Course Report

# Digital Signal Processing

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## Why the Frequency Spectrum is meaningful, Talk about some Engineering Applications of Signal Spectrum


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# 1. What is Frequency Spectrum

The frequency spectrum of a time-domain signal is a representation of that signal in the frequency domain. The frequency spectrum can be generated via a Fourier transform of the signal, and the resulting values are usually presented as amplitude and phase, both plotted versus frequency.

More specifically, we can term the Frequency spectrum to be mathematical analysis of signals with respect to frequency, rather than time. Put simply, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal.

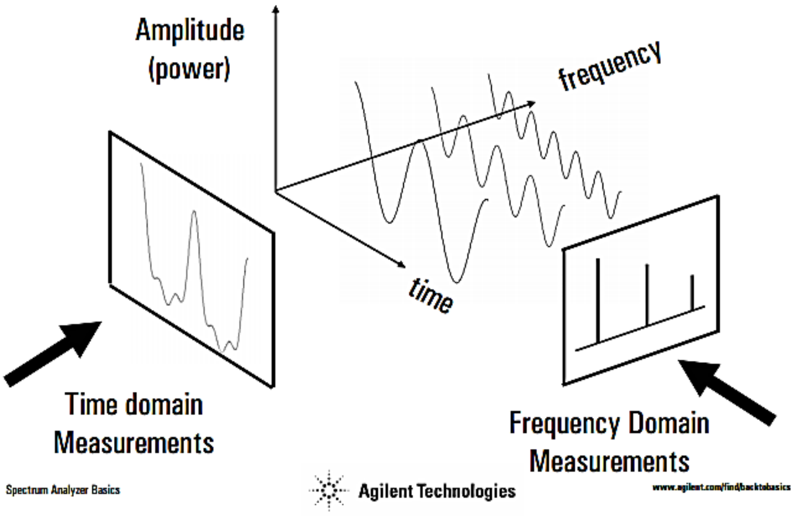


**Figure-1 : Illustration of Signals in Time and Frequency Domain. The time-domain signal depicted here, is the resultant signal from the addition of three Sinusoidal signals of various frequencies. The Frequency domain view can simply be thought of location of signals along frequency axis, and shows each Sinusoidal signal as a separate entity.**

# 2. Realization of Frequency Spectrum – Various Methodologies

***(FFT, DFT, Zoom-FFT, STFT, DWT etc)***

**2.1. Fourier Transformation.** The most basic and fundamental tool to transform signals from time domain to frequency domain, and vice versa is the Fourier Transform.

Fourier Transform is used to convert signals from time-domain to frequency domain, and is given by the equation :

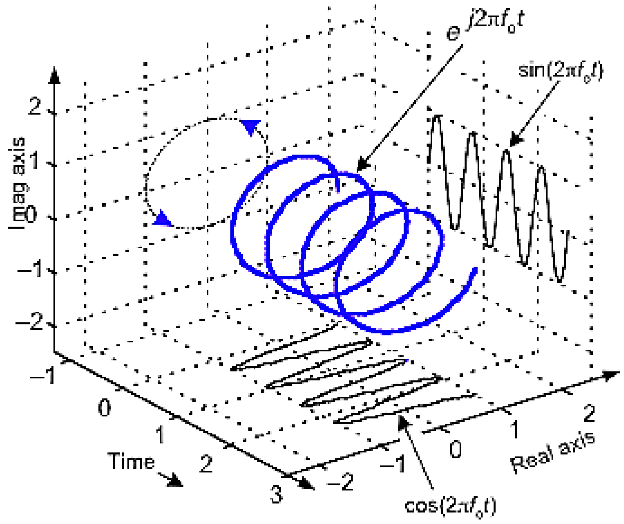


**Figure-2 : The $e^{-j\omega t}$ function in Fourier and Inverse-Fourier transformation is depicted here for clear understanding of real and imaginary parts.**

$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$ where $f(t)$ is the time- domain signal

Whereas, the inverse-fourier transform is used to convert signals from frequency domain to time domain realization, and is given the equation :

$f(t) = \int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$ where $F(j\omega)$ is the frequency domain signal

The function $e^{-j\omega t}$ is depicted in Figure-2, which is simple a complex combination of respective *Cos* and *Sin* factors.

