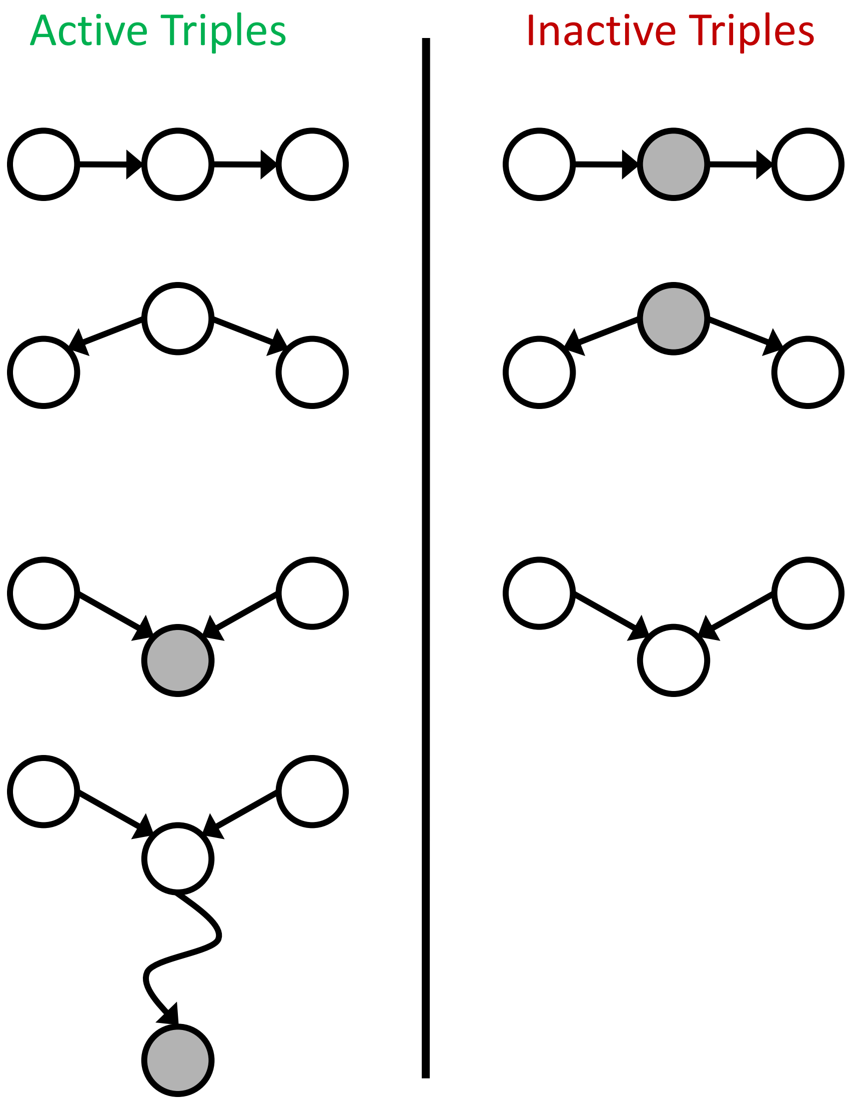
- General question: in a given BN, are two variables independent (given evidence)?

- Solution: Any complex example can be broken into repetitions of the three canonical cases

Active / Inactive Triples

- A triple is active if:
  - Causal chain A → B → C where B is unobserved (either direction)
  - Common cause A ← B → C where B is unobserved
  - Common effect (aka v-structure)
    A → B ← C where B *or one of its descendants* is observed

