Filter specifications and how to design a digital filter based on the given specifications

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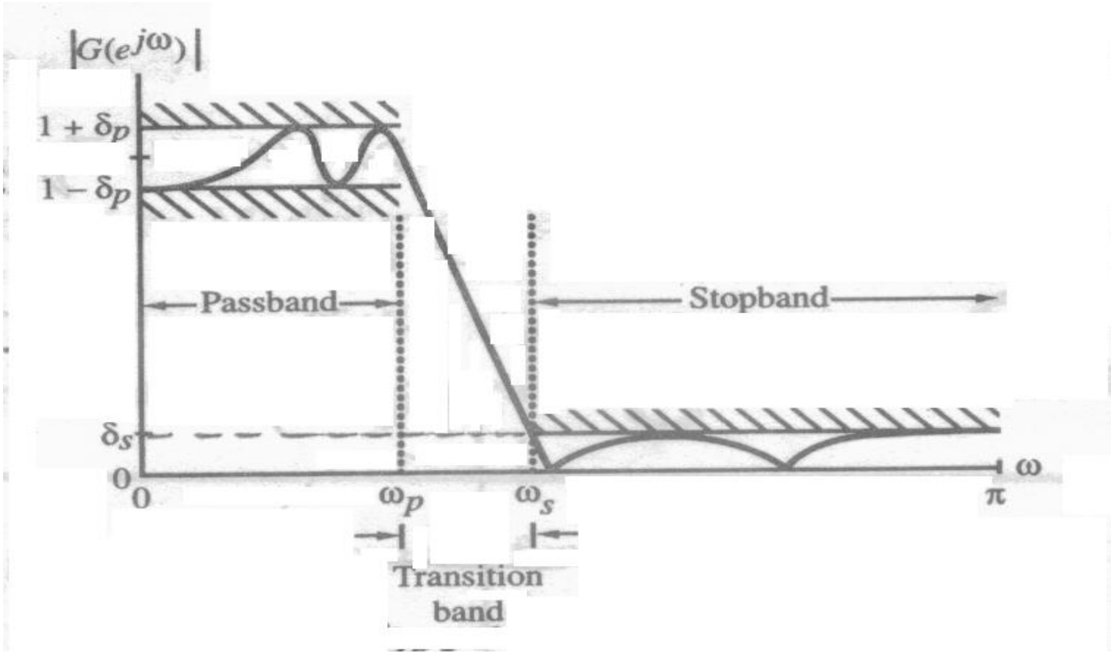
Specifications of digital filters :

- The most important step in the development of a digital filter :Determine a realizable transfer function G(z)
- Digital Filter Specifications :

(1) magnitude response specifications in the passband and the stopband are given with some acceptable tolerances.

(2) A transition band is specified between the passband and the stopband to permit the magnitude to drop off smoothly.



- Passband edge frequency ω_p
- Stopband edge frequency ω_s
- Peak ripple value of passband δ_p
- Peak ripple value of stopband δ_s
- Peak passband ripple α_p
- Minimum stopband attenuation α_s
- Sample frequency F_T

Important laws :

$$\alpha_P = -20\log_{10}(1-\delta_P)dB$$

$$\alpha_S = -20\log_{10}(\delta_s)dB$$

$$\delta_P = 1-10^{-\alpha_P/20}$$

$$\delta_S = 10^{-\alpha_S/20}$$

$$\omega_P = \frac{\Omega_P}{F_T} = \frac{2\pi F_P}{F_T}$$

$$\omega_S = \frac{\Omega_S}{F_T} = \frac{2\pi F_S}{F_T}$$

Digital filters design :

- **Selection of the Filter Type :**

(1)The objective of digital filter design is to develop a causal transfer function H(z) meeting the frequncy specifications.

(2)FIR and IIR Digital Filter :

FIR Digital Filter $$H(z) = \sum_{n=0}^{N} h[n]z^{-n}$$

IIR Digital Filter $$H(z) = \frac{p_0+p_1z^{-1}+p_2z^{-2}+\ldots+p_Mz^{-M}}{d_0+d_1z^{-1}+d_2z^{-2}+\ldots+d_Nz^{-N}}$$

The order N_{FIR} of an FIR filter is higher than the order N_{IIR} of an equivalent IIR filter meeting the same magnitude specifications.

The ratio $N_{FIR}/N_{IIR}$is typically of the order of 10 or more (the IIR filter usually is computationally more efficient).

- **Basic Approaches to Digital Filter Design :**

Step1: convert the digital filter specifications into analog lowpass prototype filter specifications

Step2: determine the analog lowpass filter transfer function $H_a(s)$

Step3: transform $H_a(s)$ into the desired digital filter transfer function G(z)

- **Why analog?**

(1)Analog approximation techniques are highly advanced.

(2)They usually yield closed-form solutions.

(3)Extensive tables are available for analog filter design .

(4)Many applications require the digital simulation of analog filters.

- **Estimation of the Filter Order :**

IIR: The order of G(z) is determined from the transformation being used to convert $H_a(s)$ into G.

