Instantaneous Frequency

Consider a generalized sinuosoidal signal given by

$$\varphi(t) = A\cos\theta(t), \theta(t) : generalized \quad angle$$

And the generalized angle for a conventional sinusoid signal is a straight line as follows

$$conventional \quad sinusoid \quad signal : A\cos(\omega_c t + \theta_0)$$
$$generalized \quad angle : \omega_c t + \theta_0$$

A hypothetical case for the general angle is that at some instantt

$$for \quad t_0 \quad and \quad \Delta t \to 0, where \quad \frac{d\theta(t_0)}{dt} = \omega_c$$
$$\varphi(t) = A\cos\theta(t) = A(\cos\omega_c t + \theta_0), t \in (t_0 - \Delta t, t_0 + \Delta t)$$

Generalizing this idea,we get

$$\omega_i(t) = \frac{d\theta(t)}{dt}, \theta(t) = \int_{-\infty}^{t} \omega_i(\alpha) d\alpha$$

Here we can tell the possibility of transimitting the information of message signal by varying the angle of a carrier.

Angle Modulation

- ***Phase Modulation:***

$$\theta(t) = \omega_c t + \theta_0 + k_p m(t), assuming \quad \theta_0 = 0$$
$$\varphi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$$
$$\omega_i(t) = \omega_c + k_p \dot{m}(t)$$

- ***Frequency Modulation:***

$$\theta(t) = (\omega_c + k_f m(t))t + \theta_0, assuming \quad \theta_0 = 0$$
$$\varphi_{FM}(t) = A\cos((\omega_c + k_f m(t))t)$$
$$\omega_i(t) = \omega_c + k_f m(t)$$

  or equally

$$\omega_i(t) = \omega_c + k_f m(t) \Rightarrow \theta(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha$$
$$\varphi_{FM}(t) = A\cos^{\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha}$$

- ***Power of Frequency Modulated Signal:***

$$P_{FM} = P_{PM} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |\Phi_{FM}(t)|^2 dt = \frac{A_c^2}{2}$$

- ***Narrow Band Angle Modulation***
  - ***Bandwidth of Narrow Band Angle Modulation***

    If PM is a narrow band signal, assuming

$$k_p m(t) \quad small \Rightarrow \sin\left(k_p m(t)\right) \approx k_p m(t), \cos\left(k_p m(t)\right) \approx 1$$

$$\Phi_{PM}(t) = A\cos\left(\omega_c t\right)\cos\left(k_p m(t)\right) - A\sin\left(\omega_c t\right)\sin\left(k_p m(t)\right) \approx A\cos\left(\omega_c t\right) - Ak_p m(t)\sin\left(\omega_c t\right)$$

$$\Phi_{FM}(t) \approx A\cos\omega_c t - Ak_f\int_{-\infty}^{t} m(\alpha)d\alpha \sin\omega_c t$$

Thus we can tell that the modulated signal=carrier+DSB-SC. And that if bandwidth of message signal is B, then the bandwidth of the modulated signal is 2B.

- ***Modulator for Narrow Band PM/FM Signal***

Suggested modulator

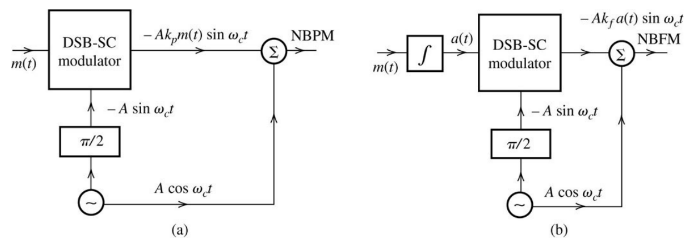


Figure 5.8 (a) Narrowband PM generator. (b) Narrowband FM signal generator.

- ***Wide Band Angle Modulation:***

  If PM/FM wide-band,

$$|k_p m(t)| \ll 1 \quad doesn't \quad hold.$$

  Carson's Rule:

$$FM : m(t) \quad bandwidth \quad B, m(t) \in [-m_p, m_p]$$

$$B_{FM} \approx 2B(\beta+1), \beta = \frac{k_f m_p}{2\pi B}$$

$$PM : m(t) \quad bandwidth \quad B, m'(t) \in [-m'_p, m'_p]$$

$$B_{PM} \approx 2B(\beta+1), \beta = \frac{k_f m'_p}{2\pi B}$$

  - ***Modulation/Generation of FM/PM Signals***

    Modulate at low carrier frequency

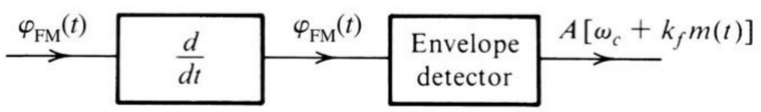
$$\widetilde{\Phi}_{PM}(t) = A\cos\left(\frac{\omega_c}{N}t + \frac{k_p}{N}m(t)\right)), \quad \frac{k_p}{N}m(t) \quad small$$

    Then pass through frequency multiplier.

- ***Demodulatin of FM Signals***
  - Envelope detection

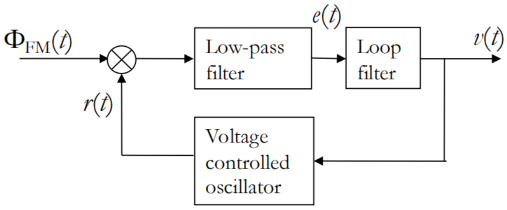
$$Key \quad idea : \omega_i = \omega_c + k_f m(t), \frac{d\Phi_{FM}(t)}{dt} = A[\omega_c + k_f m(t)]\sin\left(\omega_c t + k_f\int m(t)\right))$$

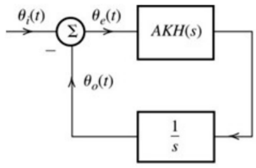
$$If \quad k_f m(t) < \omega_c, use \quad envelope \quad detection.$$



(c)

  - PLL


$\Phi_{FM}(t)$
Low-pass filter
$e(t)$
Loop filter
$v(t)$
$r(t)$
Voltage controlled oscillator


$\theta_i(t)$
Σ
$\theta_e(t)$
$AKH(s)$
$-$
$\theta_o(t)$
$\frac{1}{s}$

(a)