One of paths is inactive, it is guaranteed to be true independence

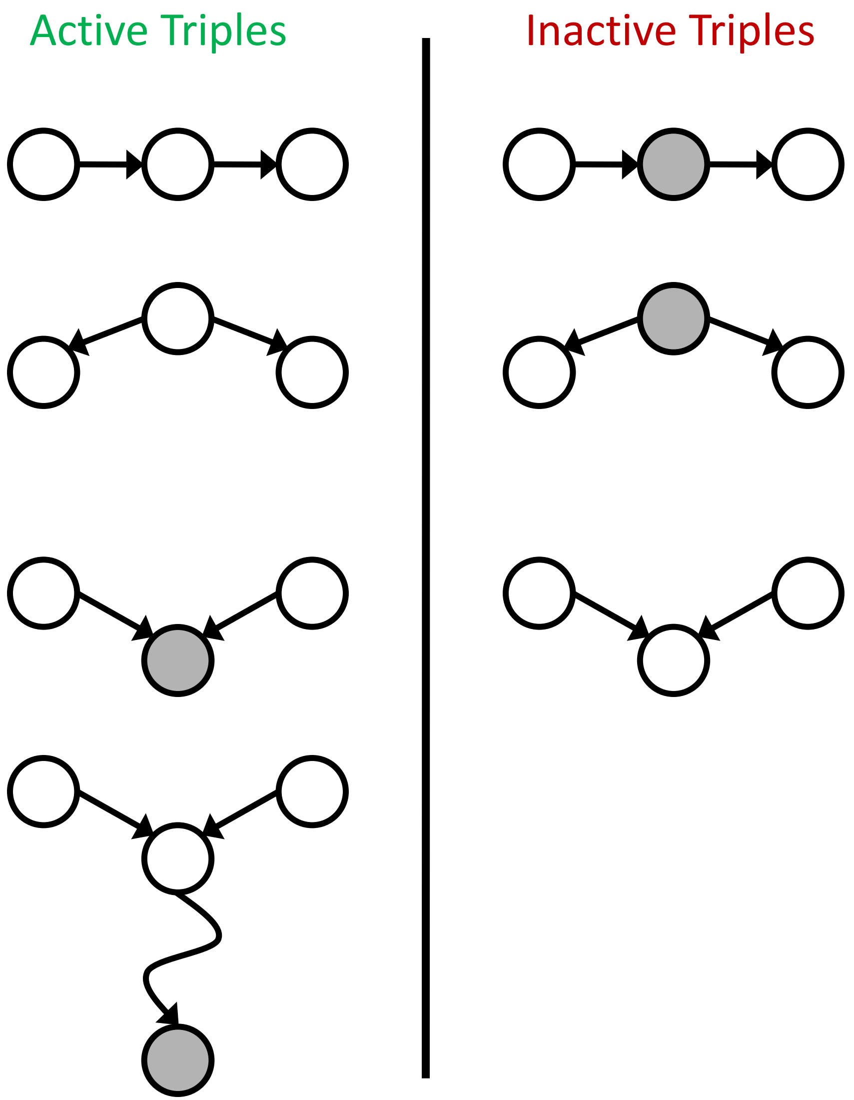


Shades Nodes:
Observed

Active / Inactive Paths

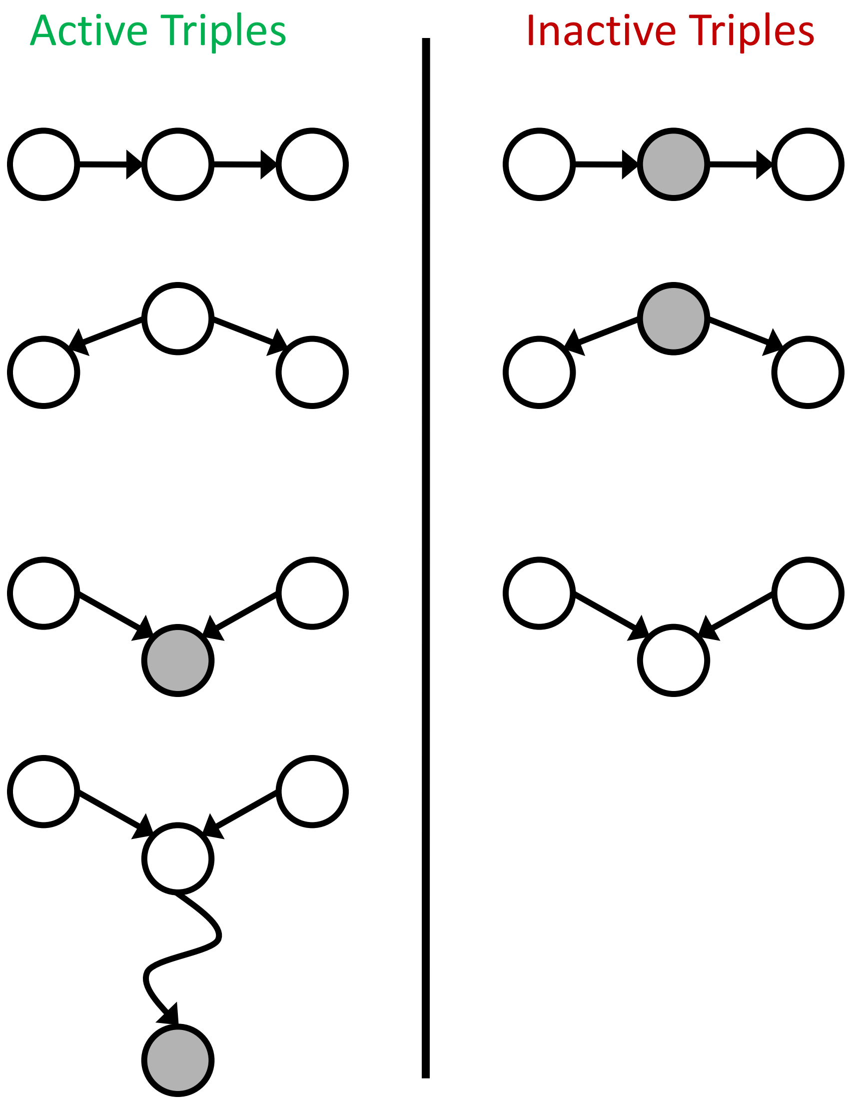
- A path is active if each triple along the path is active

Shades Nodes: Observed



D-Separation

- Question: Are X and Y conditionally independent given evidence variables {Z}?
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!



Shades Nodes: Observed

D-Separation

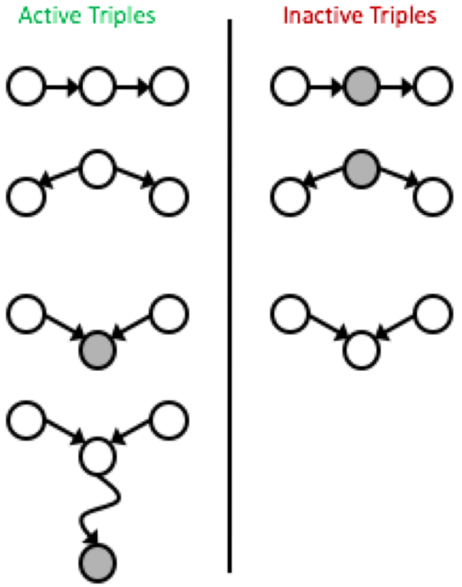
- Query: $X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$?
- Check all (undirected!) paths between X_i and X_j
  - If one or more active, then independence not guaranteed

    $$X_i \not\perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

  - Otherwise (i.e. if all paths are inactive),
    then independence is guaranteed

