Title

Design of Digital Filters

Name

Alarefi Abdulqawi Abdulmajeed Abdullah Hussien

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1. INTRODUCTION OF FILTERS

1.1 Analog and digital filters

In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range. There are two main kinds of filter, analog and digital. They are quite different in their physical makeup and in how they work. An analog filter uses analog electronic circuits made up from components such as resistors, capacitors and op-amps to produce the required filtering effect. Such filter circuits are widely used in such applications as noise reduction, video signal enhancement, graphic equalizers in hi-fi systems, and many other areas. There are well-established standard techniques for designing an analog filter circuit for a given requirement. The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. These calculations typically involve multiplying the input values by constants and adding the products together. If necessary, the results of these calculations, which now represent sampled values of the filtered signal, are output through a DAC (digital to analog converter) to convert the signal back to analog form. Note that in a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current. The actual procedure for designing digital filters has the same fundamental elements as that for analog filters. First, the desired filter responses are characterized, and the filter parameters are then calculated. Characteristics such as amplitude and phase response are derived in the same way. The key difference between analog and digital filters is that instead of calculating resistor, capacitor, and inductor values for an analog filter, coefficient values are calculated for a digital filter. So for the digital filter, numbers replace the physical resistor and capacitor components of the analog filter. These numbers reside in a memory as filter coefficients and are used with the sampled data values from the ADC to perform the filter calculations.
Comparison between digital and analog filters:

DIGITAL FILTERS	ANALOG FILTERS
1-High Accuracy 2-Linear Phase (FIR Filters) 3-No Drift Due to Component Variations 4-Flexible, Adaptive Filtering Possible 5-Easy to Simulate and Design 6-Computation Must be Completed in Sampling Period - Limits Real Tim Operation 7-Requires High Performance ADC, DAC & DSP	**1-Less Accuracy – Component Tolerances 2-Non-Linear Phase 3-Drift Due to Component Variations 4-Adaptive Filters Difficult 5-Difficult to Simulate and Design 6-Analog Filters Required at High Frequencies and for Anti-Aliasing Filters 7-No ADC, DAC, or DSP Required**

1.1 Advantages of using digital filters

The following list gives some of the main advantages of digital over analog filters.

1. A digital filter is programmable, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be

changed without affecting the circuitry (hardware). An analog filter can only be changed by redesigning the filter circuit.

2. Digital filters are easily designed, tested and implemented on a general-purpose computer or workstation.
3. The characteristics of analog filter circuits (particularly those containing active components) are subject to drift and are dependent on temperature. Digital filters do not suffer from these problems, and so are extremely stable with respect both to time and temperature.
4. Unlike their analog counterparts, digital filters can handle low frequency signals accurately. As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF (radio frequency) domain, which in the past was the exclusive preserve of analog technology.
5. Digital filters are very much more versatile in their ability to process signals in a variety of ways; this includes the ability of some types of digital filter to adapt to changes in the characteristics of the signal.
6. Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively simple and compact in comparison with the equivalent analog circuitry.

2. Digital filters:

A digital filter implements the difference equation that describes the algorithm to process the time domain signal in order to achieve filtering objectives. The objective of filtering is to remove signal in certain frequency range. The objective of the filter design is to obtain the filter coefficients so that the difference equation of the filter can be implemented. Equation (2.1) shows the standard difference equation for IIR filter (order = max (M, N)) and equation (2.2) shows the standard difference equation for FIR filter of order q. The filter coefficients are given by b_k and a_k.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k] \tag{2.1}$$

$$y[n] = \sum_{k=0}^{q} b_k x[n-k] \tag{2.2}$$

Digital filters can be divided into finite impulse response (FIR) and infinite impulse response (IIR) filter. IIR filter contains a feedback loop in the block diagram, hence the transfer function of an IIR filter contains poles, and perhaps zeros as well. FIR filter, on the other hand, does not have the feedback loop, thus its transfer function consists of only zeros. Generally there are 8 stages in the design of a digital filter:

1. Specification of the filter requirements
2. Choice of a type of filter.
3. Determination of the filter order.
4. Finding a set of coefficients.
5. Implementation.
6. Quantization.
7. Redesigning if necessary.
8. Choosing the filter structure.

