1 Question 1

Statement 1 ▶ Timescale Invariance

$$S(t_{i+1}) = S(t_i) + \mu\delta t S(t_i) + \sigma\delta t Y_i S(t_i)$$
$$S(t_{i+1}) = S(t_i) + \mu\delta t^{1/4} S(t_i) + \sigma\delta t Y_i S(t_i)$$

Please verify or dispute the timescale invariance of the two models above by numerical experiments.

Therefore, we have the following code, which verifies the timescale invariance of the models by numerical experiments. And the result of the code below with the arguments $S_0 = 1$, $\mu = 0.05$, $\sigma = 0.5$, is Figure 1.

```
function timescale_invariance_asset_path(S0, mu, sigma, ends,
    point_number, path_number)
  %{
    Compute and plot `path_number` asset paths for the given `
   S0`, `mu`, and `sigma`,
    at `numpoint_numberber` equally spaced time points in [0,
   `end`] for `end` in `ends`.

    Argument
    --------
    S0: double
    mu: double
    sigma: double
    ends: matrix_{1xn}, denote n subplots
    point_number: integer, number of points in interval
    path_number: integer, number of paths

    Return
    ------
    None, but plot a figure of subplots.

    Example
    -------
    >>> S0=1; mu=0.05; sigma=0.5; ends=[1,0.1,0.01];
   point_number=100; path_number=10;
    >>> timescale_invariance_asset_path(S0, mu, sigma, ends,
   point_number, path_number)
  %}

  if nargin<6, path_number=10; end
  if nargin<5, point_number=100; end
  if nargin<4, ends=[1, 0.1]; end
  if nargin<3, sigma=0.5; end
  if nargin<2, mu=0.05; end
```

```
30    if nargin<1, S0=1; end
31
32    figure;
33    suptitle(['S_0:', num2str(S0), '; \mu:', num2str(mu), '; \
       sigma:', num2str(sigma)]);
34
35    for i = 1 : length(ends)
36      subplot(length(ends), 1, i);
37      dt = ends(i) / point_number;
38      t = linspace(0, ends(i), point_number);
39      S = S0*cumprod( ...
40        exp( ...
41          (mu-sigma^2/2) * dt + ...
42          sigma * sqrt(dt) * randn(path_number,point_number) ...
43        ), 2 ...
44      );
45      plot(t, S);
46    end
47  end
```

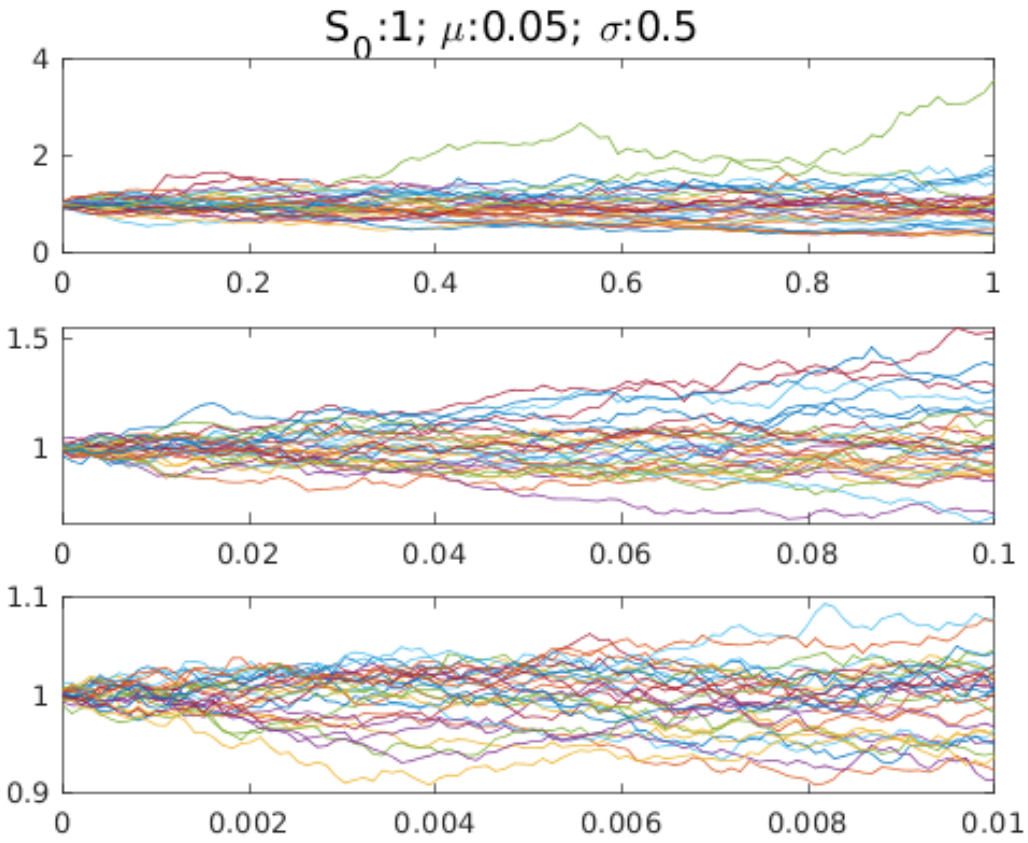

Figure 1: Timescale Invariance Asset Path