Filter Specifications And Design A Digital Filter Based On The Given Specifications

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Introduction

Digital filters are a very important part of DSP. In fact, their extraordinary performance is one of the key reasons that DSP has become so popular. As mentioned in the introduction, filters have two uses: signal separation and signal restoration. Signal separation is needed when a signal has been contaminated with interference, noise, or other signals. For example, imagine a device for measuring the electrical activity of a baby's heart (EKG) while still in the womb. The raw signal will likely be corrupted by the breathing and heartbeat of the mother. A filter might be used to separate these signals so that they can be individually analyzed.

Signal restoration is used when a signal has been distorted in some way. For example, an audio recording made with poor equipment may be filtered to better represent the sound as it actually occurred. Another example is the deblurring of an image acquired with an improperly focused lens, or a shaky camera.

Digital filter design specification

$$\frac{\mathrm{a0} + \mathrm{a1z}^{-1} + \mathrm{a2z}^{-2}}{1 + \mathrm{b1z}^{-1} + \mathrm{b2z}^{-2}}$$

$\mathrm{a0}, \mathrm{a1z}^{-1}, \mathrm{a2z}^{-2}, \mathrm{b1z}^{-1}, \mathrm{b2z}^{-2}$ defines that ehether filter is low pass filter , band pass mfilter, high pass filter and band stop filter.

In this case it is IIR filter. $\mathrm{a0} + \mathrm{a1z}^{-1} + \mathrm{a2z}^{-2}$ is IIR filter and $1 + \mathrm{b1z}^{-1} + \mathrm{b2z}^{-2}$ is FIR filter.

To design a filter first what should we choose either IIR filter or FIR FILTER. To choose the filter we should first know the performance specification of filter.

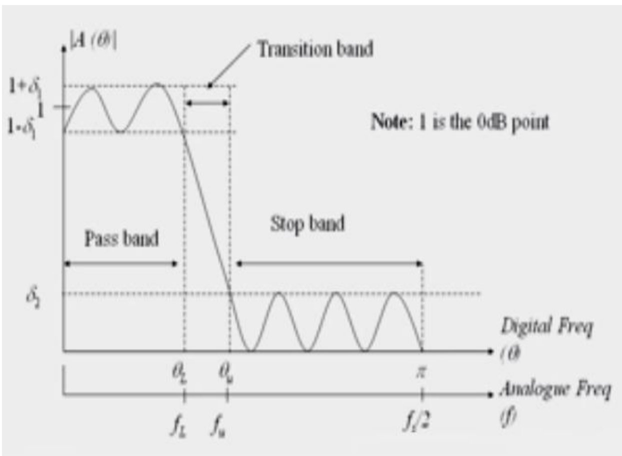


Fig. Typical amplitude characteristics of low- pass filter.

Here digital frequency π corresponds to half of sampling $\frac{fs}{2}$ frequency of analog frequency. This filter consists of pass band, transition band and stop band. If we want to make a filter very sharp is very difficult then the order of filter replaces. For designing the filter we should make sure that that pass band should be straight which is not easy. It Butterworth filter we have flat but has normally mall ripples. Here in fig.1, the 1 show the straight line and anything up and down of this are denoted by 1+ δ and 1- δ respectively. From 1 to $\delta 2$ may be -60dB, - 20dB depends upon suppression. And all ripples should be below the stop band. For designing filter we must know the lower frequency fL and fU upper frequency, sampling frequency $\frac{fs}{2}$, $\delta 2$ and ripples above and below the 1. For 0.01db of ripples means lower order filter with high ripples peaks, 0.001 means we increase the filter order and hence peaks of ripples valued less. Same case for $\delta 2$. If $\delta 2 = -20$dB of order 5th then for $\delta 2 = -60$ dB then it could be 20th order. 1 is 0dbB point. The goal of the design is to determine a transfer function H(z) so that its amplitude characteristics (Magnitude response)$|H(\theta)|$ satisfies the condition.

$$\delta 1 \leq |H(\theta)| \leq 1+\delta 1 \quad \text{for} \quad 0< \theta< \theta L \qquad \therefore \theta L \text{ is for pass band}$$

