Project 1: Implement and Prediction of LPPL Model

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1 Introduction

LPPL Model is used to describe a super-exponential (power law) accelerating behavior of asset price when just before the bubbles burst and crushes or rebounds appearing. The characteristic of abnormal increasing bubble price is a strong upward curvature with seriously unstable oscillation, which implies the price goes upward by growth rates at any time, instead of a constant growth rate which is the simple exponential increasing process [1]. This model is closer to the real market because the asset price in the real market will never grow with an ideal constant rate and this model accounts for the imitation and herding mechanisms and positive feedback among investors, so it can be a more accurate prediction indicator [2].

1.1 Definition of Bubbles

The asset price will be over-estimated and become irrational high after a series of price increases because more and more investors follow the trend to buy this popular asset, as a result, the market price of the asset will beyond its real price. The more the asset's owner expects to earn from holding or selling that asset, the higher its price. When the price reaches a relatively highest position, there is no follower who is willing to hold the asset with a higher price, then the holders start to lost confidence and leave the asset gradually, as the result, the difference between the bid and call becomes too large to remain balance, finally the bubble burst. If this unbalance between selling and holding is broken, the bubble will disrupt and a crash follows which means the price will suddenly decay after reaching the highest price. Many factors may contribute to economic bubbles, such as speculation, easy credit, finance innovation.

1.2 Positive Feedback

In the stock market, any deviation of asset price should be ultimately traced back to the behavior of investors. it is the buying and selling decisions of investors that push prices up and down, the mechanisms of positive feedback on price is that if the recent observation of the asset price is moved up, then price will follow the tendency to keep on moving up, as a result, the asset price will be grow super-exponentially.

The positive feedback leads to speculative behaviors which means that the more and more trends followers emerge in the market along with large investments accumulated by Youssefmir (1998) [3]. It will cause the market value larger than its real value presenting a greater-exponential upward tendency. At the same time, the system will become increasingly sensible until the crash.

1.3 Imitation and Herding Behavior

Herding effect theorem is the basis of positive feedback which is popular in many economics phenomena. It happens because investors are not able to attain the whole picture of a market, they can just make a buying or selling decisions by analyzing recent observations or following those expert and rational investors. The less information gained from the market, the larger the probability to follow others like a herd.

There was a mimetic contagion model of investors in the stock markets developed by Orlean (1989)[4]. The simplest version is called the Urn model. Under stimulating this model with balls, it turns out that bubble crushes are the consequences of the imitative behavior among investors.

1.4 Formula

For the original formula of LPPL model by Didier Scornette,

$$\ln p(t) = A - B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) + \varphi)) \tag{1}$$

There are seven unknown parameters in the formula above, where $p(t)$ denote the price of asset at time t,

- A is the logarithm price of greater-than-exponential bubble price when it approaches to t_c, $A > 0$.
- B denotes the direction of price change is either an upward gain ($B < 0$) or a downward loss ($B > 0$).
- C includes the degree of logarithm periodic oscillation.
- m indicates the extent of accelerating, which depends on the proportion of rational investors and herding followers ($0 < m < 1$).
- t is a certain time interval in the past that is any time before the bubble burst.
- ω is the angular frequency of oscillation during bubbles, it exists when investors show consistency on an investment strategy.
- φ is the initial phase, $0 < \varphi < 2\pi$.

A more detailed explanation about seven parameters: In our following study, we suppose the price remains finite even at t_c ,$B < 0$ for upward accelerating price while $B > 0$ for downward accelerating price and the accelerating extent near the critical point m should be $0 < m < 1$. If m is too close to 0, it implies a relatively stationary bubble while accelerating suddenly approaching the critical time, If m is too close to 1, it presents no price-increasing tendency. When it comes to w, by spectrum analysis on residual studied by Press, W.H., Teulolsky (1994), logogram frequency is about 1.1, then the corresponding angular frequency is about 7[5]. We combine these theoretical analyses and experimental results as presumptions and put into the fitting model to ensure an explainable super-exponential acceleration behavior of the bubble price.

1.5 Application

Many systems present similar super-exponential growth regimes, which can be described mathematically by power law growth. For example, planet formation in solar systems by runaway accretion of planetesimals, rupture and material failures, nucleation of earthquakes modeled with the slip-and-velocity, models of micro-organisms interacting through chemotaxis aggregating to form fruiting bodies, the Euler rotating disk, and so on.

