1 Question 1

Statement 1 ▶ Timescale Invariance

$$d_1 = \frac{\log(S/E) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$
$$d_1 = \frac{\log(S/E) + (r - \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

Prove the following identity:

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

```
from sympy import symbols, log, sqrt, pretty_print


S, E, r, σ, T, t = symbols('S, E, r, σ, T, t')

numerator_l = log(S/E) + r*(T-t)
numerator_r = σ**2*(T-t)/2
denominator = σ*sqrt(T-t)

d1 = (numerator_l + numerator_r) / denominator
d2 = (numerator_l - numerator_r) / denominator

# Process:
__Δ = d1 - d2
#   = 2*numerator_r / denominator
#   = σ**2*(T-t) / σ*sqrt(T-t)
#   = σ * sqrt(T-t)
#   =       _______
#      σ·□□ T - t

pretty_print(__Δ.simplify())
```

2 Question 2

Statement 2 ▶ Put-Call Parity

With $t = 0$, $S_0 = 5$, $E = 4$, $T = 1$, $\sigma = 0.3$ and $r = 0.05$, find the option values and verify the put-call parity.

```
from math import log, sqrt, erf, exp


S, E, r, σ, T, t = 5, 4, 0.05, 0.3, 1, 0

numerator_l = log(S/E) + r*(T-t)
numerator_r = σ**2*(T-t)/2
denominator = σ*sqrt(T-t)

d1 = (numerator_l + numerator_r) / denominator
d2 = (numerator_l - numerator_r) / denominator

N = lambda d: (1+erf(d/sqrt(2)))/2

print('␣␣␣␣d_1␣=', d1)
print('␣␣␣␣d_2␣=', d2)
print('␣N(d_1)␣=', N(d1))
print('␣N(d_2)␣=', N(d2))
print('N(-d_1)␣=', N(-d1))
print('N(-d_2)␣=', N(-d2))


C = S*N(d1) - E*exp(-r*(T-t))*N(d2)
P = E*exp(-r*(T-t))*N(-d2) - S*N(-d1)

print('␣␣␣␣␣␣␣␣␣␣␣␣␣␣P+S␣=', P+S)
print('C+E*exp(-r*(T-t))␣=', C+E*exp(-r*(T-t)))
```

3 Question 3

Statement 3 ▶ Study Note

1. Write a study note of Black-Scholes' 73 paper.
2. Find a sequence of works following this paper and sort out hot topics nowadays.

To be continued.