Similarly for stop band

$$|H(\theta)| \leq \delta 1 \quad \text{for} \quad \theta v< \theta<\pi$$

Choosing between FIR and IIR filter

FIR Filters	IIR Filters
System function contain only zeros	Contain poles and zeros (normally)
Non-recursive (no feedback)or recursive (feedback) structures are possible; the best known is non recursive (Transversal) structure.	Only recursive structure is possible; the most widely used form is the cascade connection of first-order and second order section.
FIR filters can have an exactly linear phase response. The implication of this is that no phase distortion is introduced into the signal by the filter.	The phase responses of IIR filters are nonlinear, especially at the band edges.
The effects of using a limited number of bits to implement filters such as round to noise and quantization errors are much less severe in FIR than IIR	Because of quantization of filter coefficients, a pole can in principle move from a position inside the unit circle to a position outside the unit circle and hence cause instability.
FIR filters requires more coefficients for sharp cut-off filters than IIR. Thus for a given amplitude response specification, more processing time and storage will be requires for FIR implementation	IIR requires fewer coefficients for sharp cut off filters than FIR.
Complexity is proportional to the length of impulse response.	No direct relation between the complexity and length of impulse response (which is infinite by definition) Filters with high selectivity can realized with relatively low complexity.
FIR filters have no analogue counterpart. FIR design procedures are normally iterative procedures. Design equation do not exist	Analog filters cam be readily transfored into equivalent IIR digital meeting similar specifications. IIR filters can be designed using design formula.

Design Techniques.

The method used to calculate the filter coefficients (hk for FIR, ak and bk for IIR) depends on whether the filter is IIR or FIR type. There are several method for calculating filter coefficients of which following are widely used.

FIR digital filters	**IIR digital filters**
windows	Impulse invariant transformation
Frequency sampling	Bilinear transformation
Optimization method(e.g Remez Algorithm)	Pole-zero placement method
$\frac{Y(z)}{X(z)} = h0 + h1z^{-1} + h2z^{-2} + \cdots + h_{N-1}\, z^{-(N-1)}$	$\frac{Y(z)}{X(z)} = \frac{a0 + a1z^{-1} + \cdots + an\, z^{-(N-1)}}{1 + b1z^{-1} + b2z^{-2} + \cdots + bn}$

IIR digital filters

- **Impulse invariant transformation**

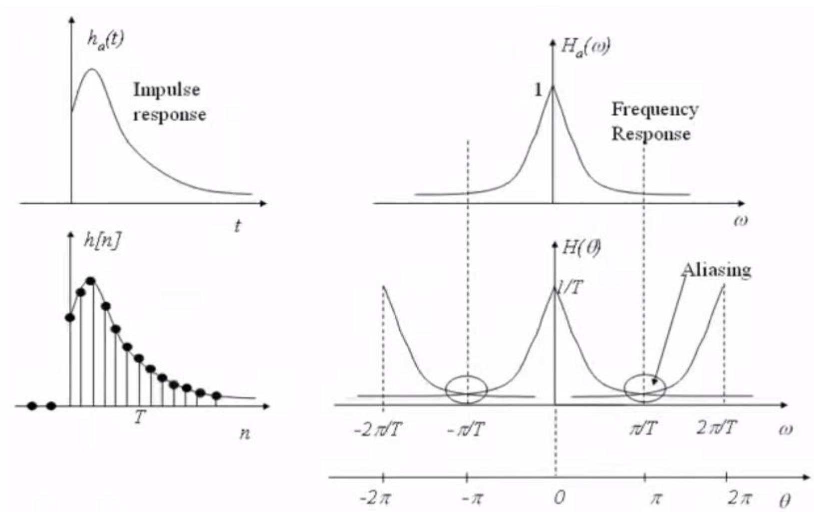
In most cases, if the FIR properties are vital then a good candidate is the optimization method, whereas, if IIR properties are desirable. Then the bilinear method will in most cases suffice. In transforming analogue filter to digital filter, we must obtain either H(z) or h[n] from the analogue filter design. In such transformation, we generally require the essential properties of analogue frequency response of resulting digital filter. This implies that we want imaginary axis of the s-plane to map into the circle of z-plane.

A second condition is that a stable analogue filter should be transformed to a stable digital filter. That is if the analogue system has two poles only in the left half s-plane, then the digital filter must have poles inside the unit circle.

