Portfolio selection with a risk-free and many risky securities.

Outline

1. Portfolio selection with a risk-free and 1 risky security.
2. Portfolio selection with a risk-free and 2 risky securities.

-> last class

3. Portfolio selection with a risk-free and many risky securities.

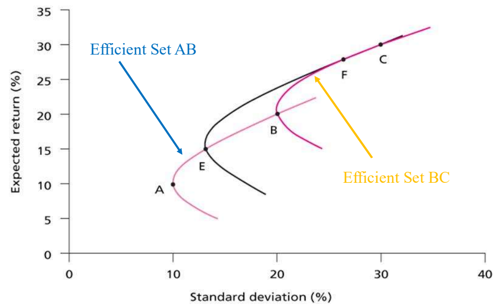
- Systematic and idiosyncratic risk
- The Single Index Model

-> today

Outline

- Investment opportunity set
  - With many risky assets (3 to…)
  - With many risky assets and a risk-free security
- Optimal portfolio choice and two-fund separation
- Diversifiable and non-diversifiable risk (systematic risk)

Three risky assets – Intuition



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Three Assets Example

- Stocks: GE, IBM, INTC
- Sample period: January, 1995 to December, 1998 (monthly returns).
- Calculate for each stock:
  - E(r) is estimated with =AVERAGE()
  - STD(r) is estimated with =STDEV()
- Calculate for each pair of stocks:
  - $\rho(r_1,r_2)$ is estimated with =CORREL()

	GE	IBM	INTC
1/31/1995	0.0098	-0.01871	0.08705
2/28/1995	0.06311	0.04679	0.14955
3/31/1995	-0.00621	0.09136	0.06426
4/28/1995	0.03704	0.15221	0.20689
5/31/1995	0.03571	-0.01453	0.09646
6/30/1995	-0.02095	0.03226	0.12806
7/31/1995	0.04656	0.13411	0.02729
8/31/1995	-0.00212	-0.04822	-0.05577
9/29/1995	0.08977	-0.08585	-0.01833
10/31/1995	-0.00784	0.0291	0.16041
11/30/1995	0.06126	-0.00386	-0.1288
5/29/1998	-0.02128	0.01592	-0.11601
6/30/1998	0.08996	-0.02287	0.03762
7/31/1998	-0.01252	0.15406	0.13912
8/31/1998	-0.10552	-0.14834	-0.15657
9/30/1998	-0.00172	0.14095	0.20457
10/30/1998	0.09976	0.15564	0.04009
11/30/1998	0.03286	0.11343	0.20718
12/31/1998	0.1325	0.11658	0.10163
Mean	3.26%	3.87%	4.89%
STD	5.81%	8.99%	11.36%
Correl(GE,IBM)	0.3588		
Correl(GE,INTC)	0.1881		
Correl(IBM,INTC)	0.5278		

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Three Assets Example

Portfolio variance formula:

$$\sigma^2(r_p) = w_1^2\sigma^2(r_1) + w_2^2\sigma^2(r_2) + w_3^2\sigma^2(r_3) + 2w_1w_2\sigma(r_1)\sigma(r_2)\rho(r_1,r_2) + 2w_1w_3\sigma(r_1)\sigma(r_3)\rho(r_1,r_3) + 2w_2w_3\sigma(r_2)\sigma(r_3)\rho(r_2,r_3)$$

Weights:			
GE	0.927214	Portfolio VAR	0.003329
IBM	0.131152	Portfolio STD	5.77%
INTC	-0.058366	Portfolio E(r)	3.25%
Sum of weights:	1.000000		

- By changing the weights, we can get the expected return and STD of every possible portfolio.
- The Portfolio VAR cell contains the formula for the variance of the portfolio, pointing to the cells with the weights, the means, the STDs, and the correlations.
- Portfolio STD is the the square root of the Portfolio VAR cell.
- Portfolio E(r) is: $w_1*E(r_1)+w_2*E(r_2)+ w_3*E(r_3)$

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Three Assets Example

- The Solver (under TOOLS) is used to solve the variance minimization problem.
- Set Target Cell: the cell that contains the variance formula.
- By Changing Cells: the cells that contain the weights of the portfolio.
- Subjects to the Constraints:
  - The cell that contains the sum of the weights is equal to 1.
  - The cell that contains the expected return is equal to the target expected return.
- Set precision and convergence (under Options) to 0.0000001 and Solve!