**2.2. FFT, DTFT, FFT, STFT, Zoon-FFT and else.** The Fourier Transform is the very basic and key principle for signal conversion in frequency-time domains. The other variants like Discrete time Fourier Transform, Discrete Fourier Transform, Short Time Fourier Transform, Zoom-FFT etc are beyond the scope of this report, only DFT and DTFT transformation are illustrated here for information purposes:

- *The Fast Fourier transform (FFT)* is an algorithm to compute the discrete Fourier transform (DFT) and its inverse.

- *The Discrete-time Fourier Transform pair* :

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$ and $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

- *The Discrete Fourier Transform (DFT)* is given by

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le N-1 \qquad \text{and} \qquad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}, \quad 0 \le n \le N-1$$

*where* $W_N = e^{-j2\pi/N}$

## 2.3. Examples/Illustrations for various transforms.

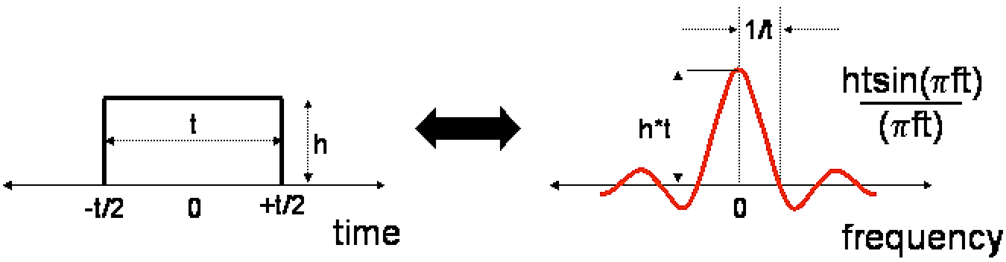


**Figure – 3 (a) : Fourier Transform of a time gated pulse signal results into a Sinc function in frequency domain, and vice versa.**

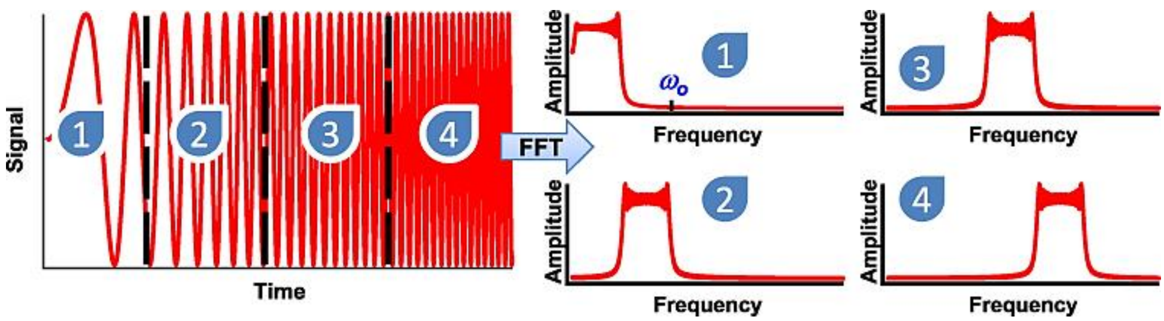


**Figure – 3 (b) : Short-Time-Fourier-Transform (STFT) can be thought of 'gated' versions of Fourier Transforms. In Figure, the signal is gated or chopped into 04 separate signals and FT is taken independently for each of the signals.**