FIR(lowpass digital filter): $N \cong \frac{-20\log_{10}(\sqrt{\delta_p \delta_s})-13}{14.6(\omega_s-\omega_p)/2\pi}$

For narrowband filter $N \cong \frac{-20\log_{10}(\delta_s)+0.22}{(\omega_s-\omega_p)/2\pi}$

For wideband filter $N \cong \frac{-20\log_{10}(\delta_p)+5.94}{27(\omega_s-\omega_p)/2\pi}$

Bilinear Transformation Method of IIR Filter Design

- Bilinear transformation is more commonly used to design IIR digital filters based on the conversion of analog prototype filters
- The Bilinear Transformation

S-plane to z-plane

$$G(z)= H_a(s)\ \Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

The transformation is a one-to-one mapping. It maps a single point in the s-plane to a unique point in the z-plane

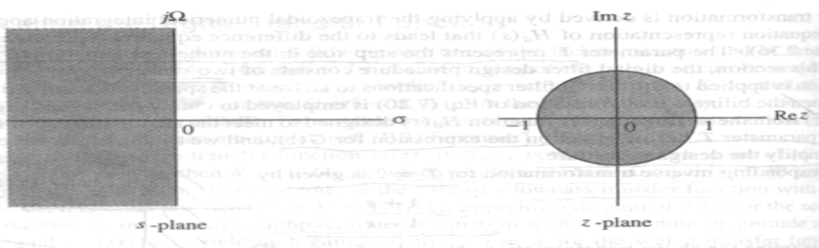
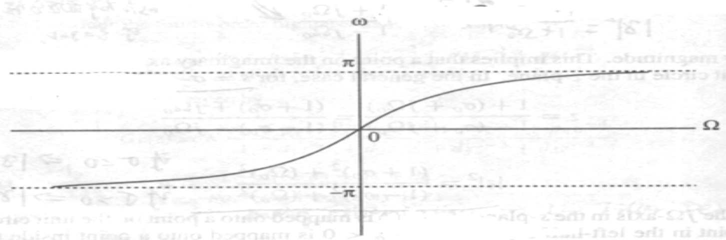
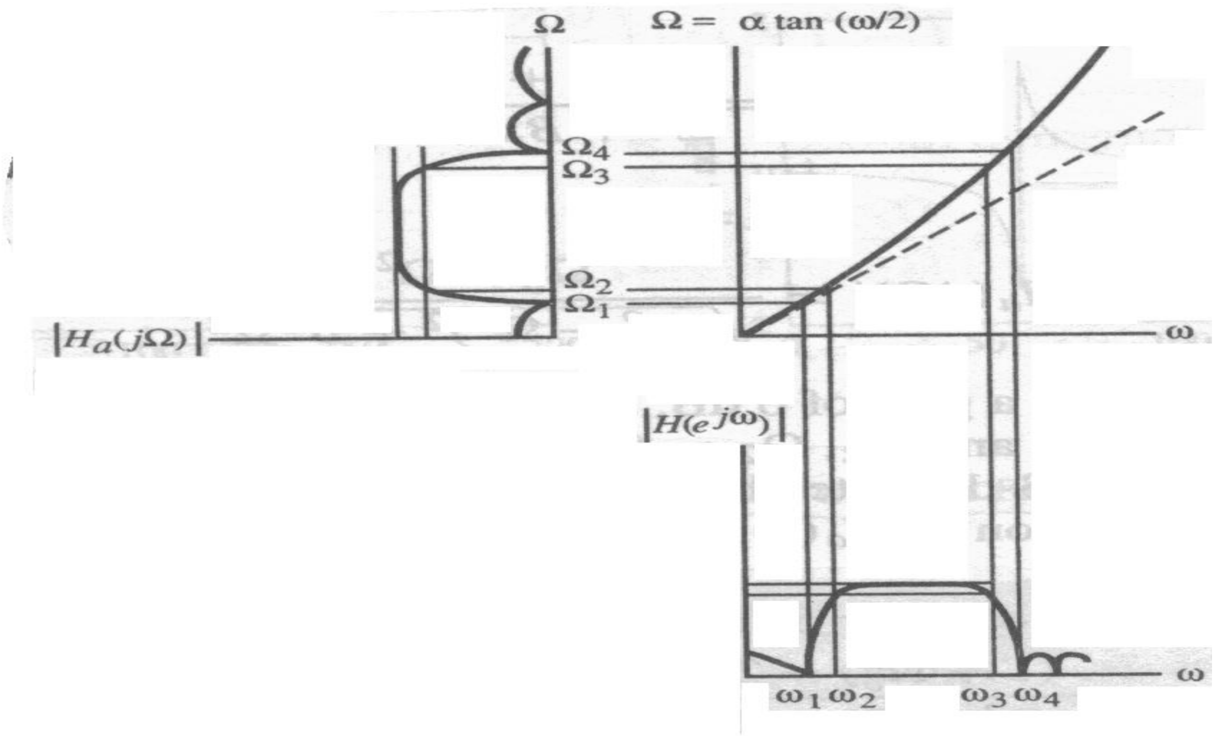


Figure 7.3: The bilinear transformation mapping.





- **Digital filter design procedure:**

Step1: the invert bilinear transformation is applied to the digital filter specifications to arrive at the specifications of the analog filter function.

Step2 : the bilinear transformation is employed to obtain the desired digital transfer function G(z) from the analog transfer function $H_a(s)$ desired to meet the analog filter specifications.

- **When T=2(T has no effect on the G(z))**

$$z = \frac{1+s}{1-s} \xrightarrow{s = j\Omega_0} z = \frac{1+j\Omega_0}{1-j\Omega_0}$$

$$\Big\downarrow\, s = \sigma_0 + j\Omega_0$$

$$z = \frac{(1+\sigma_0)+j\Omega_0}{(1-\sigma_0)-j\Omega_0} \longrightarrow |z|^2 = \frac{(1+\sigma_0)^2+(\Omega_0)^2}{(1-\sigma_0)^2+(\Omega_0)^2}$$

If $\sigma_0 < 0$ then $|z| < 1$

If $\sigma_0 > 0$ then $|z| > 1$

When $s = j\Omega$ and $z = e^{j\omega}$

$$j\Omega = \frac{1-e^{-j\omega}}{1+e^{j\omega}} = j\tan(\frac{\omega}{2})$$

$$\Omega = \tan(\frac{\omega}{2})$$