LPPL has presented pretty good examples of predicting the crisis risk, such as the Oil price bubble in early July 2008 and the bubble burst on the Shanghai stock market in early August 2009[6]. Therefore, It has great significance to forecast the critical time of bubble burst with a certain probability. And nowadays LPPL has been used commonly in the field among financial, earthquake, biochemistry.

1.6 Conclusion

we would like to use and revise the LPPL Model by analyzing the possible factors and mechanisms behind its high frequency oscillation and super exponential behaviors. In this sense, we might take other interactions or factors into account to revise the LPPL Model in order to provide a more accurate and precise indicator of crisis risk.

2 LPPL Model Implement

The formula in 1.4 of LPPL describes a nonlinear function of logarithm asset price about time, involving logarithm, periodic and super-exponential tendency with 7 unknown parameters. We chose the Levenberg-Marquardt algorithm (LM) to find out the global optimal solution about seven unknown parameters in this non-linear least squares problem.

Firstly, we aim to minimize the distance between real price data with the estimated price from the model. We measured the L_2-norm distance as aim function for better curve fitting after comparing the consequence with L_1-norm.

Secondly, we put several limits and bounds on parameters and fitted the data with MATLAB. The limits on parameters refer to the conclusive analysis of successful experiments by Professor Didier with his group.

Thirdly, we tested different time intervals(Start Date to End Date) on three different stocks — SSE Composite Index, NASDAQ, HK Seng Index. The number of iterations and the limit conditions on parameters, to some extent, did affect the consequences of critical time to get closer to the real historical crush points. The LPPL model is only valid in the super-exponential tendency.

In our report, we used the MATLAB to fit seven parameters especially the critical time, which can verify the precision of our model and algorithm.

Finally, the figures of three stocks embody the precise prediction on the critical time of bubbles ending.

Detailed results are in the Appendix.

3 Prediction of LPPL Model

We use the historical data of 000001.SS to predict the next critical time of bubbles ending, maybe crush or rebound following. The figure 1 shows that the anti-bubbles tend to rebound on the 2019-03-20.

Data

Symbol	Start Date	End Date
000001.SS	2017-1-13	2018-10-19

Results

A	B	T	m	C	ω	ϕ
7.53	0.0354	450.36	0.4723	0.0018959	-18.4191	-114.8257

Predicted rebound date is `2019-03-20`

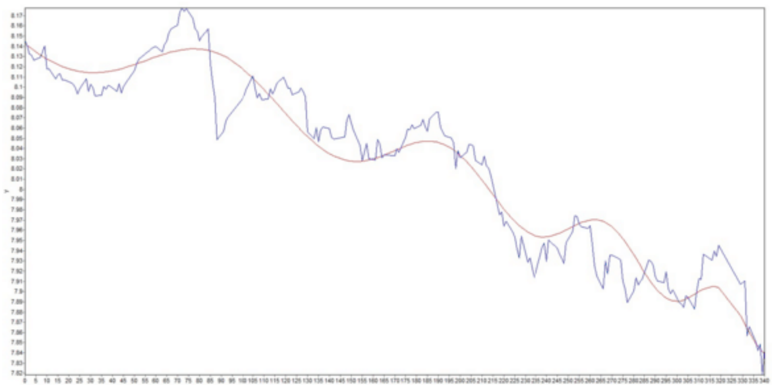

Figure 1: Prediction of 000001.SS

4 View of Ted Talk of Didier Scornette

Professor Didier Scornette has pioneered in studying in the critical time of financial crisis, they started the Financial Crisis Observatory in order to study if critical time of financial bubbles can be detected in advance and developed a theory called "Dragon Kings", which compared those extremely special and large events to "dragon" and "kings". In the real market, the asset prices normally move up in a natural exponential way, however, the extreme situation is inevitable and not rarely although it only possesses 1 percentage. The financial crisis like "Great Recession" brought incredible tragedy to the world market and risk management tools at present are not strong enough to diagnose these outliers, even will misguide the professional analysts. Thus, it is essential to find out a powerful theorem and model to make it controllable and predictable. Fortunately, the Scornette ' s group has identified that the information about the critical time of the system eventually burst is contained in the early development of the previous formal super-exponential growth. The normal prices move upward with a greater-exponential growth tendency. The next growth is pushed forward by positive feedback. The Dragon King theorem provides a possible diagnostics method of crisis for investors, to some extent, agents can prepare and take measures in advance to prevent the crisis like the Great Recession occurring again.