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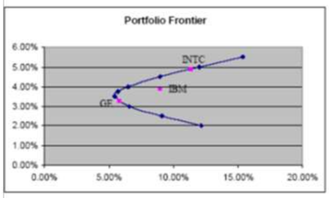
Three Assets Example

- By minimizing the variance for a given set of expected returns we can generate the Portfolio Frontier.

STD	Mean	GE	IBM	INTC
12.18%	2.00%	1.6531	0.1991	-0.8522
9.10%	2.50%	1.3627	0.1719	-0.5347
6.58%	3.00%	1.0724	0.1447	-0.2171
5.46%	3.50%	0.7821	0.1175	0.1004
5.72%	3.75%	0.6369	0.1039	0.2592
6.50%	4.00%	0.4917	0.0903	0.4179
8.98%	4.50%	0.2014	0.0631	0.7355
12.04%	5.00%	-0.0889	0.0359	1.0530
15.35%	5.50%	-0.3793	0.0087	1.3706



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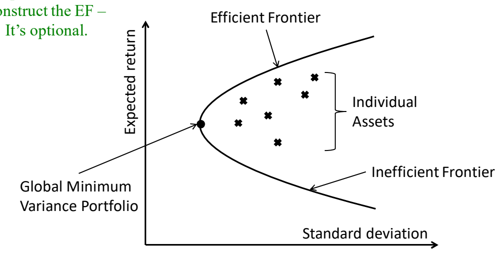
Investment Opportunity Set with Many Assets

Handout 13 explains how to construct the EF – It's optional.



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Step 3B: Optimal Portfolio Selection with Many Risky Assets and a Risk-Free Asset (on board)

1. Create the set of possible mean-s.d. combinations from different portfolios of risky assets
2. Find the "tangency portfolio," i.e. the portfolio with the highest Sharpe ratio:

$$SR_i = \frac{E[R_i]-R_f}{\sigma_i}$$

3. Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

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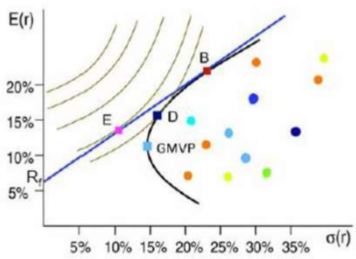
Optimal Portfolio Selection with Many Risky Assets and a Risk-Free Asset



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Two-Fund Separation

- All investors hold combinations of the same two "mutual funds":
  - The risk-free asset
  - The tangency portfolio
- An investor's risk aversion determines the fraction of wealth invested in the risk-free asset.
- But, all investors should have the rest of their wealth invested in the tangency portfolio.

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Application: Portfolio Optimizer

- Calculates optimal portfolio with 5 risky assets and 1 riskless asset.
- Why does the optimal portfolio load up on 2?
- Why hold asset 4 at all?
- Importance of correlation:
  - $\rho_{4,5} = 0 \rightarrow 0.7$;
  - $\rho_{4,5} = 0.7 \rightarrow 0.9$.

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Portfolio Variance and SD

- With 2 securities (N=2), the portfolio variance is:

$$\sigma_p^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2$$

  - In general, the portfolio variance is:

$$\sigma_p^2 = \sum_{i=1}^{N}\omega_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N}\omega_i\omega_j\rho_{ij}\sigma_i\sigma_j$$

  - The standard deviation of the portfolio is:

$$\sigma_p = \sqrt{\sigma_p^2}$$

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Risk Reduction in Equally-Weighted Portfolios: Independent Returns

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N independent stocks (independent = zero correlation)
- The variance of the portfolio return is

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 \Longrightarrow \frac{1}{N}\{\frac{1}{N}\{\sum_{i=1}^{N}\sigma_i^2\}\}$$

$$= \frac{1}{N}\begin{bmatrix}\text{average} \\ \text{variance}\end{bmatrix}$$

- As the number of assets increase, the risk is diversified away. (The *insurance principle*.)