    $$X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

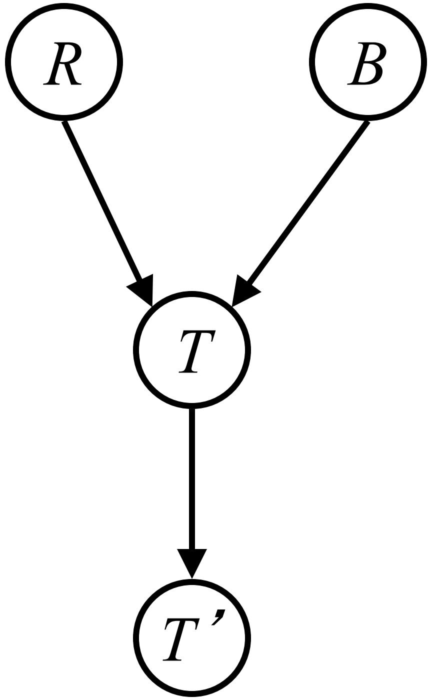
Example



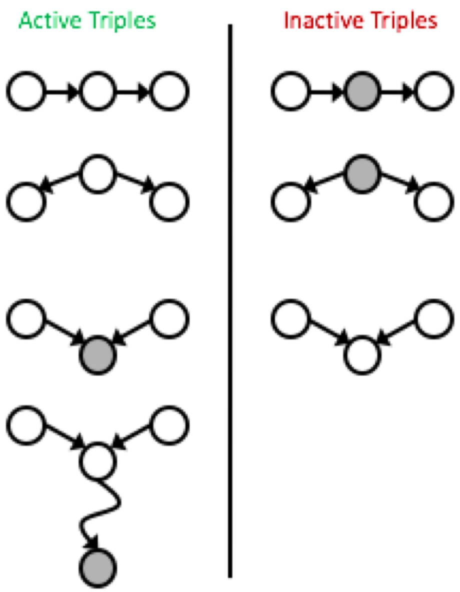
$R \perp\!\!\!\perp B$ *Yes*

$R \perp\!\!\!\perp B | T$

$R \perp\!\!\!\perp B | T'$



Example



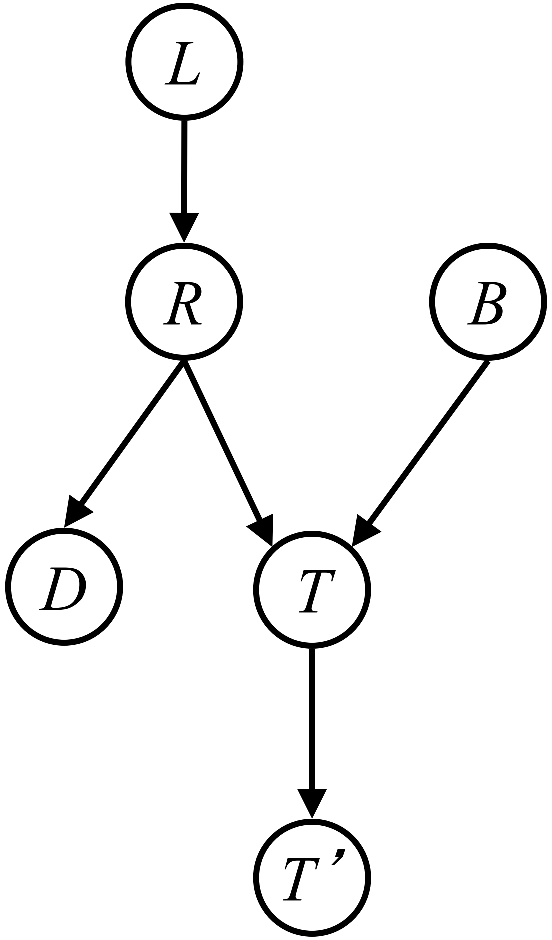
$L \perp\!\!\!\perp T' | T$ Yes

$L \perp\!\!\!\perp B$ Yes

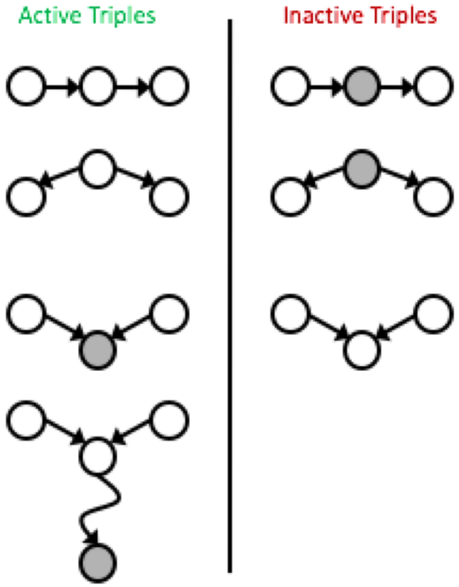
$L \perp\!\!\!\perp B | T$

$L \perp\!\!\!\perp B | T'$

$L \perp\!\!\!\perp B | T, R$ Yes



Example

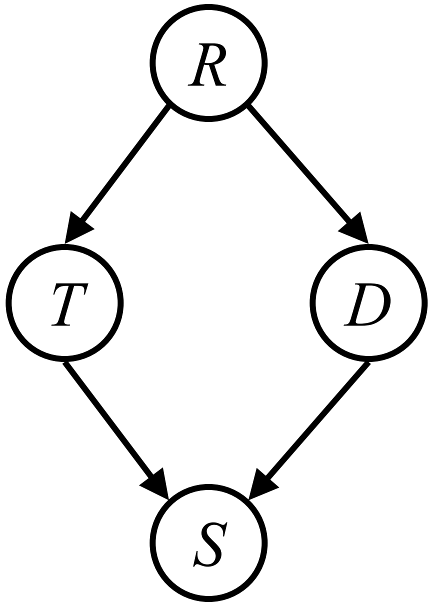


- Variables:
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I'm sad
- Questions:

$T \perp\!\!\!\perp D$

$T \perp\!\!\!\perp D | R$ *Yes*

$T \perp\!\!\!\perp D | R, S$

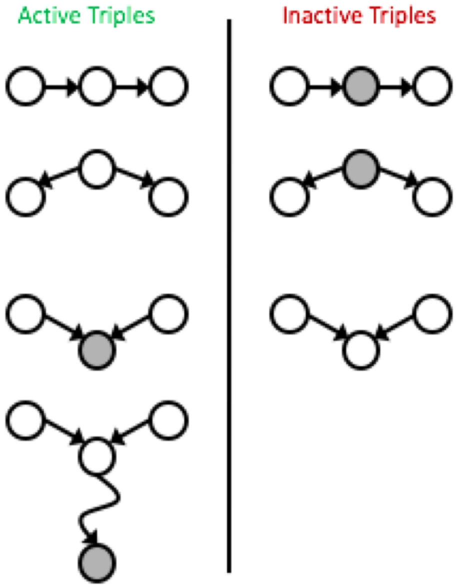


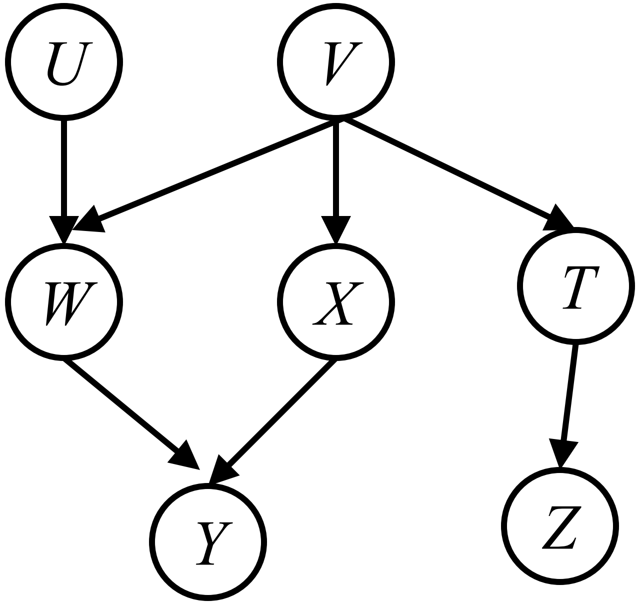
Question

- V _||_ Z

Guaranteed to be true

Not Guaranteed to be true ✓



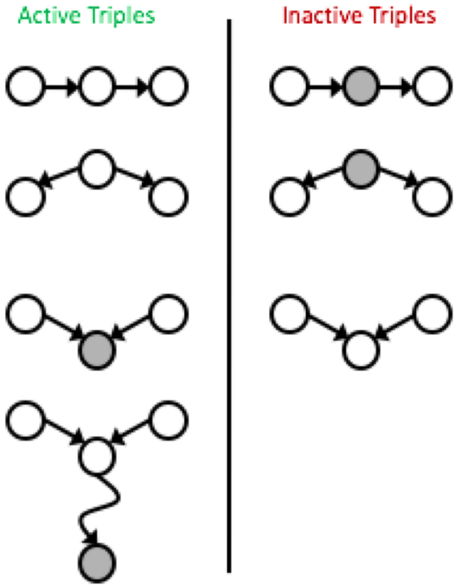


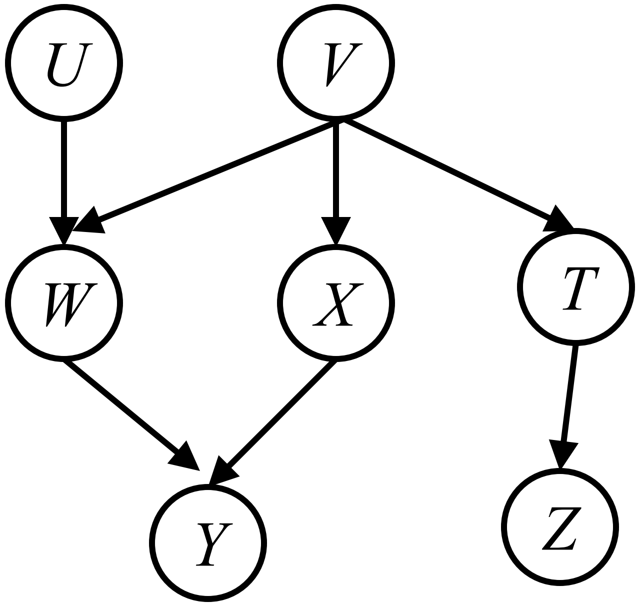
Question

- V _||_ Z | T

Guaranteed to be true

Not Guaranteed to be true



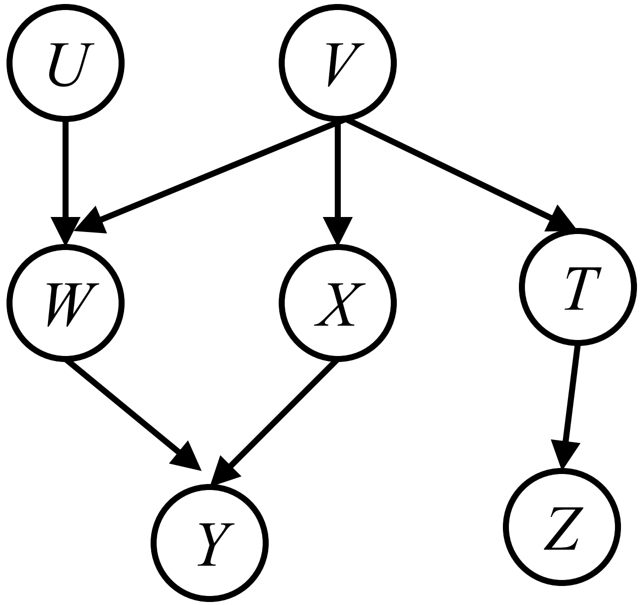


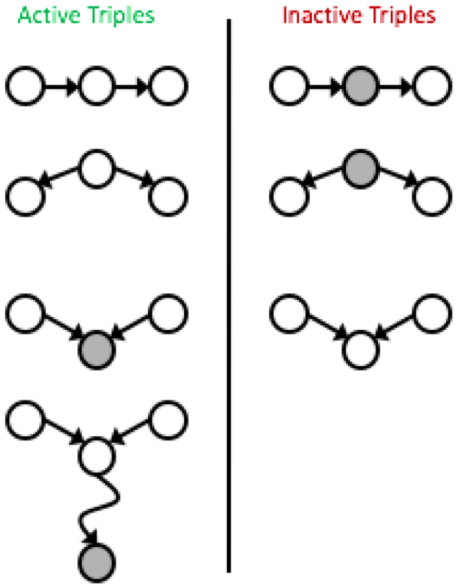
Question

- U _||_ V

Guaranteed to be true

Not Guaranteed to be true