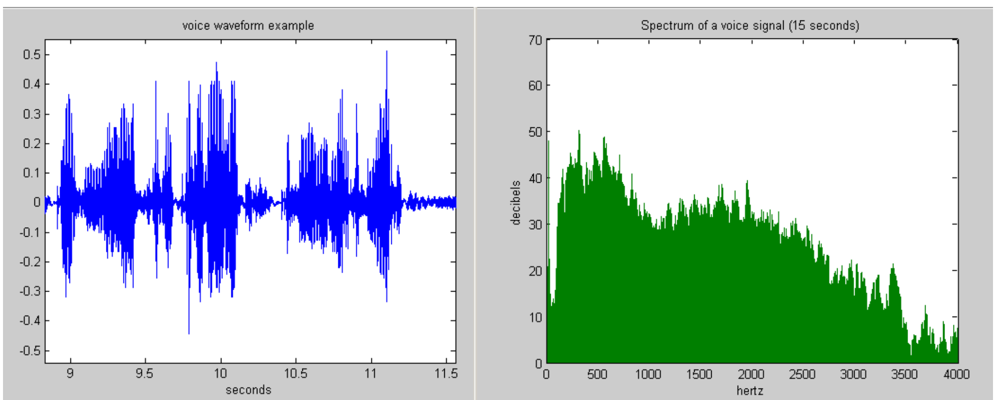


**Figure – 3 (c) : Example of voice waveform and its frequency spectrum**

# 3. Why Frequency Spectrum in Meaningful – Daily life and Engineering Examples

## 3.1. Spectral Allocation (ISM Band, Military Radar bands, etc)

The industrial, scientific and medical (ISM) radio bands are radio bands reserved internationally for the use of radio frequency (RF) energy for industrial, scientific and medical purposes other than telecommunications. Similarly, military has its own designation of bands. In air, the each and every frequency band is highly occupied and pre-dedicated. The spectral allocation is only possible because of Fourier tools.

| Frequency range | | Bandwidth | Center frequency | Availability |
|---|---|---|---|---|
| 6.765 MHz | 6.795 MHz | 30 kHz | 6.780 MHz | Subject to local acceptance |
| 13.553 MHz | 13.567 MHz | 14 kHz | 13.560 MHz | Worldwide |
| 26.957 MHz | 27.283 MHz | 326 kHz | 27.120 MHz | Worldwide |
| 40.660 MHz | 40.700 MHz | 40 kHz | 40.680 MHz | Worldwide |
| 433.050 MHz | 434.790 MHz | 1.74 MHz | 433.920 MHz | subject to local acceptance |
| 902.000 MHz | 928.000 MHz | 26 MHz | 915.000 MHz | subject to local acceptance |
| 2.400 GHz | 2.500 GHz | 100 MHz | 2.450 GHz | Worldwide |
| 5.725 GHz | 5.875 GHz | 150 MHz | 5.800 GHz | Worldwide |
| 24.000 GHz | 24.250 GHz | 250 MHz | 24.125 GHz | Worldwide |
| 61.000 GHz | 61.500 GHz | 500 MHz | 61.250 GHz | Subject to local acceptance |
| 122.000 GHz | 123.000 GHz | 1 GHz | 122.500 GHz | Subject to local acceptance |
| 244.000 GHz | 246.000 GHz | 2 GHz | 245.000 GHz | Subject to local acceptance |