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Risk Reduction in Equally-Weighted Portfolios: The General Case

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N stocks.
- The variance of the portfolio return is:

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 + \frac{2}{N^2}\sum_{i=1}^{N}\sum_{j>i}^{N}\underbrace{\mathrm{cov}(R_i, R_j)}_{\rho_{ij}\,\sigma_i\,\sigma_j}$$

$$= \frac{1}{N}\left[\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\right] + \left(1-\frac{1}{N}\right)\left[\frac{1}{N(N-1)/2}\sum_{i=1}^{N}\sum_{j>i}^{N}\mathrm{cov}(R_i, R_j)\right]$$

$$= \frac{1}{N}\begin{bmatrix}\text{Average}\\ \text{Variance}\end{bmatrix} + \left(1-\frac{1}{N}\right)\begin{bmatrix}\text{Average}\\ \text{Covariance}\end{bmatrix}$$

Risk in Equally-Weighted Portfolios: The General Case

What happens when N goes to infinity?

- Variance of portfolio return
  → average covariance of returns
- Risk of portfolio → <u>non-diversifiable</u> risk
- The key lesson is that the variance or the volatility of an individual asset is not a good indicator of its riskiness!
- The only thing that matters is its covariance with other assets.
- Even if the asset has a low return and high standard deviation, it may be a great asset (when?)
- Example: Asset 4 (mean 4% ; corr -30% with all the other)

Classifications of Risk

- Part that cannot be diversified away is called:
  - “covariance risk,” “systematic risk” or “non-diversifiable risk”
  - E.g. market risk, macroeconomic risk, industry risk
- Part that can be diversified away (in a large portfolio) is called:
  - “variance risk,” “idiosyncratic risk,” “non-systematic risk,” “diversifiable risk” or “unique risk”
  - E.g. Individual company news.

Systematic Vs. Unsystematic Risk

- Systematic Risk:
  - Caused by the tendency of returns on stocks to move together due to changes in:
    - Macroeconomic variables (interest rate, boom, inflation..)
    - Technological changes
    - Political situation (war, peace, taxation…)
  - Also called Undiversifiable Risk since it is reflected in the covariances of stocks' returns and therefore does not disappear in a well-diversified portfolio with many stocks.

Systematic Vs. Unsystematic Risk

- Unsystematic Risk:
  - Caused by firm-specific events or characteristics that affect the return of a single stock but not the returns of other stocks (management change, regulatory change, new competitor…)
  - Also called Diversifiable Risk since it is reflected only in the variance of stock's returns and therefore disappears in a well-diversified portfolio with many stocks.

Systematic vs. Idiosyncratic Risk

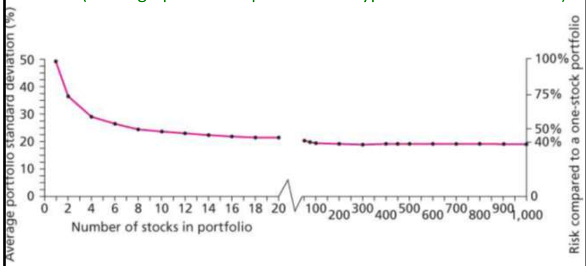
- When held in a portfolio some of the risk of a stock disappears.
- Or, the risk contribution the stock makes to the portfolio is LESS than the risk of the stock if held in isolation:

$$\begin{pmatrix}\text{Total risk in}\\ \text{a stock}\end{pmatrix} = \begin{pmatrix}\text{Systematic}\\ \text{risk}\end{pmatrix} + \begin{pmatrix}\text{Idiosyncratic}\\ \text{risk}\end{pmatrix}$$

- Investors need to be compensated for holding which kind of risk?