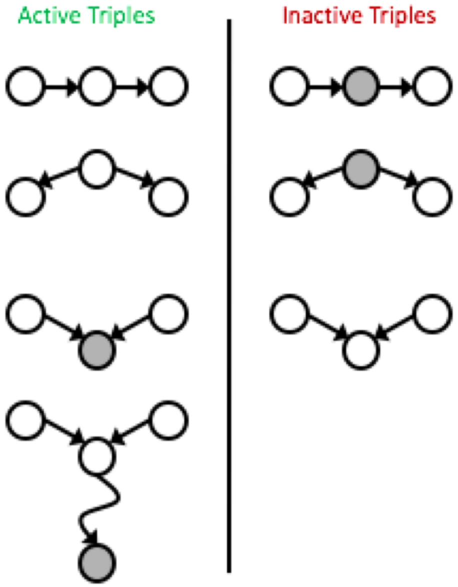


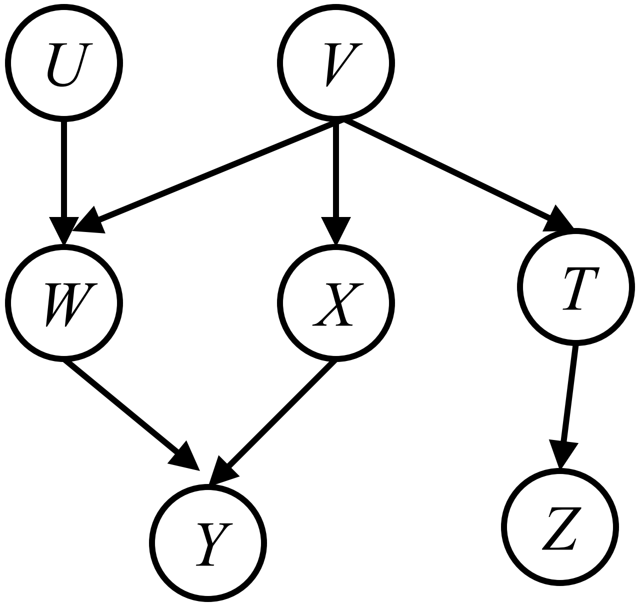
Question

- U _||_ V | W

Guaranteed to be true

Not Guaranteed to be true ✓



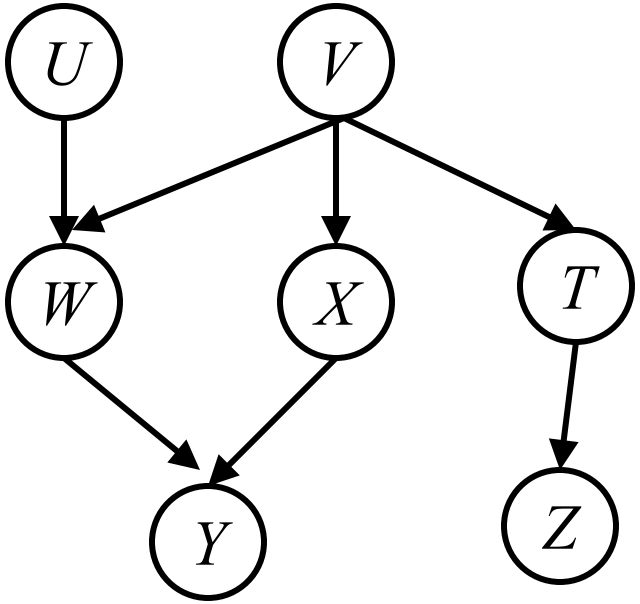


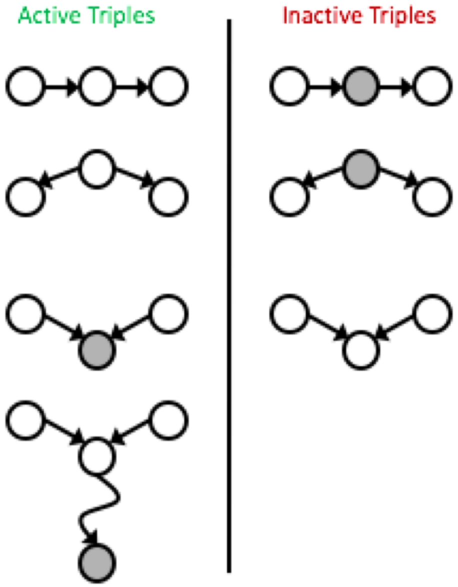
Question

- U _||_ V | X

Guaranteed to be true

Not Guaranteed to be true





Question

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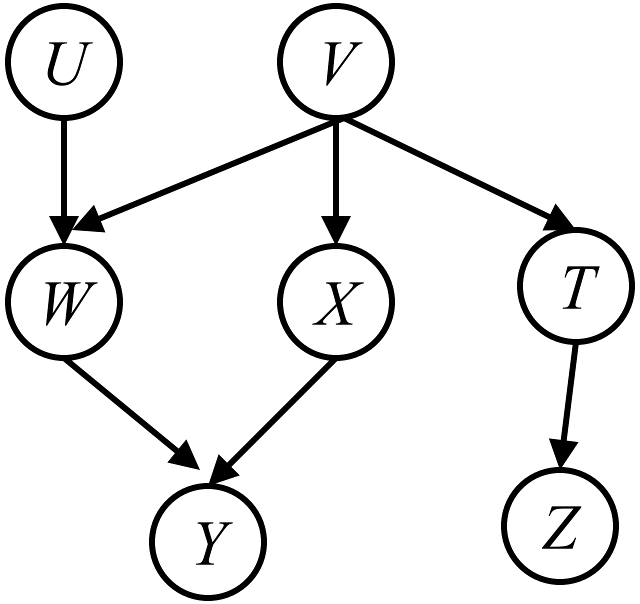

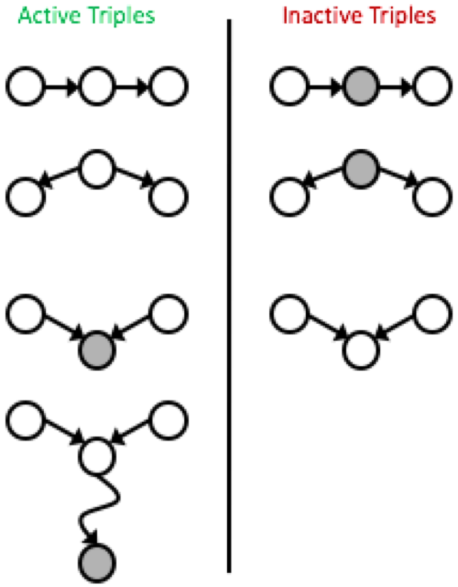

Question

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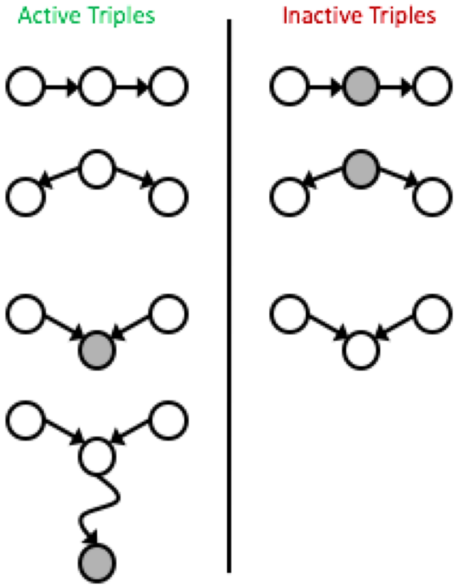