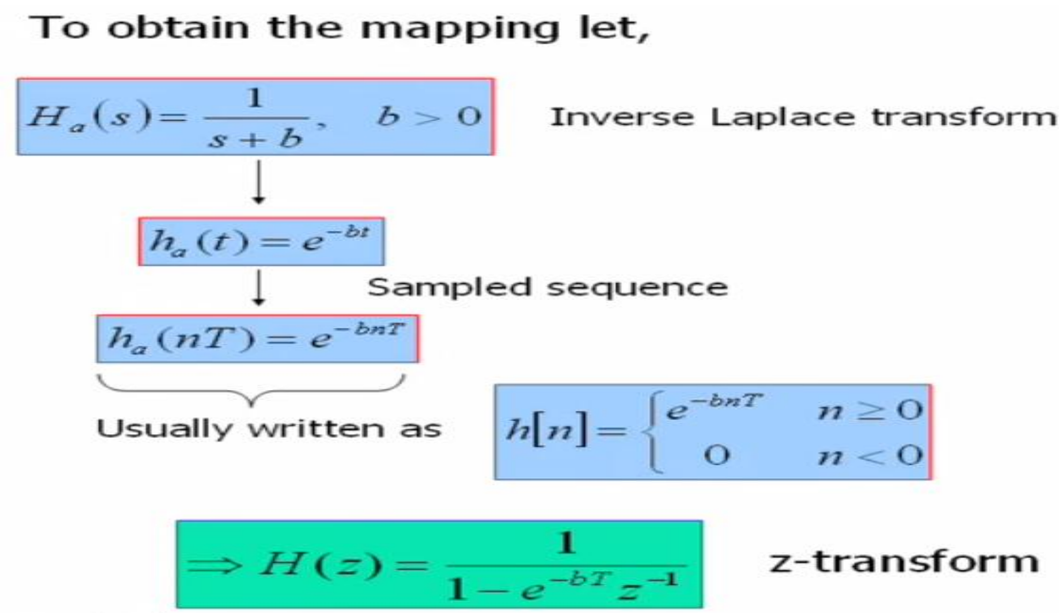
- In this method we start from an analogue filter of impulse response ha(t) and the system function. Then take the Laplace inverse transform.
- The objective of the design is realize an IIR filter with an impulse response h[n] which satisfies,

h[n]=h_a(nT) where t=nT ∴ T is sampling frequency

- The characteristics property preserved by this transformation is that the impulse response of resulting digital filter is a sampled version of the analogue filter shown in fig below



- The sampling frequency affects the frequency of impulse invariant discrete filter. A sufficient high sampling frequency is necessary for the frequency response to be close that of the equivalent analogue filter.
- Thus due to aliasing, the frequency response of digital filter will not be identical to that of analogue filter.



Actually we want to get $H(z) = \frac{1}{1-az^{-1}}$ where $a = e^{-bt}$.It is seen that H(z) is obtained from analogue filter function Ha(s) by using the mapping relationship. That is impulse invariant transformation which is exactly mapping the analogue filter response to the digital filter response. And the frequency response also must be same but it is not exactly same because of aliasing. If we want to impulse response to be matched we used impulse invariant transformation.

For example

$$\frac{1}{s+b} \rightarrow \frac{1}{1-e^{-bT}z^{-1}}$$ b>0 T-sampling period

Here we transfer only poles not zeros, in analogue domain into digital domain by this transformation technique.