**Figure-4 (a) : Example of Spectral Allocation – ISM Bands**

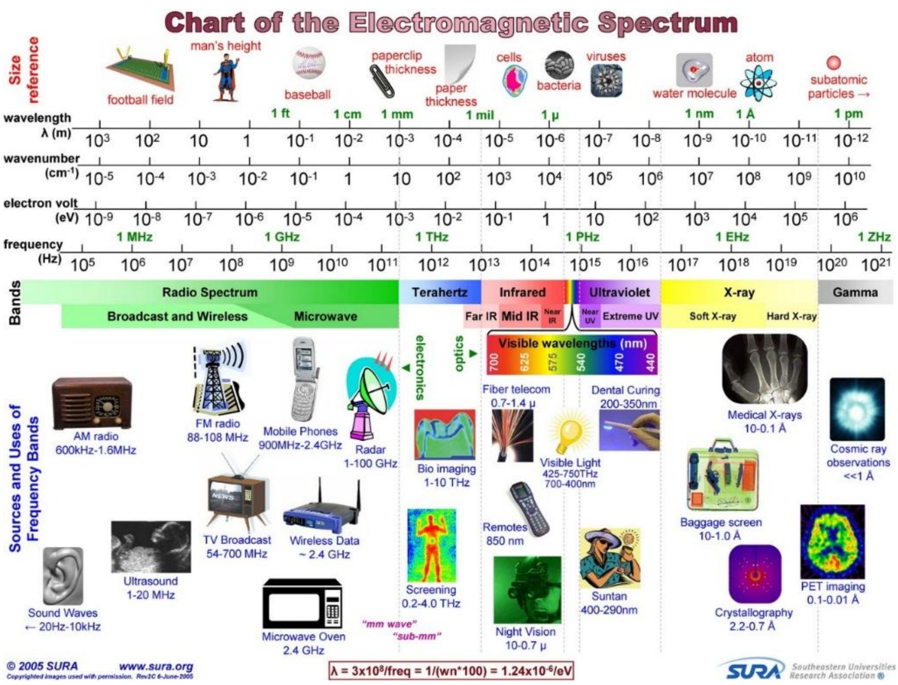


**Figure-4 (b) : In air, the spectrum is fully occupied. The proper allocated for each and every user is only possible through ensuring spectral integrity.**

## 3.2. The Spectrum is fully Packed – Analysis through Fourier Tools

The governmental agencies keep the spectrum allocation and regulate the use of spectrum. Therefore, each and every vendor has to reside within their own allocated spectrum which is only possible through Frequency Domain analysis techniques.


**Figure-5 (a) : Spectrum Analyzer is one of the most widely used Spectral Analysis tool to investigate the signal characteristics in the air.**

Apart from basic spectral view, one of the most promising technique to do the time-signal analysis, or analyze the waterfall spectral display, as shown in Figure below:

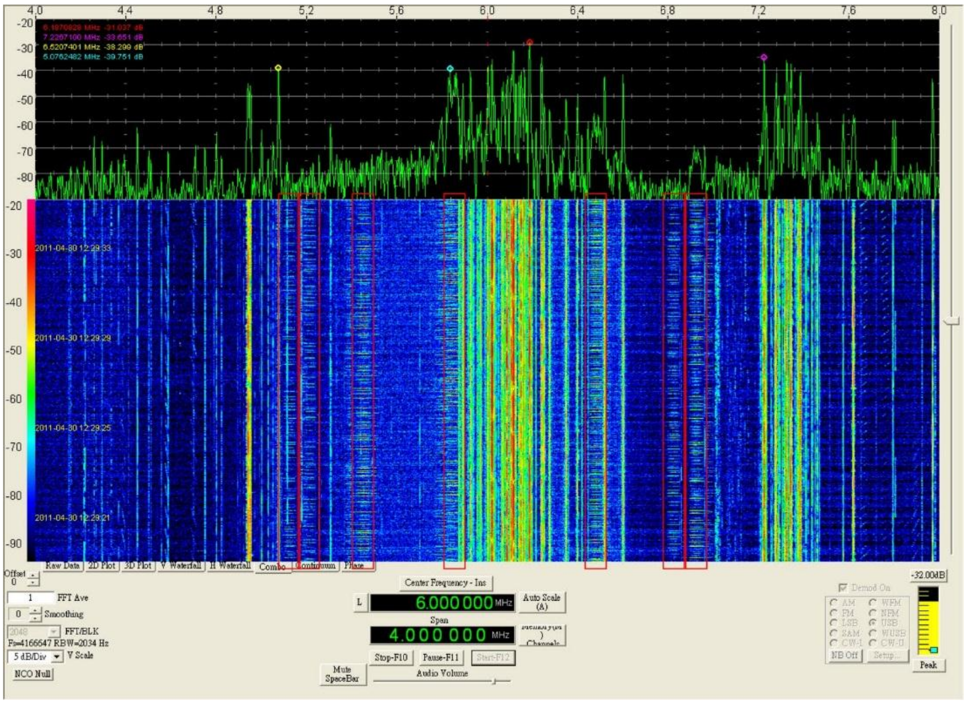


**Figure-5 (b) : Spectral Waterfall display can be used to infer the characteristics of time-varying signals along with spectral information**

## 3.3. Analysis of Signal Parameters in Communication and Telecommunications Systems

***(harmonics, IMDs, ICI, P1dB, IP3, Two-Tone, Channel Spectral Mask etc)***

Designers and engineers in Telecommunication and Communication systems need to adhere to the standardized limits for systems. In this reference, the parameters like Harmonics, Inter-modulation distortion, Inter-Carrier-Interference (like in OFDM communications), and analysis methods like 1-dB compression point P1dB, Third Point Intercept IP3, and Two-Tone tests etc are all performed in Frequency spectrum.