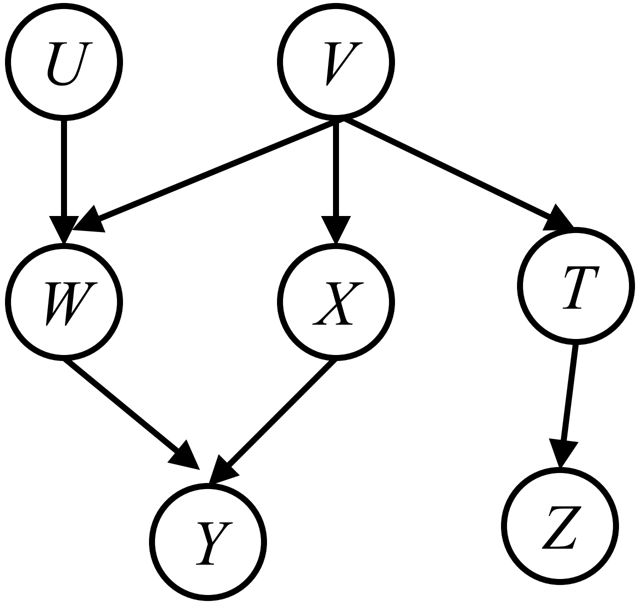

Question

- X _||_ W

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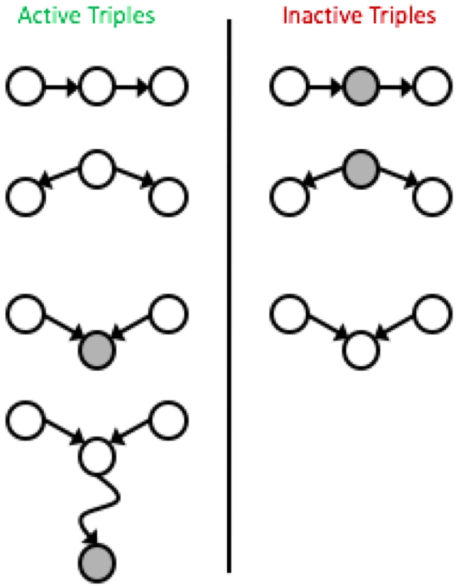
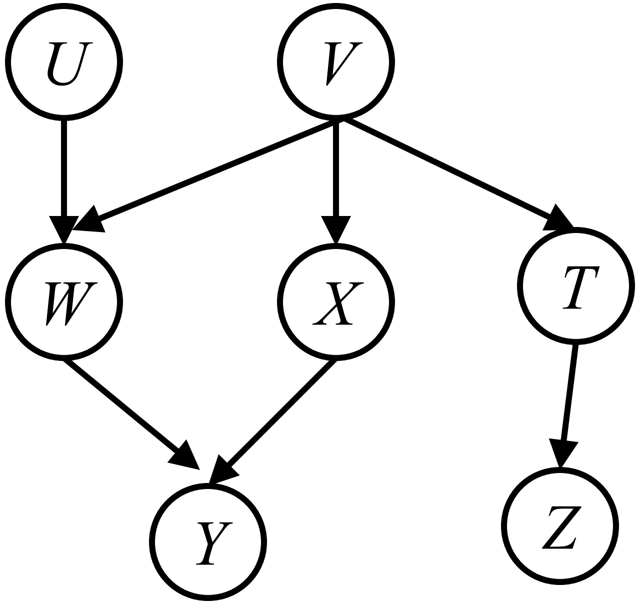