2.1 Selecting the Filter Type and Order

There are two main types of filters, namely FIR and IIR. These differ in their characteristics and in the way they are designed. Since the design algorithm depends strongly on the choice of IIR vs. FIR filter, the designer should make this decision as early as possible. Although the desired frequency response specifications can be approximated with either type of filter, deciding which of the two filter types to use depends on many factors including the implementation hardware, as well as the magnitude and phase characteristics of the resulting filter. The difference between FIR&IIR is shown below:

FIR filters:

1. Linear phase response
2. Stability with quantized coefficients
3. Higher order required than using IIR filters

IIR filters:

1. Better attenuation properties
2. Closed form approximation formulas
3. Nonlinear phase response
4. Instability (limit cycle oscillations) with finite word length computation

-NFIR/NIIR is typically of the order of tens (or more)

2.2 IIR filters

IIR filters have traditional analog counterparts (Butterworth, Chebyshev, Elliptic, and Bessel) and can be analyzed and synthesized using more familiar traditional filter design techniques.

Infinite impulse response filters get their name because their impulse response extends for an infinite period of time. This is because they are recursive, i.e., they utilize feedback. Although they can be implemented with fewer computations than.

Causal IIR filters are causal LTI systems that are described (input-output relation):

$$y[n] = \sum_{k=0}^{N} h[k]x[n-k] \qquad (2.3)$$

Advantages of IIR:

1. Analog filters can be readily transformed into equivalent IIR digital filters. This is impossible with FIR filters as they have no analog counterpart.
2. Require less filter coefficients than FIR to achieve similar frequency response
3. In many applications, linearity of phase response is not an issue.

2.2.1 IIR Filter structure:

Filter structures in which the multiplier coefficients are precisely the coefficients of the transfer function:

$$H(z) = \frac{P(z)}{D(z)} = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}} \tag{2.4}$$

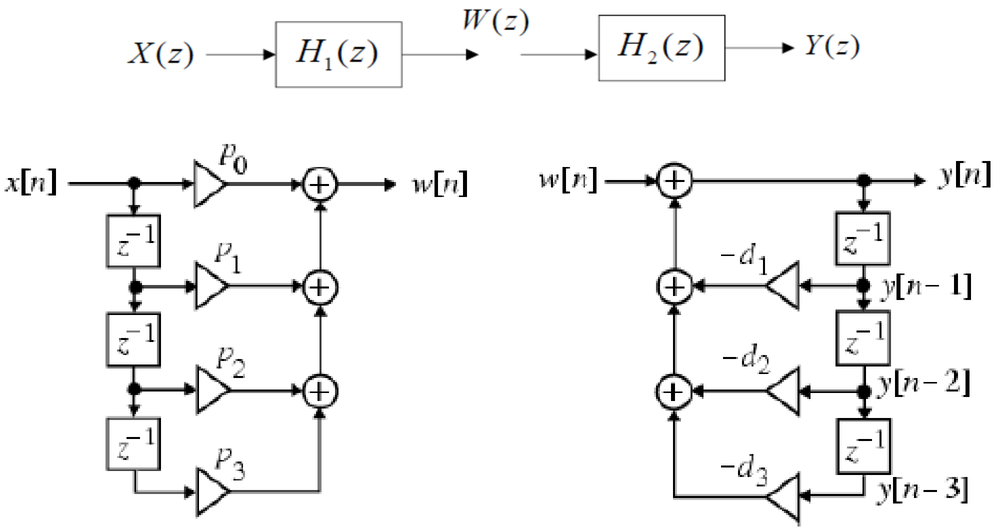

Fig2.1 H1 (z) realizes the zeros and H2 (z) realizes the poles of the transfer function H(z)

