Computer Science C73 — September 7, 2022
Scarborough Campus — University of Toronto

Homework Assignment #1
Due: September 14, 2022, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use data structures and algorithms previously described in class, or in prerequisites of this course, without describing them. You may also use any result that we covered in class, or is in the assigned sections of the official course textbooks, by referring to it.*
• *Unless we explicitly state otherwise, you may describe algorithms in high-level pseudocode or point-form English, whichever leads to a clearer and simpler description.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on* ***the correctness and efficiency of your answers, and the clarity, precision, and conciseness of your presentation****.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC73. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (20 marks) Let $\mathcal{I}$ be a sequence of closed intervals. A set of numbers C is a ***cover*** of $\mathcal{I}$ if every interval in $\mathcal{I}$ contains at least one number in C (that is, for each $I = [s, f]$ in $\mathcal{I}$, there is some $c \in C$ such that $s \leq c \leq f$). C is a ***minimum cover*** of $\mathcal{I}$ if it is a minimum cardinality cover of $\mathcal{I}$.

a. (5 marks) Consider the following greedy algorithm.

1 $C := \varnothing$
2 **while** $\mathcal{I} \neq \varnothing$ **do**
3   let c be a number that is contained in the largest number of intervals in $\mathcal{I}$
4     $C := C \cup \{c\}$
5   delete from $\mathcal{I}$ all intervals that contain c
6 **return** C

(In line 3, assume that ties are broken arbitrarily. That is, if there are multiple numbers that are contained in the same maximum number of intervals in $\mathcal{I}$, then we pick any one of these numbers as our c.)

Prove that this algorithm does not always find a minimum cover of $\mathcal{I}$. For full marks, your counterexample should be such that if in any iteration of the algorithm there are multiple choices for c, then ***every*** choice results in a suboptimal C at the end. (Thus, there is no hope of fixing this greedy algorithm by using a more refined choice of c as a number that is contained in the maximum number intervals ***and*** satisfies some additional property.)

b. (15 marks) Describe a greedy algorithm that given a sequence of intervals $\mathcal{I}$ (in arbitrary order) returns a minimum cover of $\mathcal{I}$. (Your algorithm should be given in clear high-level pseudocode or point-form English; do not present detailed code.) Prove the correctness of your algorithm.

Question 2. (15 marks) Consider the following process: We are given a positive integer n. We start with $x = 1$. At each step of the process we can apply to x one of two operations: INCREMENT x by 1 (i.e., $x := x + 1$) or DOUBLE x (i.e., $x := 2x$). We want to reach the given number n (i.e., make $x = n$) by applying repeatedly such steps. For example, if $n = 1024$, we can do this in 1023 increment steps, or in 10 double steps, or in some combination of these two kinds of steps.

Our problem is the following: Find a sequence of increment or double operations so that starting from 1 we reach n ***in the fewest possible number of steps***. Describe a greedy algorithm that solves this problem. Your algorithm should take as input the positive integer n, and should produce as output a sequence each element of which contains I (increment) or D (double), so that if we apply these operations to x in the given order starting with $x = 1$, we end with $x = n$. Analyze the running time of your algorithm and prove its correctness.