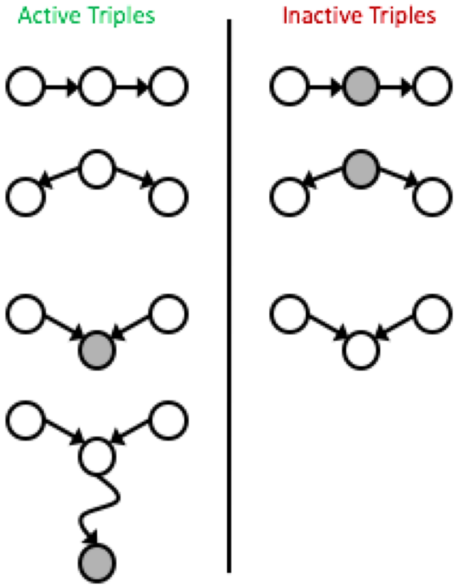


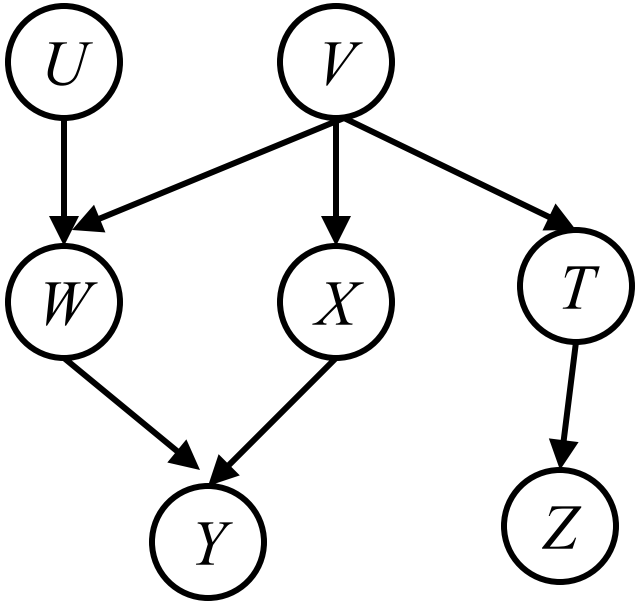
Question

- X _||_ W | U

Guaranteed to be true

Not Guaranteed to be true



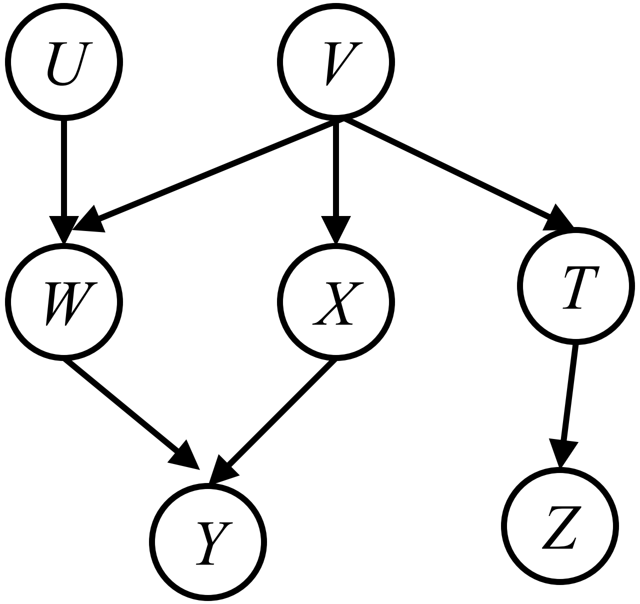


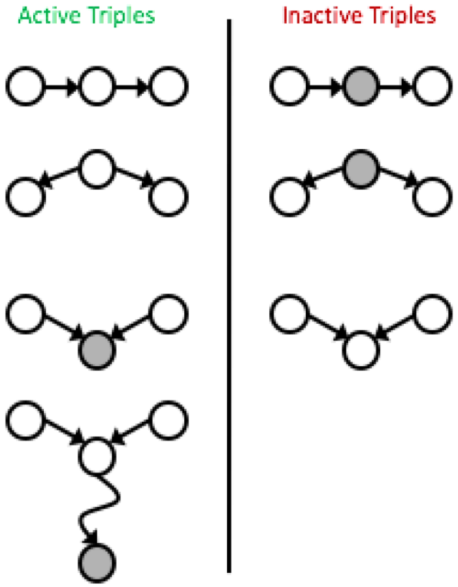
Question

- X _||_ T | V

Guaranteed to be true

Not Guaranteed to be true





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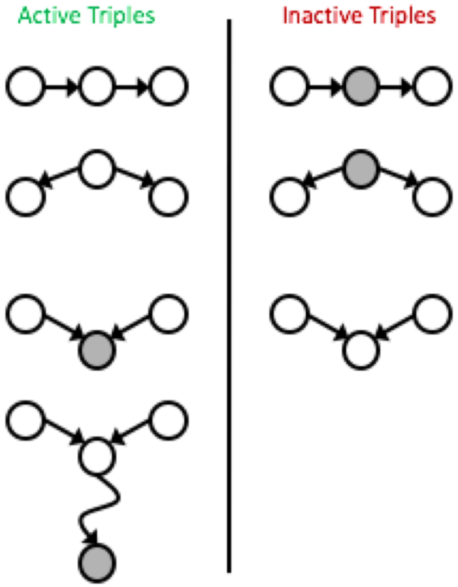