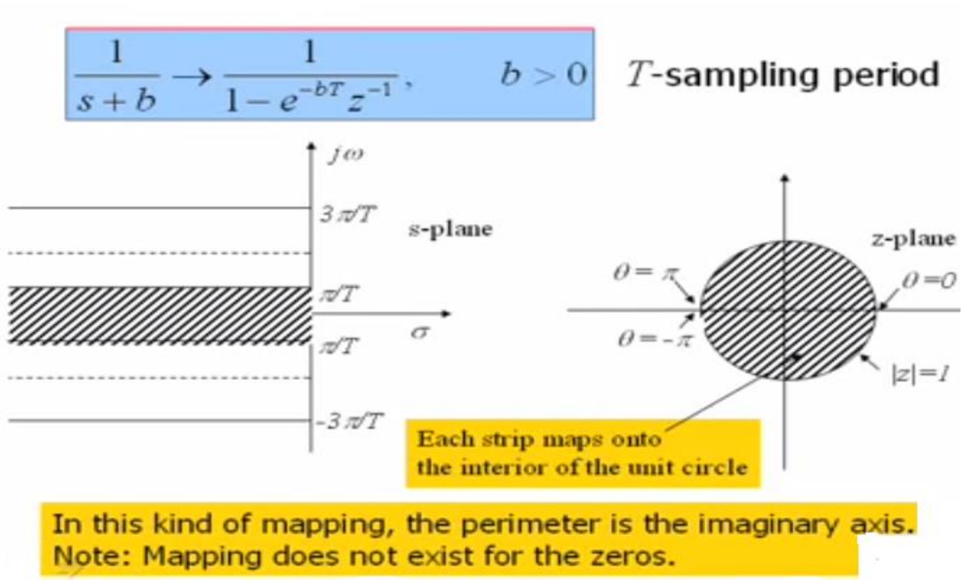


Figure shows that everything in the region π/T and – π/T in s-plane is mapped to unit circle of z- plane. 3π/T and $-\pi$/T also mapped if the spectrum is repeated.

Sampling frequency affects the frequency response of digital filter obtained using impulse invariant transformation. A sufficient frequency is necessary for the frequency to be closer to that of the equivalent analogue filter. Further low degree of aliasing can be achieved by making the sampling frequency high.

- **Bilinear Transformation**

The bilinear transformation yields stable digital filters from stable analogue filters. Also the bilinear transformation avoids the problems of aliasing encountered with the use of impulse invariant transformation, because it maps the entire imaginary axis in the s-plane on the unit circle in the z-plane.

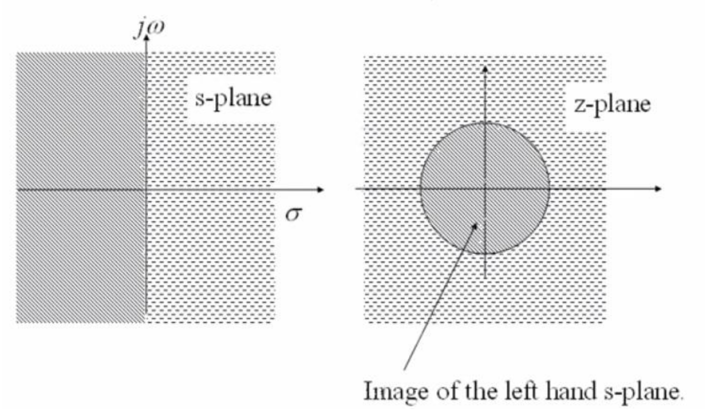


Fig. s-plane and z-plane mapping

Figure shows that anything in right side (stable) of s-plane should be mapped inside the unit circle (stable) and anything outside from the circle should be mapped outside the unit circle. In impulse invariant transformation, if spectrum repeats then due to aliasing everything mapped to unit circle. But in bilinear transformation there is no aliasing. The price paid for the avoidance of aliasing is a distortion in the frequency axis. Consequently, the design of digital filters using the bilinear transformation is only useful when the distortion can be compensated.

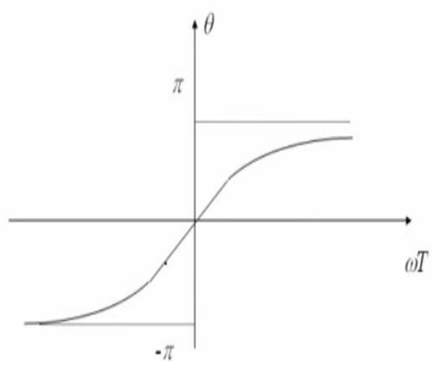


Fig. θ and ωT has nonlinear relationship

From above figure it is clear that there is nonlinear relationship between ω and θ. This effect is called "wrapping". The great advantage of wrapping is that no aliasing of the frequency characteristics can occur in the transformation of an analogue filter to discrete filter. We must however check carefully just how the various characteristics frequencies of the continuous characteristics frequencies of discreet filter, illustrated below in figure. The effect of "wrapping" in the conversion of $|H(\omega)| \rightarrow |H(\theta)|$. In designing digital filter by this method we must **pre-wrap** the given filter specifications to find the continuous filter.