2.2.2 IIR Filter Design

An analog filter transfer function Ha(s) is transformed into the desired digital filter transfer function G(z)

$$H_a = (s) = \frac{P_a(s)}{D_a(s)} \Longrightarrow G(z) = \frac{P(z)}{D(z)} \tag{2.5}$$

The basic idea behind the conversion of an analog prototype transfer function Ha(s) to a digital filter transfer function G (z) is to apply a mapping from the s-domain to the z-domain so that the essential properties of the analog frequency response are preserved

Requirements for the mapping are:

a. The imaginary axis (jΩ) of the s-plane is mapped onto the unit circle in the z-plane.
b. Stable Ha(s) must be transformed into a stable G (z).

2.2.3 Impulse Invariance Method

Straight-forward approach for obtaining the digital filter:

The impulse response of the digital filter is made identical to the impulse response of an analog prototype filter at sampling instants

Analog transfer function: Ha(s)'

$$h_a(t) l^{-1}\{h_a(s)\}$$
(2.6)

The impulse response of the digital filter is obtained by sampling:

$$g[n] = h(nT),\ n = 0,1,2,... \tag{2.7}$$

The digital filter transfer function G(z) is:

$$G(z) = Z\{g[n]\} = Z\{h_a[nT]\} \tag{2.8}$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a\left(s + j\frac{2\pi k}{T}\right)\Bigg|_{s=\frac{1}{T}\ln z}$$

The frequency responses are obtained by substituting z=$e^{j\omega}$ and s=jΩ:

$$G\left(d^{j\omega}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a(j\Omega + j\frac{2\pi k}{T}) \tag{2.9}$$

According to the sampling theorem G($e^{j\omega}$) is aperiodic version of Ha($e^{j\omega}$)

Thus, the impulse invariance mapping has the desired properties:
1) Frequency axis jΩ corresponds to unit circle
2) Stability is preserved

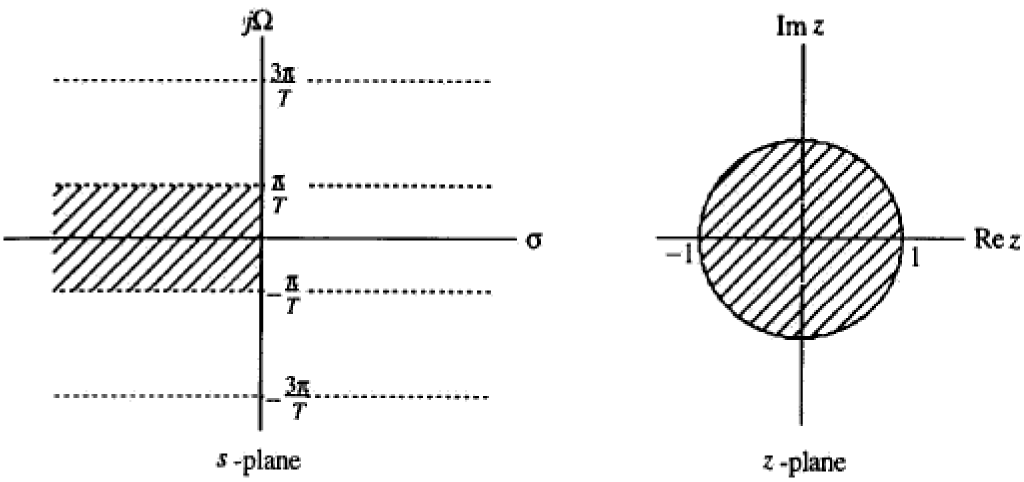

Fig 2.2 Mapping of s plane to z plane under Impulse Invariance transformation

2.2.4Bilinear Transform Method

The bilinear transformation is the one that is the most often used for designing IIR filters. It is defined as