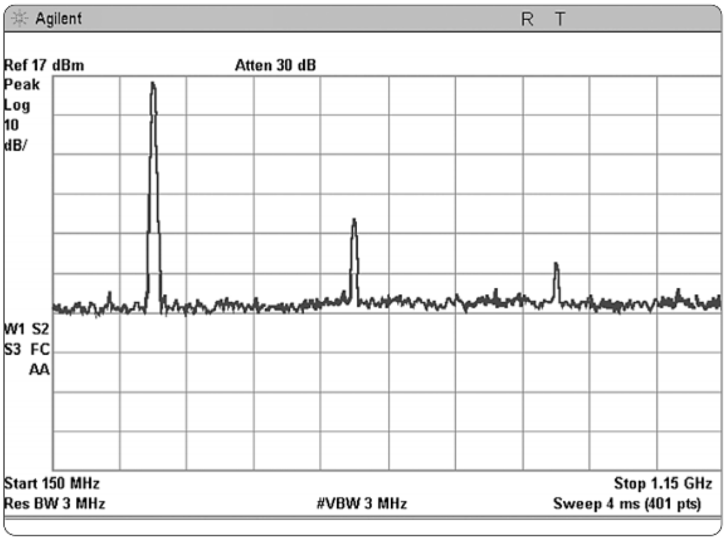


**Figure-6 (a) : Harmonic distortion test of a transmitter as analyzed on Frequency Spectrum**

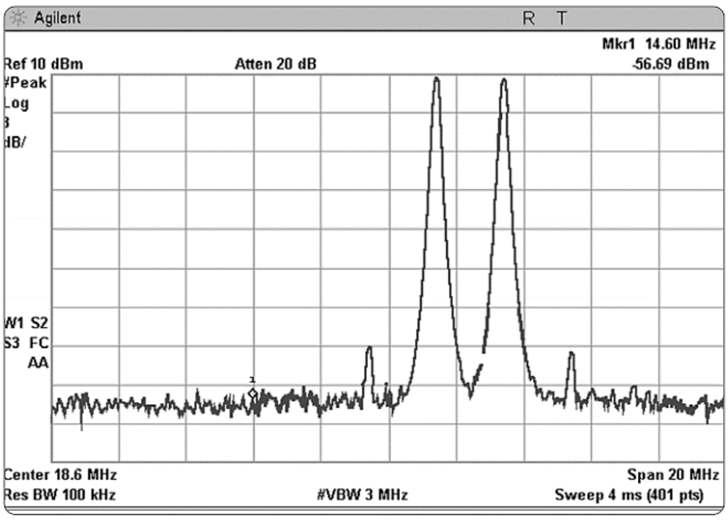


**Figure-6 (b) : Two-tone test for an RF power amplifier**

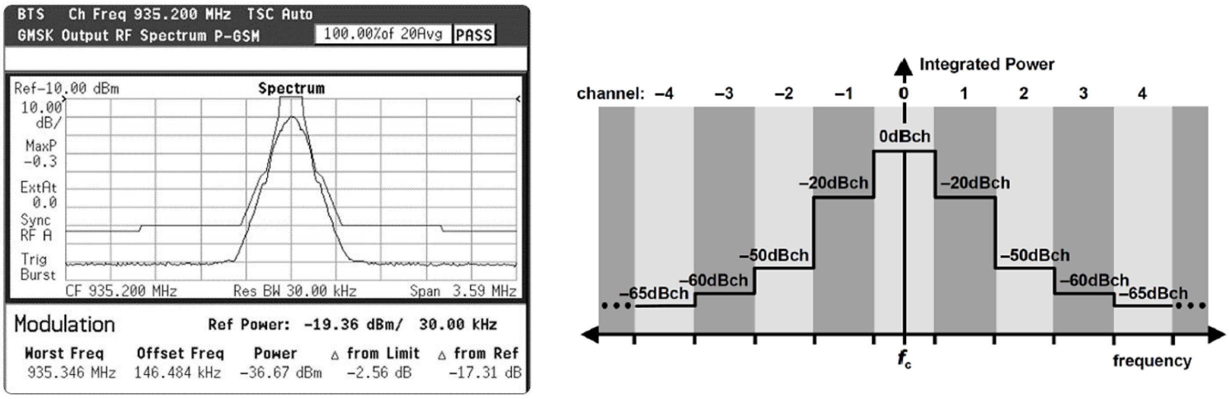


**Figure-6 (c) : Channel Spectral Mask is defined for each transmitter to limit the EIRP and interference to side channels**