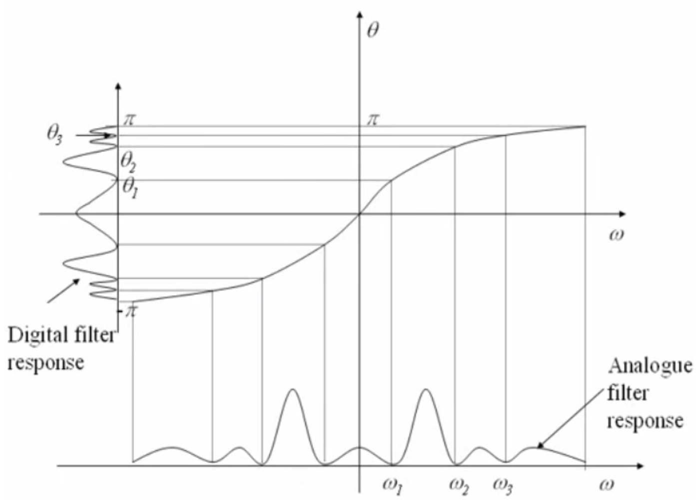


Fig Analogue and Digital Filter response

A set of transformations can be formed that take a low-pass digital filter and turn it into high pass. Band pass and band stop or another low pass digital filter.

- **Digital to digital transformation**

We can obtain low pass band pass and band stop filters by selecting the appropriate transformation. Similarly A set of transformations can be formed that take a low-pass digital filter and turn it into high pass. Band pass and band stop or another low pass digital filter. Transformations are given below

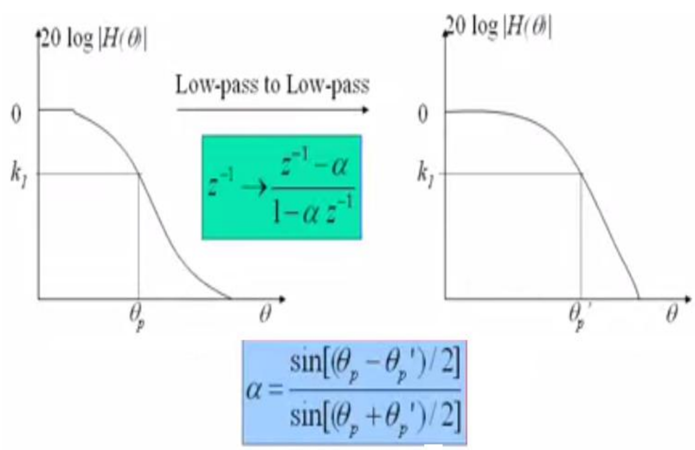
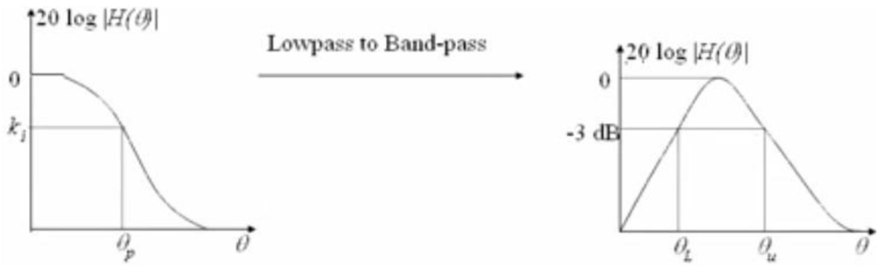


Fig. Low-pass to Low-pass transformation

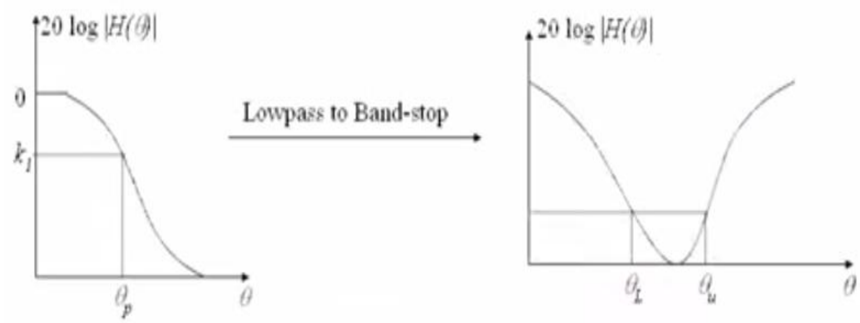


$$z^{-1} \rightarrow -\frac{z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1}$$

