Computer Science C73
Scarborough Campus

September 20, 2023
University of Toronto

Homework Assignment #3
Due: September 27, 2023, by 11:59 pm

- ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
- *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
- *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
- *For any question, you may use data structures and algorithms previously described in class, or in prerequisites of this course, without describing them. You may also use any result that we covered in class, or is in the assigned sections of the official course textbooks, by referring to it.*
- *Unless we explicitly state otherwise, you may describe algorithms in high-level pseudocode or point-form English, whichever leads to a clearer and simpler description. Do not provide executable code.*
- *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on* ***the correctness and efficiency of your answers, and the clarity, precision, and conciseness of your presentation****.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC73. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (10 marks) Consider Huffman's algorithm.

a. Give an example of a (small) set of symbols and their associated frequencies so that there is exactly one symbol with frequency 1/3, all other symbols have frequency (strictly) less than 1/3, and Huffman's algorithm may produce a codeword of length 1.

b. Prove that for any set of symbols, if every symbol has frequency (strictly) less than 1/3, Huffman's algorithm cannot produce a codeword of length 1.

Question 2. (10 marks) Describe a version of Karatsuba's integer multiplication algorithm where instead of dividing the binary representation of each integer into two segments, you divide it into three segments. Justify the correctness of your algorithm, analyze its running time, and compare it to the running time of the version we saw in class. To receive credit, your algorithm should run in sub-quadratic time in the number of bits of the integers being multiplied.

Question 3. (15 marks) We are given an array $A[1..n]$ of distinct integers so that each element is at most k positions away from its correct position in sorted order. That is, for every $i \in [1..n]$, the i-th smallest element of A is in $A[j]$, where $j \geq \max(1, i-k)$ and $j \leq \min(i+k, n)$. Describe an algorithm that

sorts such an array in $O(n \log k)$ time. Justify the correctness of your algorithm and analyze its running time. For simplicity, assume that both n and k are powers of 2.