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \tag{2.10}$$

The s-plane transfer function Ha(s) gives a z-plane transfer function

$$G(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

(2.11)

To avoid aliasing, the mapping from s-plane to z-plane should be one-to-one, i.e., a single point in the s-plane should be mapped to a unique point in the z-plane and vice versa:

1) The entire jΩ-axis should be mapped onto the unit circle

2) The entire left-half s-plane should be mapped inside the unit circle

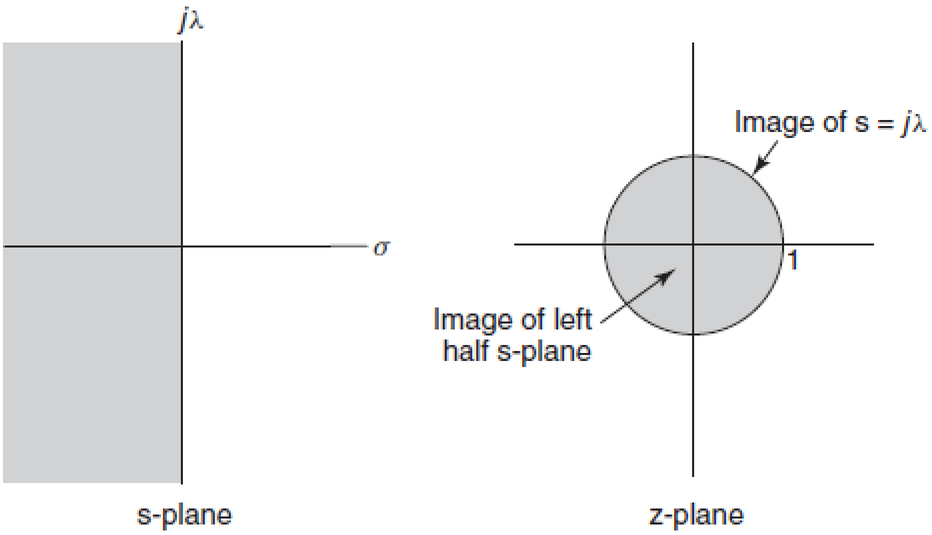

Fig 2.3 Mapping of s plane to z plane under bilinear transformation.

2.2.5 Classical IIR Filter Types

The four standard classical analog filter types are known as

(1) Butterworth: The magnitude-squared function of an Nth order Butterworth lowpass filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}} \qquad (2.12)$$

Where Ωc is the cutoff frequency. The Butterworth filter is optimal according to a flatness criterion. For a specified filter order and cut-off frequency, the magnitude response of the Butterworth filter is the solution that attains the maximum number of derivatives equal to 0 at Ω = 0 and 1 (ω= 0 and π for the digital filter).

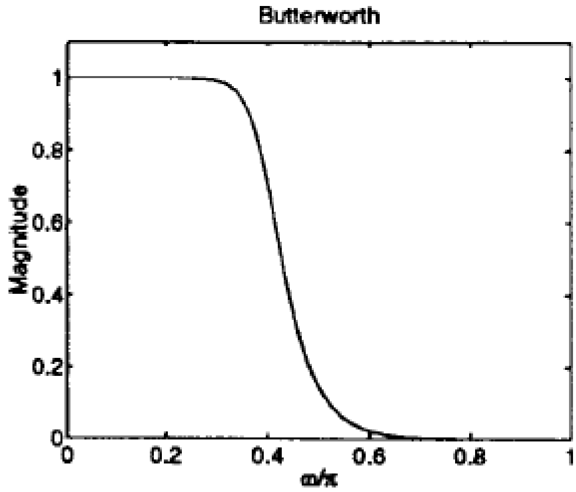

Fig 2.4 IIR Butterworth filter

(2) Chebyshev I and II: The magnitude response of a Type I Chebyshev filter is equiripple in the passband and monotonic in the stopband. The magnitude-squared response is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+\epsilon^2 T^2{}_N(\Omega/\Omega_c)^{2N}} \qquad (2.13)$$

Where TN(x) is the Nth degree Chebyshev polynomial in *x*, _ is a parameter specified by the allowable passband ripple, Ωc is the filter cutoff frequency, and *N* is the filter order.