$$\alpha = \frac{\cos[(\theta_u + \theta_L)/2]}{\cos[(\theta_u - \theta_L)/2]}$$

$$k = \cot((\theta_u - \theta_L)/2) \cdot \tan\left(\frac{\theta_p}{2}\right)$$

Fig. Low-pass to Band-pass transformation



$$z^{-1} = \frac{z^{-2} - \frac{2\alpha}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha}{k+1} z^{-1} + 1}$$

Fig. Low-pass to Low-stop transformation

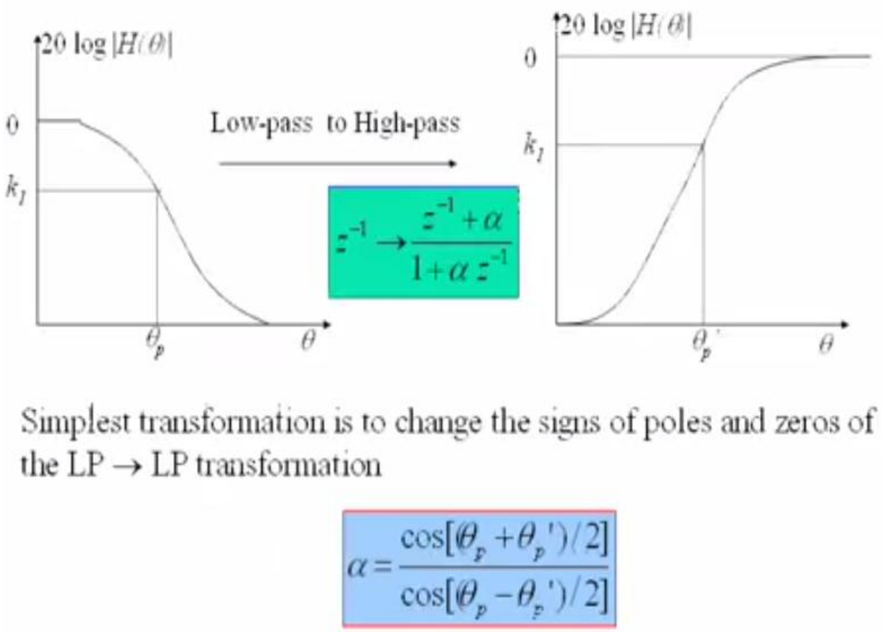


Fig. Low-pass to High-pass transformation

FIR filter design

To know who to know FIR filter design must know the windowing function. Basic function of windowing is selection. There are different types of windows which are given below in figure

