How to Calculate the Total Amount of Your Salary?

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Outline

Introduction

Example 1

- Many jobs offer an annual cost-of-living increase to keep salaries consistent with inflation.
- A recent college graduate finds a position as a sales manager earning an annual salary of $ 26,000.
- He is promised a 2% cost of living increase each year.

- So how could we determine how much this college graduate will get after *n*, which is a natural integer, years' hard work?
- This question arises naturally as payment is one of the most significant factors people will consider when finding a job.

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The Geometric Progression

- Let's review the definition of **arithmetic progression** (Doe(2019)) first.

Definition 1

In an **arithmetic progression**, the difference between n^{th} term and $(n-1)^{th}$ term will be a constant which is known as the common difference of the arithmetic progression.

- What if the ratio of n^{th} term to $(n-1)^{th}$ term in a sequence is a constant?

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The Geometric Progression

Definition 2

A **geometric progression** is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the **common ratio**.

Definition 3

If we define the initial term of a sequence $\{a_n\}$ as a_1, then $\{a_n\}$ is a **geometric progression** ($a_n \neq 0$ for any n) if and only if for any positive integer n,

$$\frac{a_{n+1}}{a_n} = r,$$

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Summation of Geometric Progression

Definition 4

The sum of the terms in a geometric sequence is called a **geometric series**.

Theorem 1

The formula for S_n is

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1,$$

where S_n is the geometric series.

- How to derive this theorem?

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- How to derive this theorem?

Summation of Geometric Progression

Proof.

- Recall that a geometric progression is a sequence in which the ratio of any two consecutive terms is the common ratio, r. We can write the sum of the first n terms of a geometric series as

$$S_n = a_1 + ra_1 + r^2a_1 + \cdots + r^{n-1}a_1. \tag{3.1}$$

- We will begin by multiplying both sides of the equation by r,

$$rS_n = ra_1 + r^2a_1 + r^3a_1 + \cdots + r^na_1. \tag{3.2}$$

- Next, if we subtract (3.2) from (3.1),

$$S_n - rS_n = (a_1 + ra_1 + r^2a_1 + \cdots + r^{n-1}a_1) - (ra_1 + r^2a_1 + r^3a_1 + \cdots + r^na_1).$$

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Proof.

- Notice that when we subtract, all but the first term of the top equation and the last term of the bottom equation are canceled.
- And it is trivial to obtain the formula for S_n,

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1. \tag{3.3}$$

- Notice that we have assumed that the common ratio r does not equal to 1.

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- The number of terms in infinite geometric progression will approach infinity ($n = \infty$).
- The formal definition of infinity has been discussed in Zhang(2018).
- Sum of infinity geometric progression can only be defined at the range of $-1 < r < 1 (r \neq 0)$ exclusive.

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Application to the Salary Example

Example 2 (Salary Problem)

- There will be a 102% annual increment. So the common ratio r is 1.02.
- If we consider the first 5 years' total amount of the salary, we could calculate S_5 by directly substituting $n = 5$, $r = 1.02$, $a_1 = 26,000$ into Equation (3.3).
- We will get $S_5 = 137,907.06$. I.e.This graduate will get $137,907.06 after 5 years' working.

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- We derived the formula of the geometric series.
- Furthermore, with a proper choice of the common ratio, we showed that this formula can be generalized to infinite case.
- A real problem is analyzed to illustrate the proposed formula and answered the question in the title: by using the formula proposed in this paper, we could calculate the total amount of the salary if it has a constant multiple of the common ratio each payment period.

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