## 3.4. FFT Analysis in Magnetic Resonance Imaging

In Magnetic Resonance Imaging (MRI), an image of a cross-section of tissue can be made by producing a well-calibrated magnetic field gradient across the tissue so that a certain value of magnetic field can be associated with a given location in the tissue. Since the signal frequency is proportional to that magnetic field, a given signal frequency can be assigned to a location in the tissue. The MRI signals are detected as a function of time, and then converted to signal strength as a function of frequency by means of Fourier transformation.

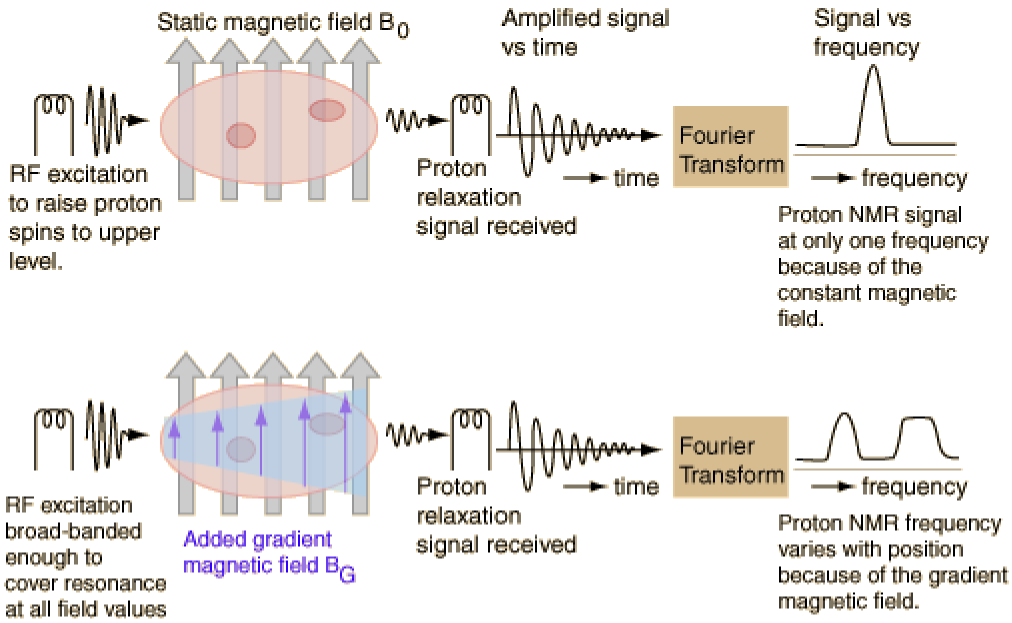


**Figure-7 : The MRI signals are detected as a function of time, and then converted to signal strength as a function of frequency by means of Fourier transformation.**

### 3.5. Microsoft Research : Use of FFT to distinguish movement of hands via Doppler processing

The software, called SoundWave, emits an inaudible sound from your laptop's speakers. Around 20kHz, it is high-pitched enough that most people can't hear it - but the laptop's microphone can. When a person moves his hand toward and away from the laptop, the sound bounces off the hand and changes pitch. That's the Doppler effect in action. The system utilizes FFT to analyze the Doppler in the frequency as shown in figure below:

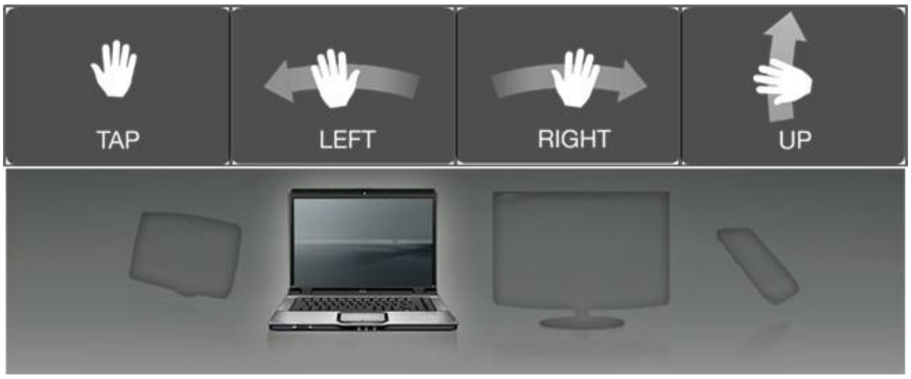


**Figure-8 (a) : The SoundWave is intended to control digital interactive media by use of gestures and movements through Doppler effect which is analyzed through spectral analysis techniques (FFT).**

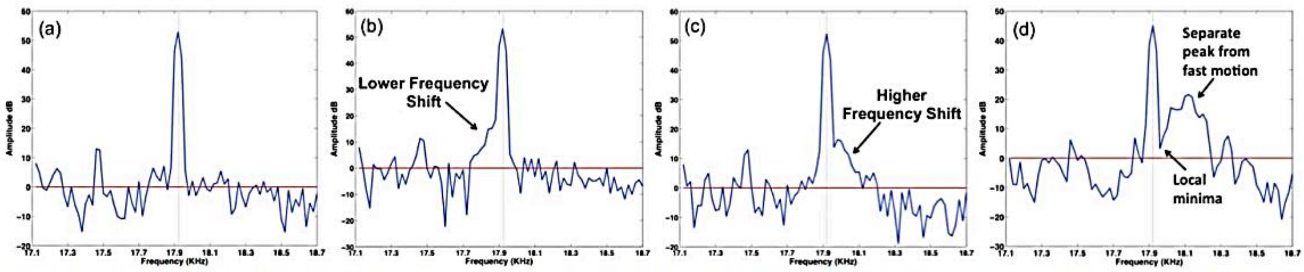


**Figure 2: (a) Pilot tone with no motion. (b and c) Increase in bandwidth on left and right due to motion away from and towards the laptop respectively. (d) Shift in frequency large enough for a separate peak. A single scan would not capture the true shift in frequency and would terminate at the local minima. A second scan compensates for the bandwidth of the shifted peak.**

**Figure-8 (b) : The Figure illustrates how a slight movement of hands shift the spectral portions of the original wave (Doppler effect)**

### 3.6. Astronomy

The RF signals from Radio/Radar-telescopes use the radio waves or radar instead of light. These radar signals are treated just like other ordinary time varying voltage signal and are be processed digitally by Fourier tools just like signals.