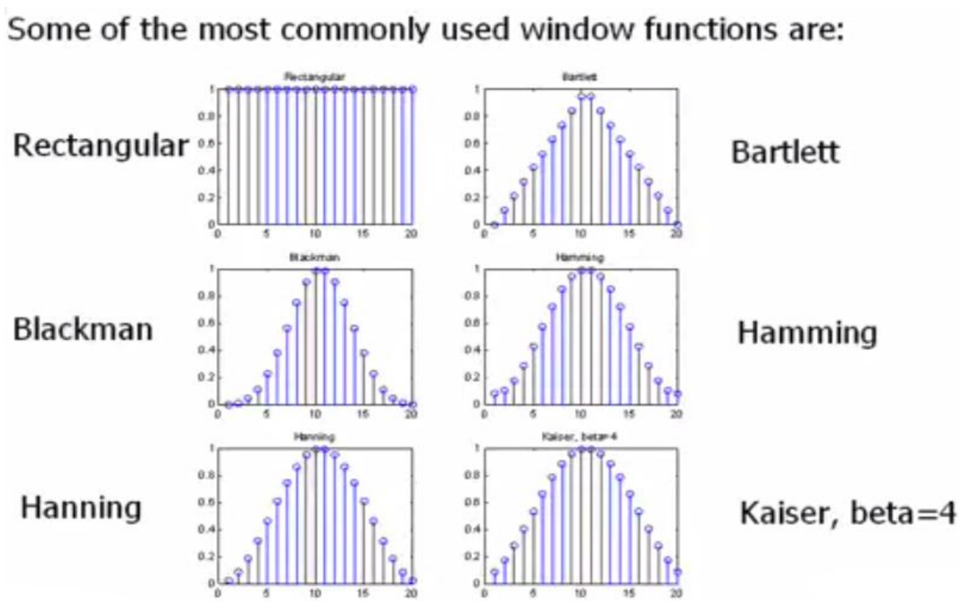
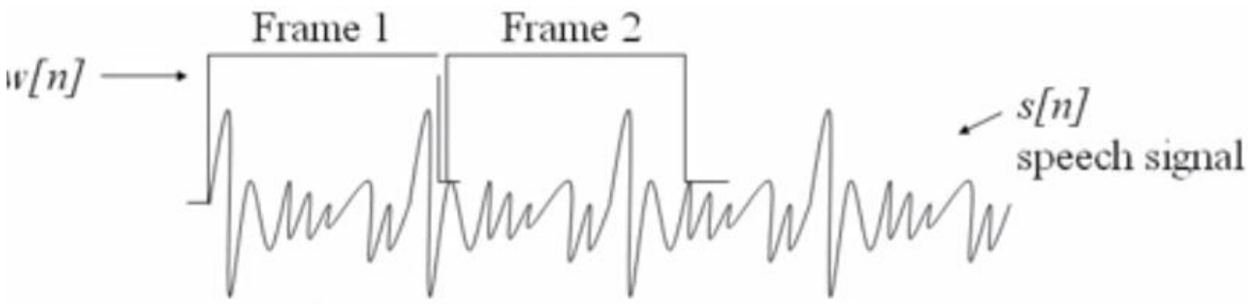


Fig. Different windowing techniques

To analyze the truncation process, it is modeled as a multiplication of the desired sequence by finite duration window sequence denoted by w[n]. Truncation of sequence s[n] is equivalent to placing a rectangular time window around s[n].



Y[n]=s[n].w[n] in frequency indomian Y(θ)=S(θ)*W(θ). Thus when window is applied, the frequency domain convolution causes distortion in the spectrum Y(θ). It can be shown that rectangular windows creates ripples innthe spectrum Y(θ). To reduce the distortion, we use Hamming or Kaiser Window.

FIR design Methods

The most essential feature of FIR filters is, by definition, the finite length of impulse response. A filter is said to have a linear phase response of its response of its response satisfies one following relationship.

$$\varphi(\theta) = -a\ \theta \qquad (1)$$

$$\varphi(\theta) = b - a\ \theta \qquad (2)$$

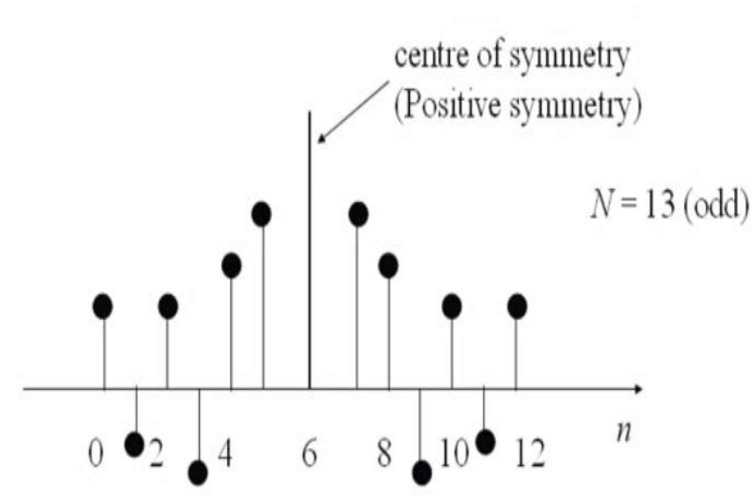
Where a and b are constants.

It can be shown that for condition (1) above to be satisfied the impulse response of the filter must have positive symmetry.

h[n]= h[N-n-1], $\boldsymbol{a} = \frac{N-1}{2}$ where N denotes the filter length.