Percentage Risk Reduction

- What is the percentage reduction in risk we should expect from adding stocks to our portfolio?
- (In the graph 100% represents the typical risk of NYSE firms)



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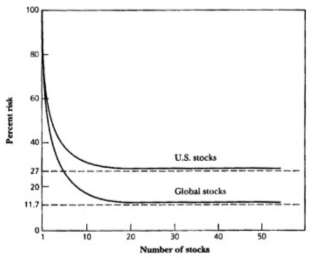
Percentage Risk Reduction

- What is the percentage reduction in risk we should expect from adding stocks to our portfolio?
- (In the graph 100% represents the typical risk of a US stock)



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Markets: Diversification in Crisis?

- How do you diversify when all asset classes are moving in tandem, as in fall 2008?

Correlation Analysis

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Diversification in Practice

- **Bottom line:** In down markets, correlations tend to go up, which lowers the gains from diversification.
- That does not mean that diversification is wrong or useless. It still helps to hedge your risk because it allows you to eliminate idiosyncratic risk.
- However, what it does mean is that the gains from diversification are now **smaller** than they used to be because more of the risk is systematic.
- The fact of the matter is that the world economy is more integrated than it has ever been in the past. These global economic interdependencies are wonderful (think about the welfare benefits of trade), but they necessarily imply that there is less diversification possible across countries than say 10 or 20 years ago.

Implementation issues with Markowitz model

- With N stocks, one needs:
  - N estimates of expected returns
  - N estimates of variances
  - N(N-1)/2 estimates of correlation between all pairs of returns
- For N = 500, this amounts to 125,750 parameters that need to be estimated.
- Solution 1, Index models: all co-movement of returns is captured by a few common factors
- Solution 2, CAPM: implied expected returns in equilibrium

Single Index Model

- **Assumption:** All co-movement is driven by the fact that stocks are driven by the same economy! Then only need to know covariance between each stock and economy, and we also know covariance between all stocks.
  - Note: Maybe not perfect assumption. For example, stocks in same industry co-move as well due to industry-specific risks.

Single Index Model

- **How to separate idiosyncratic from systematic risk for security i?**
- Mutual fund (S&P 500) return: $R_M = \Sigma_i\, w_i\, R_i$
- Regression analysis: $R_i = \alpha_i + \beta_i R_M + e_i$
  - e_i: Idiosyncratic component of the return, idiosyncratic risk (variance risk) = σ^2_e.
  - $\beta_i R_M$: Systematic component of the return, systematic risk (covariance risk) = $(\beta_i)^2\, \sigma^2_M$.
  - **Definition:** $\beta_i = Cov(R_M, R_i)/\sigma^2_M$
- See Handout 12 for more information

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Single Index Model

- **A *single index model* relates security returns to their betas, thereby measuring how each security varies with the overall market.**
  - R_i = Constant + Common-Factor News + Firm-Specific News

$$R_i = \alpha_i + \beta_i R_M + e_i$$

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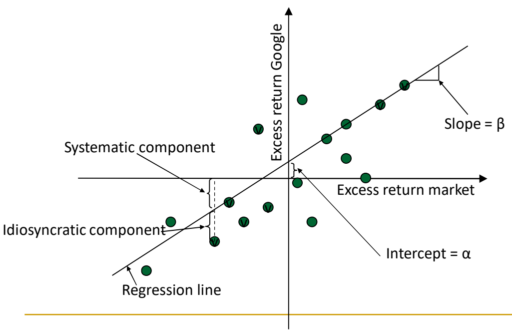
Single Index Model estimation



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Concepts to Know

- Optimal portfolio selection with many risky assets
  - Investment opportunity set
  - Efficient frontier
  - Optimal portfolio
- Diversification
  - Independent versus general case
- Systematic versus idiosyncratic risk