DIGITAL FILTER DESIGN

DIGITAL SIGNAL PROCESSING REPORT

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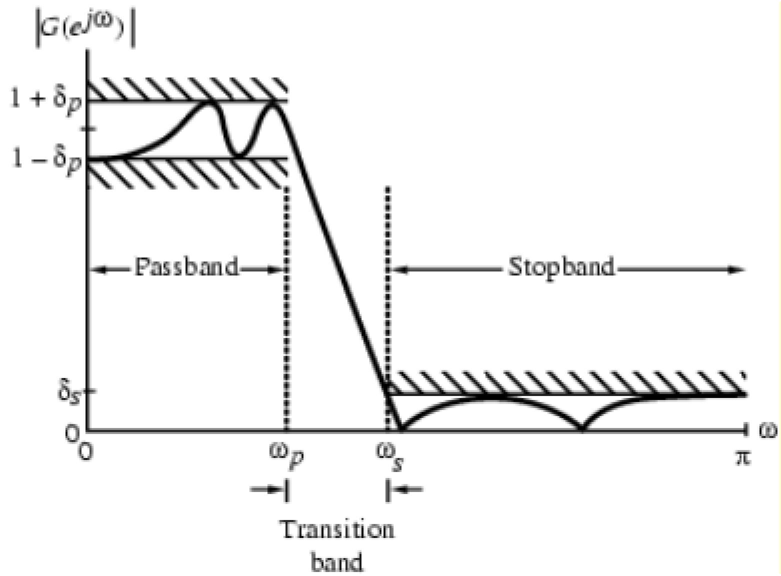
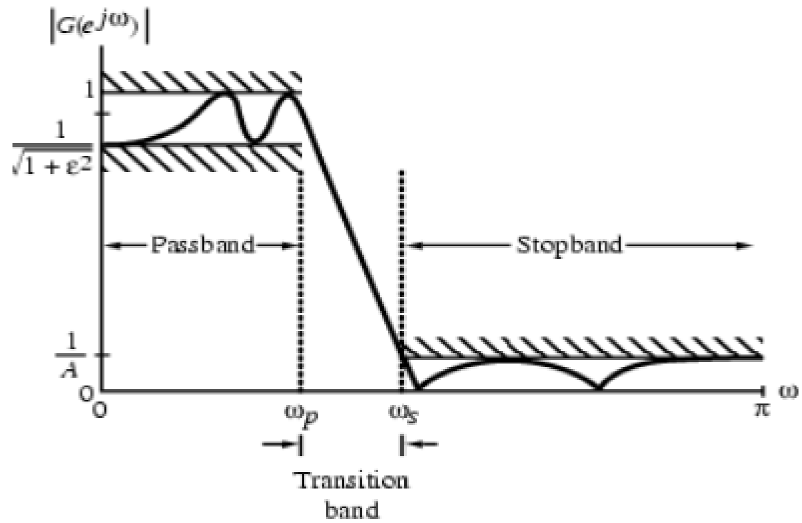
FILTER SPECIFICATIONS



- **Pass band Edge Frequency:** The pass band is the band of frequency components that are allowed to pass. As indicated in the figure above, the pass band is defined by **0**$\leq \omega \leq \omega_p$. The frequency ω_p is the pass band edge frequency.
- **Stop band edge frequency:** The stop band is the band of frequency components that are suppressed, defined by $\omega_s \leq \omega \leq$ **π.** The frequency ω_s is the stop band edge frequency.
- **Pass Band Ripple** and **Minimum Stop Band Attenuation:** The limit of tolerances in the pass band and stop band, $\boldsymbol{\delta_p}$ and $\boldsymbol{\delta s}$ are called ripples. These are specified in dB in terms of the peak pass band ripple $\boldsymbol{\alpha_p}$ and the minimum stop band attenuation $\boldsymbol{\alpha_s}$.

  $\alpha_p = -20\log_{10}(1-\delta_p)$ dB

α_s=-20log$_{10}$(δ_s) dB



The magnitude response specifications may also be given in normalized form

Here, the pass band ripple is denoted as $\frac{1}{\sqrt{1+\varepsilon^2}}$ and the maximum stop band ripple is denoted by 1/A

INFINITE IMPULSE RESPONSE DIGITAL FILTER DESIGN

There are two methods of designing an Infinite Impulse response Digital Filter. They are Impulse Invariance method and bilinear transformation.