When the condition give in (2) is satisfied, the impulse response of the filter has negatively symmetry.

h[n]= -h[N-n-1], $a\,\frac{N-1}{2}$ is centre of symmetry and b=$\frac{\pi}{2}$



Design of FIR filters using windows

The easiest way to obtain an FIR filter is to simply truncate the impulse response on IIR filter. If hd[n] represents the impulse response of a desired IIR filter, then an FIR filter with impulse response h[n] can be obtained as follows.

$$h[n]=\begin{cases} hd[n] & N1 \le n \le N2 \\ 0 & otherwise \end{cases}$$

in general h[n] can be thought of as being formed by the product hd [n] and a window function w[n] as follows

$$h[n]=h_d[n]\cdot w[n]$$

$$H(\theta)=H_d(\theta)*W(\theta)$$

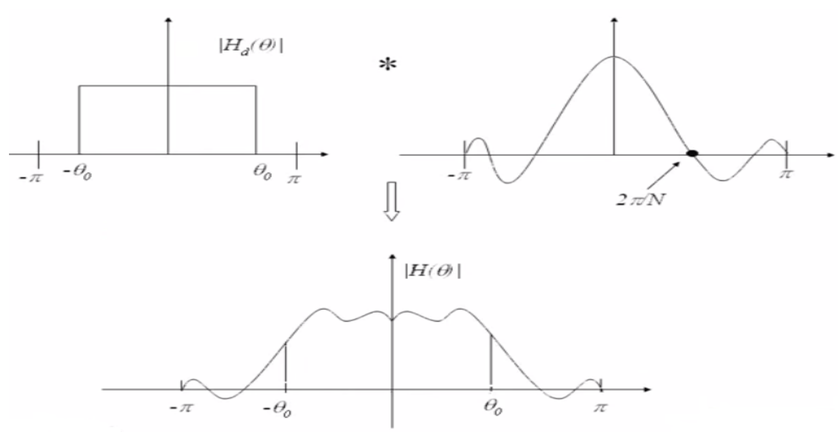


Fig. Design of FIR filters using windows

It is seen that the convolution produces a measured version of ideal low pass frequency response $H_d\ (\theta)$. In general, the wider the main lobe of w(θ), the more spreading, where as the narrower the main lobe (larger N) the closer the |H(θ)| comes to |$H_d\ (\theta)$|.

Design procedure

An ideal low pass filter with linear phase of slope-β and cut-off ωc can be characterized in the frequency domain by

$$H_d(\theta) = \begin{cases} e^{-j\theta\beta} & |w| \le \omega_c \\ 0 & w_c < |\omega| \le \pi \end{cases}$$

The corresponding impulse response h_d[n] can be obtained by taking the inverse Fourier transform of to |$H_d(\theta)$|. Shown as

$$h_d[n] = \frac{\sin[\omega_c(n-\beta)]}{\pi(n-\beta)}$$

A causal FIR filter with impulse response of h[n] can be obtained by multiplying h_d[n] by a windowing begning at the origion and ending at N-1 as follows

$$h[n] = \frac{\sin[\omega_c(n-\beta)]}{\pi(n-\beta)}\omega[n]$$

For h[n] to be a linear phase, β must be selected so that the resulting h[n] is symmetric. as$\frac{\sin[\omega c(n-\beta)]}{\pi(n-\beta)}$ is symmetric about n= β and the window is symmetric about n=$\frac{N-1}{2}$ where $\beta = \frac{N-1}{2}$ symmetric about β.

Frequency Sampling Filter:

Although it is implied that all FIR filter are non-recursive, this is not the case. To illustrate the approach, let us consider the following filter having a causal finite-duration unit-sample response containing N elements of constant value.

$$h[n] = \begin{cases} g_0/_N & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

References

1. Alan V. Oppenheim, Ronald W. Schafer "**Discrete-Time Signal Processing"**
2. Sanjit. K.Mitra **"Digital Signal Processing: A Computer-based Approach"**
3. Steven W. Smith "**Digital Signal Processing: A Practical Guide for Engineers"**
4. Dag Stranne "**Digital Signal Processing: DSP and Applications"**
5. VIJAY MADISETTI **"The Digital Signal Processing Handbook"**