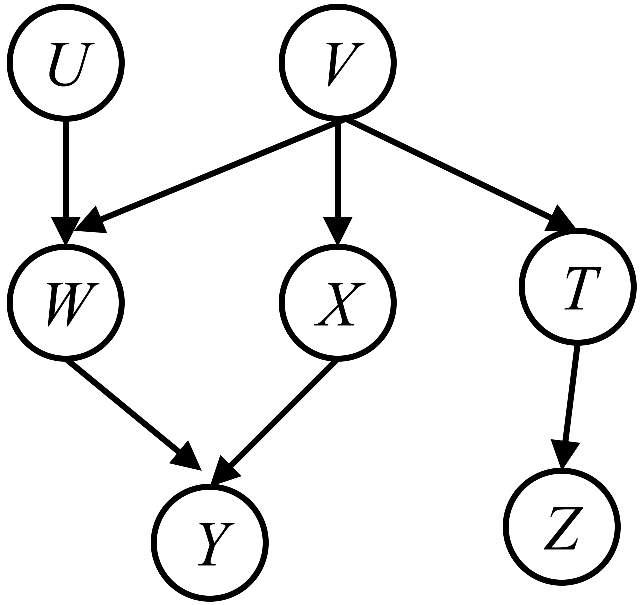

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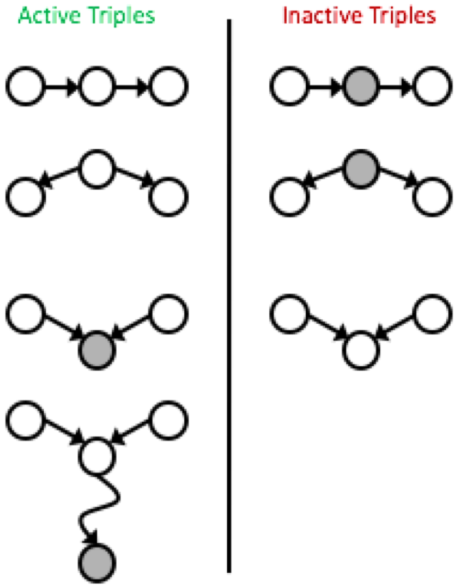
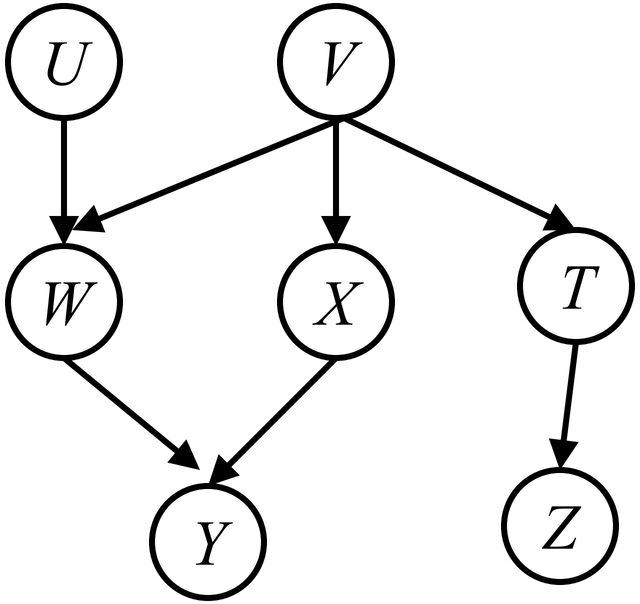
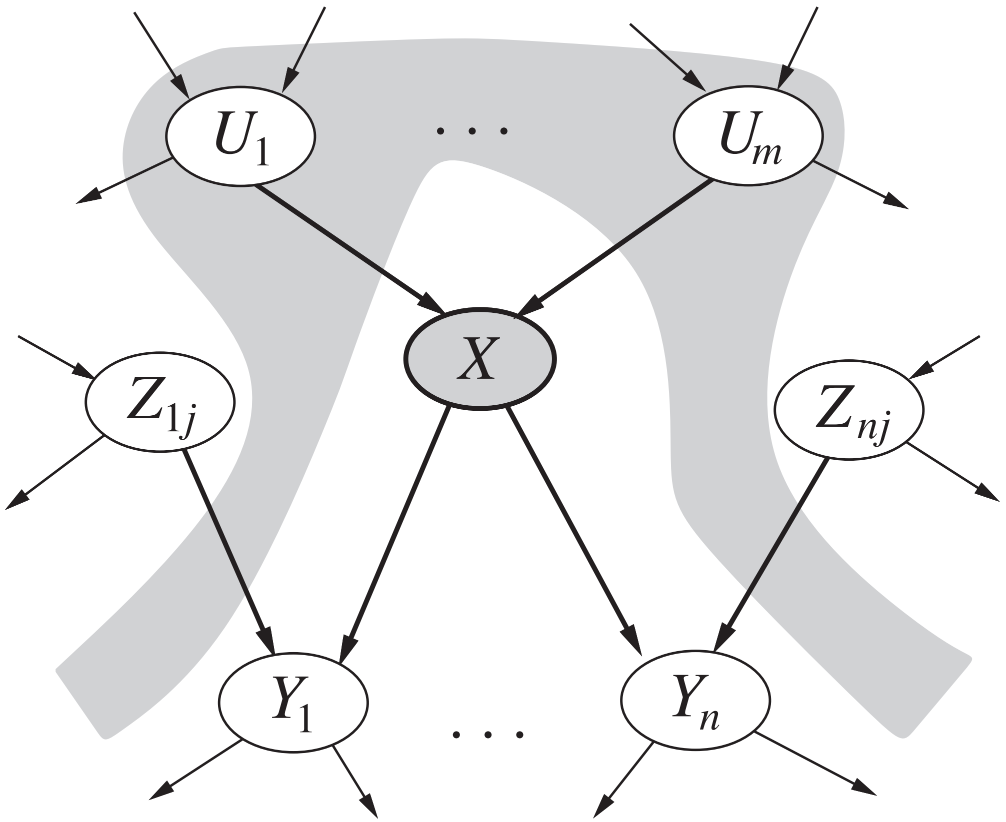




Conditional Independence Semantics

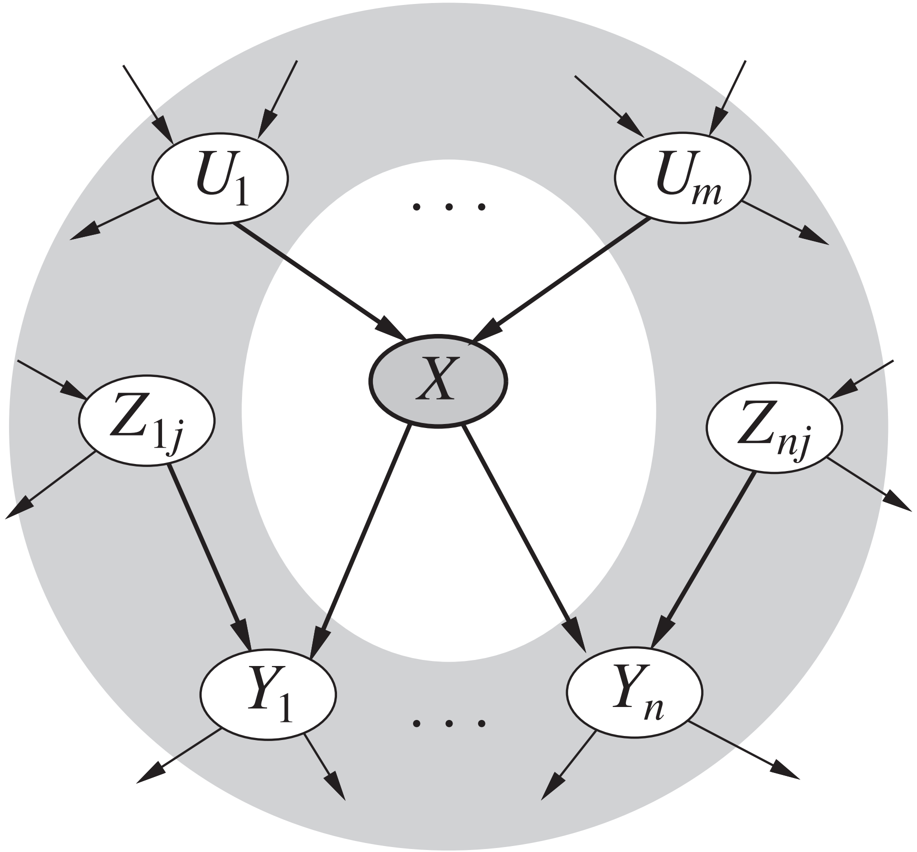
- Every variable is conditionally independent of its non-descendants given its parents



A node X is conditionally independent of its non-descendants (e.g., the Z_{ij}s) given its parents (the U_is shown in the gray area).

Markov Blanket

- A variable's Markov blanket consists of parents, children, children's other parents
- Every variable is conditionally independent of all other variables given its Markov blanket



Bayes' Nets

- ✓ Representation
- ✓ Conditional Independences
- Probabilistic Inference
  - Enumeration (exact, exponential complexity)
  - Variable elimination (exact, worst-case exponential complexity, often better)
  - Probabilistic inference is NP-complete

Reading

- Read Sections 14.1, 14.2, and 14.4 in the AIMA textbook (Third Edition)