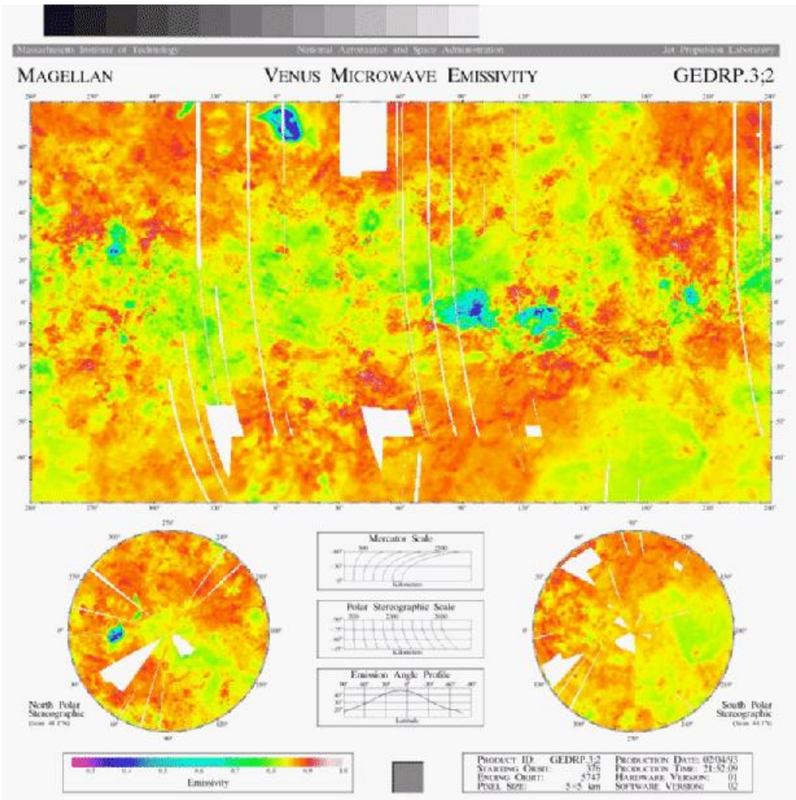


**Figure-10 : Global map of Venus showing emissivity of the Venus's surface from Magellan Satellite Radar, processed by FFT.**

## 3.7. Geological Engineering

For mapping the layers of earth, seismic energy is downward propagated into the earth. The reflected seismic waves are recorded and processed by FFT to produce the analysis results and images.

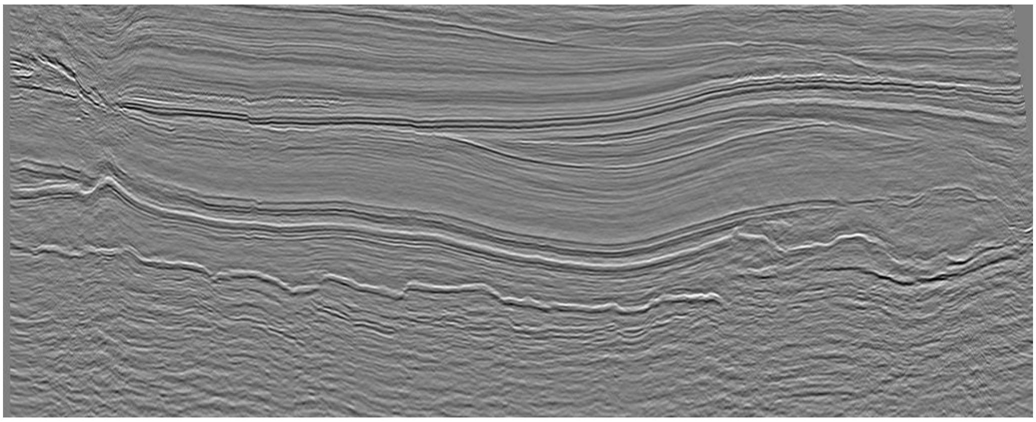

**Figure-11 : The ground penetrating waves are incident on specific area. These waves travel down the layers of soil and resonate at specific layers or materials. The echoes from these waves are processed by FFT.**

### 3.8. Image Compression using Spectral Techniques

The impetus of Frequency Spectrum in not only limited to circuit analysis, signals and systems, rather, it is one of the most valueable and powerful tools for image analysis, especially the compression. The Discrete Wavelet Transform is used in JPEG-2000 compression.

**Figure-12 : An FBI-digitized fingerprint using DWT. Original Image (left) and Same Image at 26:1 compression (Right)**

### 3.9. Computational Complexity of FFT

The following table depicts that how the computational complexity of a boolean expression for complex numbers (Length = N) can be reduced down using FFT tools.

| Number of Points | Direct Computation of the DFT | | Radix-2 FFT | |
|---|---|---|---|---|
| | Complex Multiplies | Complex Additions | Complex Multiplies | Complex Additions |
| $N$ | $N^2$ | $N^2-N$ | $(N/2)\log_2 N$ | $N\log_2 N$ |
| 4 | 16 | 12 | 4 | 8 |
| 16 | 256 | 240 | 32 | 64 |
| 64 | 4096 | 4032 | 192 | 384 |
| 256 | 65536 | 65280 | 1024 | 2048 |
| 1024 | 1048576 | 1047552 | 5120 | 10240 |

**Figure-9 : The computational complexity, especially of complex numbers, in Boolean algebra is reduced manifolds using FFT. The above table illustrates the difference between computational cycles of same numbers using DFT and FFT.**

## 4. Conclusion

In present day technological and wireless world, the frequency spectral tools and methods are bolts and nuts of each and every system, ranging from house-hold microwave to deep-space satellites. In this short report, the main focus has been laid to depict the important and applications of frequency spectral tools not only in engineering and science, but, in every daily life.

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