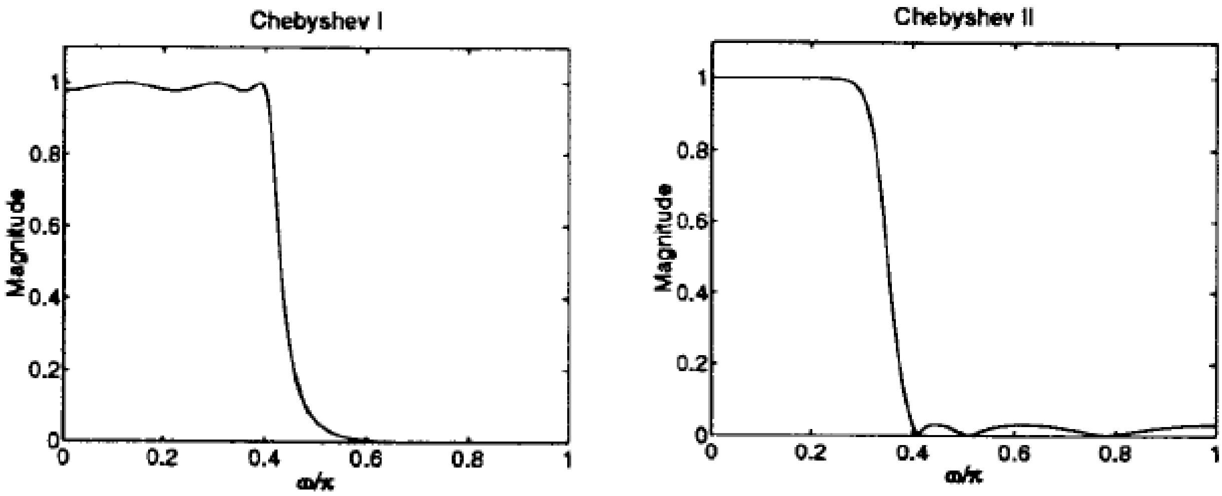

Fig 2.5 IIR Chebyshev I and II filter

(3) Elliptic: The magnitude response of an Elliptic filter is equiripple in both the passband and
stopband. It is optimal according to a weighted Chebyshev criterion.
The magnitude-squared response of an Elliptic filter is given by:

$$|H_a(j\Omega)|^2 = \frac{1}{1+\epsilon^2 E^2{}_N(\Omega)} \qquad (2.14)$$

Where EN(Ω) is a Jacobian elliptic function above. Elliptic filters are so called because elliptic functions are used in the formula.

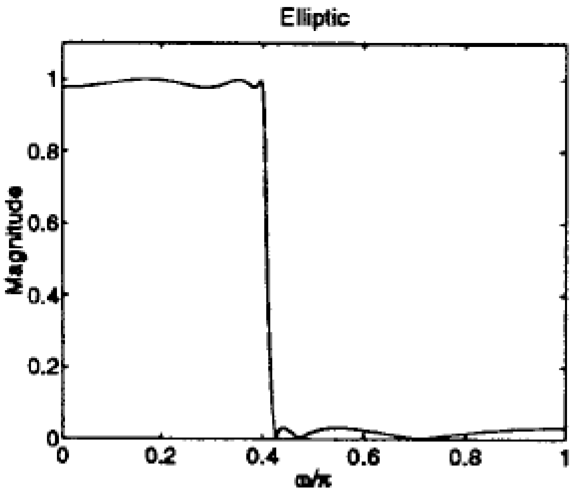

Fig 2.6 IIR Elliptic filter

2.3 FIR filters

FIR filters, IIR filters do not match the performance achievable with FIR filters, and do not have linear phase. Also, there is no computational advantage achieved when the output of an IIR filter is decimated because each output value must always be calculated