It can be applied to many other fields where run the similar regime that affected by positive feedback and contains the extreme consequence, such as to find the critical time interval of machine rupture occurs depending slight noise of emission, the precursor sign of baby maturation, glacier collapse, blockbusters, earthquake forecasting and so on. They have successfully predicted lots of financial bubbles in the past 20 years applying this model, taking the Chinese bubbles crushed in December 2007 while rebounded in August 2009 as examples.

Although the prediction method presented in the TED Talks seems to be an excellent way to reveal the abnormal phenomena, the explanatory power of a formula can be questioned due to seven unknown parameters within a complicated formula of LPPL. In the course of dealing with practical problems, fitting data and returning a group of parameters is not easy. It can not explain the real meanings of the value of all parameters, analysts can only refer to thousands of outputs from successful experiments. It is the secret of model fitting that matters the excellent prediction consequence.

5 References

References

[1] Sornette, D. (2003). Why stock market crash.

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[5] Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1992). Numerical recipes in FORTRAN (Cambridge.)

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A Solver of LPPL in MATLAB

Title

```
% ---------- Interface ----------
function Para = LPPL(Times, Prices, X0)
    % Pre-process
    N      = length(Times);
    Times  = transpose(Times(:));
    Prices = Prices(:);
    X0     = X0(1);
    % ------- TRAIN -------
    Para = Train(Times, Prices, X0);
    % ------- PREDICT -------
    Y    = Predict(Para, 1:N);
    % ------- FIGURE -------
    plot(Times, Prices, Times, Y);
    legend('exact', 'fitted');
end


% ---------- FUNCTIONS ----------
% ------- TRAIN -------
function Para = Train(Times, Prices, t0)
    %{
    Train function solve the optimization problem and get
        the optimal
    paramater.
    ----------------------------
    Args:
        Times:  Time sequence (suggested to be 1:N).
        Prices: Prices list.
        t0:     The start point for the optimization.
    Returns:
        Para:   Struct instance.
    %}
    ObjFunc      = @(t) Func2(t, Times, log(Prices));
    OptimProblem = createOptimProblem('fmincon',...
        'objective', ObjFunc,...
        'lb',        max(Times),...
        'x0',        t0,...
        'options',   optimset('Display', 'iter'));
```

```
    [tmin, Fmin] = fmincon(OptimProblem);

    [~, Para]   = Func2(tmin, Times, log(Prices));
    Para.tc     = tmin;
    Para.m      = Para.m;
    Para.omega  = Para.omega;

    [~, tmp] = Func1(tmin, Para.m, Para.omega, Times, log
       (Prices));
    Para.A    = tmp.A;
    Para.B    = tmp.B;
    Para.C1   = tmp.C1;
    Para.C2   = tmp.C2;

    Para.TrainRes = Fmin;

    function [Value, Para] = Func2(t, Times, LogPrices)
        ObjSubFunc = @(x) Func1(t, x(1), x(2), Times,
            LogPrices);
        OptimSubProblem = createOptimProblem('fmincon'
            ,...
            'objective', ObjSubFunc,...
            'lb',        [0, 0],...
            'ub',        [1, Inf],...
            'x0',        [0.5, 2],...
            'options',   optimset('Display', 'off'));
        [Xmin, Value] = fmincon(OptimSubProblem);

        Para.m     = Xmin(1);
        Para.omega = Xmin(2);
    end

    function [Value, Para] = Func1(t, m, omega, Times,
       LogPrices)
        dt = t - Times;
        N  = length(dt);
        X  = zeros(N, 4);

        X(:,1) = 1.0;
        X(:,2) = dt .^ m;
        X(:,3) = X(:,2)' .* cos(omega*log(dt));
        X(:,4) = X(:,2)' .* sin(omega*log(dt));
```

```

        Coef    = regress(LogPrices, X);
        Para.A  = Coef(1);
        Para.B  = Coef(2);
        Para.C1 = Coef(3);
        Para.C2 = Coef(4);

        Value = sum((LogPrices - X*Coef).^2);
    end
end


% ------- PREDICT -------
function Y = Predict(Para, X)
    dt    = Para.tc - X;
    [M,N] = size(X);
    Y     = Para.A * ones(M,N) + Para.B * dt.^ Para.m + ...
            Para.C1 * dt .^ Para.m .* cos(Para.omega .* log(dt)) + ...
            Para.C2 * dt .^ Para.m .* sin(Para.omega .* log(dt));
    Y     = exp(Y);
end
```