Computer Science C73 — September 8, 2021
Scarborough Campus — University of Toronto

Homework Assignment #1
(worth 4% of the course grade)
Due: September 15, 2021, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use data structures and algorithms previously described in class, or in prerequisites of this course, without describing them. You may also use any result that we covered in class, or is in the assigned sections of the official course textbooks, by referring to it.*
• *Unless we explicitly state otherwise, you may describe algorithms in high-level pseudocode or point-form English, whichever leads to a clearer and simpler description.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness and efficiency of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC73. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (15 marks) We live in an one-dimensional world where everything is a point on a straight line, and the location of each point is defined by its distance from a fixed point on that line, called the ***origin***: negative for points to the "left" and positive for points to the "right" of the origin. We are given the locations of m firefighters in an array $F[1..m]$ and the locations of n fires in an array $R[1..n]$.

The Fire Chief assigns firefighters to fires subject to the following constraints: (a) a firefighter is assigned to at most one fire, (b) at most one firefighter is assigned to a fire, and (c) a firefighter can be assigned to a fire only if the fire is located within distance $d \geq 0$ (to the left or to the right) of the firefighter's location.

Give an efficient greedy algorithm that helps the Fire Chief assign firefighters to as many fires as possible under these constraints. Your algorithm takes as input the arrays F and R and produces as output a set of pairs of the form (f, r), where $1 \leq f \leq m$ and $1 \leq r \leq n$, indicating that firefighter f is assigned to fire r. Prove the correctness of your algorithm and analyze its running time.

Hint: Who should be assigned to the leftmost fire (if anyone can)?

Question 2. (20 marks) Let $\mathcal{I}$ a set of intervals over the real numbers. (For specificity say these are closed intervals, but this is not important.) A subset $\mathcal{C}$ of $\mathcal{I}$ is a ***cover of*** $\mathcal{I}$ if for every interval $I \in \mathcal{I}$

there is an interval $I' \in \mathcal{C}$ such that I and I' intersect (even at one point). We wish to find a ***minimum cardinality*** cover of $\mathcal{I}$ — i.e., a cover of $\mathcal{I}$ with as few intervals as possible.

a. True or false? For every set of intervals $\mathcal{I}$ and every minimum cardinality cover $\mathcal{C}$ of $\mathcal{I}$, no two intervals in $\mathcal{C}$ intersect. Justify your answer.

b. Describe an $O(n^2)$ greedy algorithm that, given a set $\mathcal{I}$ of n intervals, returns a minimum cardinality cover of $\mathcal{I}$. Prove the correctness of your algorithm and analyze its running time.