A causal FIR filter of order N is characterized by a transfer function H (z) given by

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n} \tag{2.15}$$

For IIR filters, it is necessary to ensure that the derived transfer function G(z) is stable

• In the case of FIR filters, the stability is not an issue as the transfer function is a polynomial in z-1 and the stability is always guaranteed

• Unlike the IIR filter design problem, it is always possible to design FIR digital filters with exactly linear phase response

FIR filter design does not have any connection with analog filters

• The design of FIR filters is based on a direct approximation of the specified magnitude response, with the usually added requirement that the phase response be linear

• A causal FIR transfer function H(z) of length N+1 is

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n} \tag{2.16}$$

The corresponding frequency response is

$$H(e^{j\omega}) = \sum_{n=0}^{N} h[n]e^{-j\omega n} \tag{2.17}$$

As a result, the design of an FIR filter of length can be accomplished by finding either the impulse response sequence {h[n]} or N+1 samples of its frequency response H($e^{j\omega}$). To ensure the linear-phase design, the symmetry condition of the impulse response must be satisfied

$$h[n] = \pm h[N-n] \tag{2.18}$$

2.3.1 FIR Digital Filter Structures

A direct form realization of an FIR filter can be readily developed from the convolution sum description as indicated below for $N = 4$

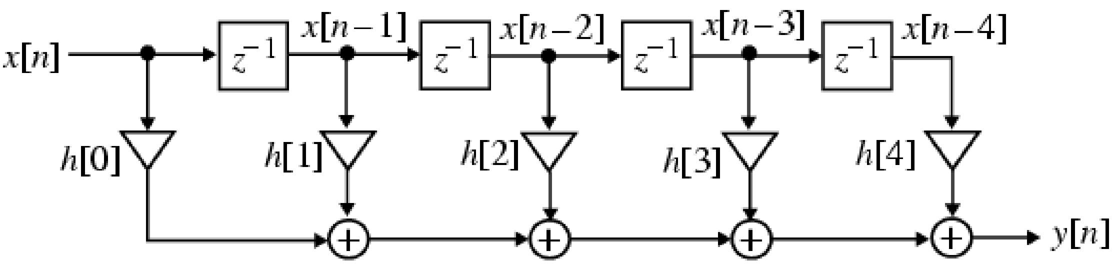

Fig 2.7 FIR Digital Filter Structures

An analysis of this structure yields

$$\begin{aligned} y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] \\ + h[3]x[n-3] + h[4]x[n-4] \end{aligned} \tag{2.19}$$

2.3.2 Basic Approaches to FIR Filter Design

Basic approaches in designing FIR filters:

1) Truncating the Fourier series representation of the desired frequency response => Window method.

2) Frequency sampling Length N FIR filter, N distinct equally spaced frequency samples of the desired frequency response constitute the N-point DFT of its impulse response.

2.3.3 Truncating the Impulse Response

Let $H_d(e^{j\omega})$ denote the desired frequency response function

$H_d(e^{j\omega})$ is periodic function of ω with period 2π and can be expressed as a Fourier series

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n] e^{-j\omega n} \tag{2.20}$$

The Fourier coefficients {hd[n]} are the impulse response samples

$$h_d = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega \qquad ,-\infty \leq n \leq \infty \tag{2.21}$$

Thus, given $H_d(e^{j\omega})$ we can compute h_d[n] and the corresponding h_d(z)

Usually, $H_d(e^{j\omega})$ is piecewise constant with ideal (or sharp) transitions between bands => {h_d [n]} sequence is of infinite length and noncausal. The objective is to find a finite-duration impulse response {h_t [n]} of length 2M+1 whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$ .Using the Parseval's relation:

$$\varphi = \sum_{n=-\infty}^{\infty} |h_t[n] - h_d[n]|^2 = \sum_{n=-M}^{M} |h_t[n] - h_d[n]|^2 + \sum_{n=-\infty}^{-M+1} h_d[n]^2 + \sum_{n=m+1}^{\infty} h_d[n]^2 \tag{2.22}$$

Now, φ is minimum when h_t [n] = h_d [n] for -M < n < M, i.e., the best finite-length approximation is obtained by truncating the impulse response.