Impulse Invariance Method

The common approach in designing an IIR filter using impulse invariance method is:

1) Convert the digital filter specifications into an analog prototype low pass filter specification. $\Omega_p = \frac{\omega_p}{T}$ $\quad \Omega_s = \frac{\omega_s}{T}$
2) The next step is to determine the order of the low pass filter. The transfer function of the low pass filter is given as $|Ha(j\Omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$ Where N= order of the filter. The appropriate formula for finding N is used, depending on whether a Butterworth or a Chebyshev filter approximation is required. Eg: if a Butterworth filter is required N= $\frac{1}{2}\frac{log_{10}[(A^2-1)/\varepsilon^2]}{log_{10}(\frac{\Omega_s}{\Omega_p})} = \frac{log_{10}(\frac{1}{k_1})}{log_{10}(\frac{1}{k})}$

From $10log_{10}\left(\frac{1}{1+\varepsilon^2}\right) = peak\ passband\ ripple$ and $10log_{10}\left(\frac{1}{A^2}\right) =$ *minimum stop band attenuation*, A and ε can be determined. For reduced computational complexity, the filter order should be the smallest integer greater than or equal to the estimated value. If N=3, then the order of the Butterworth filter is $H_{an}(s) = \frac{1}{(s+1)(s^2+s+1)}$. $H_{an}(s)$ is the normalized transfer function but we have a specific Ω_c. We then use the transformation $H_a(s) = H_{an}(\frac{s}{\Omega_c})$.

3) The next step is to transform $H_a(s)$ from its analog state to a digital transfer function G(z), by first performing a partial fraction expansion of $H_a(s)$. $H_a(s)= \sum_{i=1}^{N} \frac{A_i}{s-s_i}$ and then it can transformed into a digital filter using the relation, $G(z)= \sum_{i=1}^{N} \frac{A_i T}{1-e^{s_i T} z^{-1}}$, which uses the mapping rule $z=e^{st}$

Bilinear Transformation

The Bilinear Transformation method is similar to the impulse invariance method. The steps involved in designing a digital filter using the bilinear transformation method is as follows:

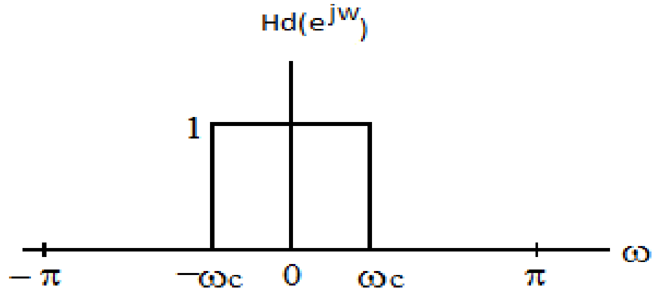
1) Convert the digital filter specifications into an analog prototype low pass filter specification. $\Omega_p = tan\frac{\omega_p}{2}$ $\Omega_s = tan\frac{\omega_s}{2}$
2) The next step is to determine the order of the low pass filter. The transfer function of the low pass filter is given as $|Ha(j\Omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$ Where N= order of the filter. The appropriate formula for finding N is used, depending on whether a Butterworth or a Chebyshev filter approximation is required. Eg: if a Butterworth filter is required $N= \frac{1}{2}\frac{log_{10}[(A^2-1)/\varepsilon^2]}{log_{10}(\frac{\Omega_s}{\Omega_p})} = \frac{log_{10}(\frac{1}{k_1})}{log_{10}(\frac{1}{k})}$. The order of the low pass filter using the bilinear transformation and the impulse invariance method are different. For impulse invariance N may be 2.5 and for bilinear transformation $\bar{N}$ may be 2.66. But because we choose the greatest integer greater than or equal to the estimated value, the order will be 3. Ω_c is also determined using $|Ha(j\Omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}} = \left(\frac{1}{1+\varepsilon^2}\right)$. . If N=3, then the order of the

Butterworth filter is $H_{an}(s) = \frac{1}{(s+1)(s^2+s+1)}$. $H_{an}(s)$ is the normalized transfer function but we have a specific Ω_c. We then use the transformation $H_a(s) = H_{an}(\frac{s}{\Omega_c})$.