2.3.4 Impulse Response of Ideal Lowpass Filter

The ideal lowpass filter has a zero-phase frequency response

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

1. Choose a pass band edge frequency for design that is midway through the transition width.

2. Find the impulse response h_L[n] for the ideal low pass filter with this pass band edge frequency.

3. Choose a window w[n] that gives adequate stop band attenuation, and determine the number of terms *N* needed to give the required transition width. An odd number should be chosen, so that the impulse response will be perfectly symmetrical around its middle point.

4. Find the impulse response h[n] = h_L[n]w[n] for the FIR filter.

5. Shift the impulse response to the right by (N-1)/2 samples.

The impulse response produced by this design method can be used to create a difference equation for the filter and also a plot of the filter shape.

2.3.5 Fixed Window Functions

Using a tapered window causes the height of the sidelobes to diminish, with a corresponding increase in the main lobe width resulting in a wider transition at the discontinuity

Hanning:

$$W[n] = 0.5+0.5\cos\left[2\pi n/(2M+1)\right], \quad -M \le n \le M \qquad (2.23)$$

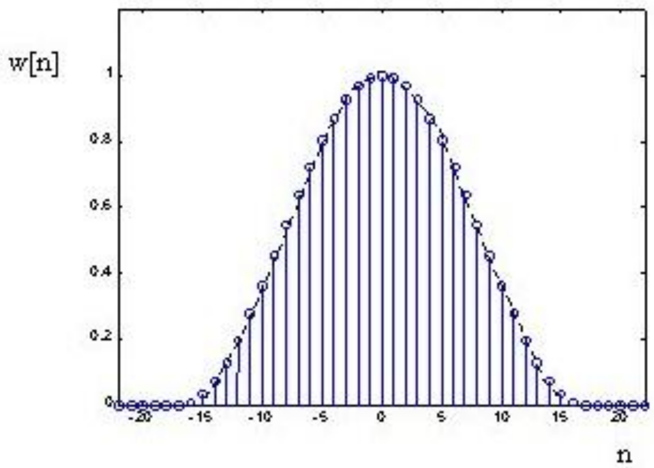

Fig 2.8 Hanning Fixed Window Function

Hamming:

$$W[n] = 0.54+0.46\cos\left[2\pi n/(2M+1)\right], \quad -M \le n \le M \qquad (2.24)$$

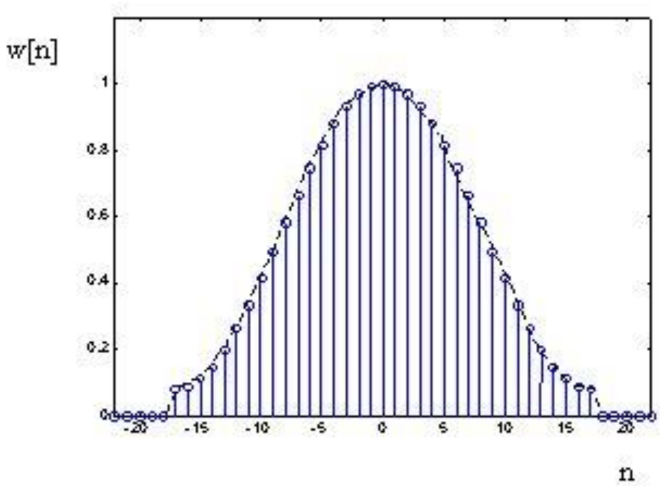

Fig 2.9Hamming Fixed Window Function

Blackman:

$$W[n] = 0.42 + 0.5\cos[2\pi n/(2M+1)] + 0.08\cos[4\pi n/(2M+1)] \qquad (2.25)$$

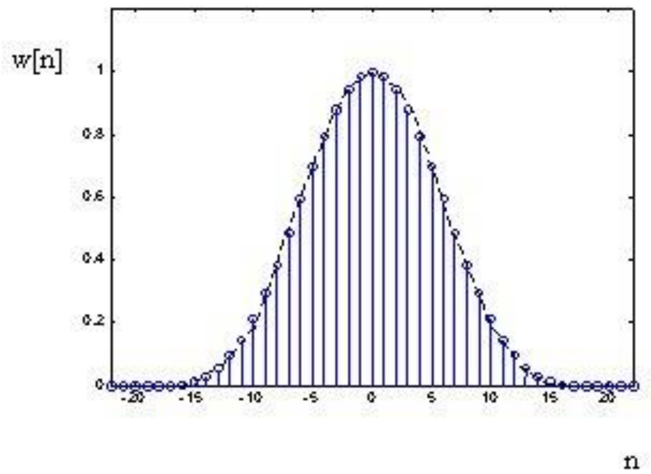

Fig 2.10 Hamming Fixed Window Function

Magnitude spectrum of each window characterized by a main lobe centered at $\omega = 0$ followed by a series of sidelobes with decreasing amplitudes

Parameters predicting the performance of a window in filter design are Main lobe, width Relative sidelobe level, Main lobe width Δ_{ML}- given by the distance between zero crossings on both sides of main lobe, Relative sidelobe level A_{sl} - given by the difference in dB between amplitudes of largest sidelobe and main lobe.

Design Steps for Windowed Low Pass FIR Filters:

(1) Choose a pass band edge frequency in Hz for the filter in the middle of the transition width.

$$f_c = (f_p + f_s)/2 \qquad (2.26)$$

(2) Calculate $\Omega_c = 2\pi f_c/f_T$ and substitute into $h_1[n]$, the infinite impulse response for an ideal low pass filter:

$$h_1[n] = \sin(n\Omega_c)/n\pi \qquad (2.27)$$

(3) Choose a window based on the specified, and calculate the number of nonzero window terms.

(4) Calculate FIR h[n] from $h[n] = h_1[n]w[n]$, notice the response is noncausal.

(5) Shift the impulse response to the right by (N-1)/2 to make the filter causal.

2.3.6 FIR Filter Design Based on Frequency Sampling Approach

The method of using window is an approximation in time-domain. The idea can be used in frequency-Domain.

Let

$$H(k) = H(e^{j\frac{2\pi}{N+1}k}) \tag{2.28}$$

Denote the sampling values of N+1-point FIR filter $H(e^{j\omega})$ designed.

We do

$$H(k) \xrightarrow{approximaiton} H_d(k) = H_d(e^{j\frac{2\pi}{N+1}k})$$

$$H(e^{j\omega}) \xrightarrow{approximaiton} H_d(e^{j\omega}) \tag{2.28}$$

The Design Steps of Frequency Sampling:

(1) From ideal filter, the values of $H(k)$ are computered (linear phase).

(2) From the IDFT of $H(k)$, $h[n]$ is derived.

3. Conclusion:

The design and realization of digital filters involve a blend of theory, applications, and technologies. For most applications, it is desirable to design frequency-selective filters which alter or pass unchanged different frequency components.
With recursive IIR filters, we can generally achieve a desired frequency response characteristic with a filter of lower order than for a non-recursive filter (especially if elliptic designs are used). A recursive filter has both poles and zeroes which can be selected by the designer, hence there are more free parameters than for a non-recursive filter of the same order (only zeroes can be varied).
FIR filters constitute a class of digital filters having a finite-length impulse response. An FIR filter can be realized using nonrecursive as well as recursive algorithms. However, the latter are not recommended due to potential stability problems while nonrecursive FIR filters are always stable.

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