3) The next step is to apply the bilinear transformation to Ha(s) to get the desired digital transfer function. $G(z) = Ha(s)|_{\frac{1-z^{-1}}{1+z^{-1}}}$

FINITE IMPULSE RESPONSE DIGITAL FILTER DESIGN (FIR DIGITAL FILTER DESIGN)

The FIR digital filter design is independent of analog filter design.



$H_d(e^{j\omega})$is piecewise constant with sharp transitions between the bands. It therefore means $H_d[n]$is of infinite length and non-causal. The objective is to find a finite duration $H_t[n]$ of length 2M+1, whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$. This can be done in both frequency and time domain. I want to do this in the time domain.

Because we approximating $H_t(e^{j\omega})$ to be like $H_d(e^{j\omega})$ there will be an error term defined as

$$\Delta(e^{j\omega}) = \mathrm{Ht}(e^{j\omega}) - \mathrm{Hd}(e^{j\omega})$$

$\Delta(e^{j\omega})$ should be as small as possible. If $\Delta(e^{j\omega})$ is zero, then it means the designed filter is the desired filter. There are two ways of evaluating $\Delta(e^{j\omega})$. We can check the peak of $|\Delta(e^{j\omega})|$. If the peak is small then it means all other points are also small. $\underset{\omega}{max}|\Delta(e^{j\omega})| \rightarrow min$. We can also use average evaluation. $\frac{1}{2\pi}\int_0^{2\pi} |\Delta(e^{j\omega})|^2 d\omega \rightarrow min$. The former is for the worst case scenario. The latter includes all points but is more difficult to calculate mathematically.

We need to choose $\mathrm{Hd}(e^{j\omega})$ to minimize $\Delta(e^{j\omega})$. To minimize $\Delta(e^{j\omega})$ is equivalent to minimizing $\frac{1}{2\pi}\int_0^{2\pi} |\mathrm{Ht}(e^{j\omega}) - \mathrm{Hd}(e^{j\omega})|^2 d\omega$. Because I stated earlier that I will be using time domain analysis, using parseval's relation this integral sum then becomes $\sum_{n=-\infty}^{\infty} |ht[n] - hd[n]|^2 \rightarrow min$. This summation can be separated into three parts, thus

$$\sum_{n=-\infty}^{-M-1} |hd[n]|^2 + \sum_{n=-M}^{M} |ht[n] - hd[n]|^2 + \sum_{n=M+1}^{\infty} |hd[n]|^2$$

The first and the last term cancel out because hd[n] is known. The parseval minimum value of $\sum_{n=0}^{N-1} |ht[n] - hd[n]|^2$ is zero if ht[n]=hd[n] for -M≤n≤M. That is the optimal solution. We then truncate to finite length. (This produces unwanted ripples increasing in height near discontinuity.). It is then modified to ht[n]=hd[n]*W_R[n]. This method is called windowing. It is just the multiplication of hd[n] and the window function. It is called the rectangular window.

$W_R[n]=\begin{cases} 1, 0 \le |n| \le M \\ 0, \ \ otherwise \end{cases}$

Another requirement is needed before we can start before the design. The phase must be linear. A causal FIR filter with an impulse response h[n] can be derived from h_t[n] by delaying. h[n]=ht[n-M].

With this, the design of the Finite Impulse Response Filter can start. The steps are outlined below:

1) A pass band edge frequency in Hz is chosen. This pass band edge frequency should be midway of the transition width. Thus, $f_c = (f_p + f_s)/2$
2) Using this pass band edge frequency, the impulse response h[n] for the ideal low pass filter can be found. This is done by calculating $\Omega_c=2\pi f_c/f_T$ and then substituting it into the equation $h_L[n] = \sin(n\Omega_c)/n\pi$.
3) Using the stop band attenuation, choose a window that satisfies the minimum stop band attenuation and determine the number of terms *N* needed to give the required transition width. An odd number should be chosen, so that the impulse response will be perfectly symmetrical around its middle point. If more than one window satisfies our stop band attenuation, we need to choose the window that will give the lowest order.
4) Calculate the finite impulse response h[n] from h[n] = h_L[n]w[n] for the filter. Its response at this stage is non causal. It should be made causal.
5) Shift the impulse response to the right by (N-1)/2 samples to make the filter causal.