Overview of the Communication System:

$$message \quad signal \rightarrow sampling \rightarrow quantization \rightarrow modulation \rightarrow simple \quad channel \\ \rightarrow demodulation \rightarrow D/A converter \rightarrow reconstruction$$

Sampling

- ***Mathematical Representation:***

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT), x_s(t) = x(t)s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT), x(t) : message \quad signal$$

$$spectrum : X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega - \frac{2\pi k}{T}))$$

We can tell that ***the spectrum of the sampled signal*** is summation of an infinite number of replicas of the given spectrum shifted by integer multiples of

$$\frac{2\pi}{T}$$

- ***Nyquist Frequency***

For a band-limited signal,

$$x(t) \overset{F}{\leftrightarrow} X(j\Omega)$$
$$\exists \Omega_m > 0, X(j\Omega) = 0, \forall |\Omega| > \Omega_m$$

to acquire a loss-free transmission communication system, we must have

$$\Omega_s \geq \Omega_N = 2\Omega_m$$
$$\Omega_s : sampling \quad rate, \Omega_N : Nyquist \quad frequency$$

Pulse Code Modulation(PCM)

- ***Process of Pulse Modulation:***
  - Sampling:continuous-time signal -> discrete-time sequence
  - Pulse modulation:
  1. Tx:convert the sequence into a modualted pulse train
  2. Rx:reconstruct the signal by tracking certain parameter of the pulse train.
- ***Benefit of Pulse Modulation:***
  Enables time division multiplexing(TDM)
- ***Three Steps:***
  1. Sampling
  2. Quantization: Round off the sample values to the closest level
  3. Coding: Index the finite number of levels and use bit streams to represet indices of samples
- ***Uniform Quantization:***
- Assume the message signal is between

$$m(t) \in [-m_p, m_p]$$

Divide the message signal into L intervals, each of width

$$\frac{2m_p}{L}$$

And each sample is approximated using the mid-point of the interval where it falls.

$$SNR = \frac{P_m}{P_q} = \frac{12P_m}{\Delta^2}, \Delta = \frac{2m_p}{L}$$

$$Quantization \quad Noise : N_q(P_q) = \frac{\Delta^2}{12}$$

- ***Nonuniform Quantization:***
  - ***Motivation:***

    We want to increase SNR as much as possible for the quantization communication system. Given that

    $$Quantization \quad Noise : N_q(P_q) = \frac{\Delta^2}{12}, \Delta = \frac{2m_p}{L}$$

    the quantization noise is proportional to the square of the step size. So that using ***smaller steps(larger L)*** for samller amplitudes, called **nonuniform quantization**, gives a better way of compressing noise.

    The compressor maps input signal increments

    $$\Delta m$$

    into larger increment

    $$\Delta y$$

    for small input signals and vice versa for large input signals.Hence, a given interval contains a larger number of steps when m is small.
  - ***Two Compression Laws:***
    - ***A-law:***

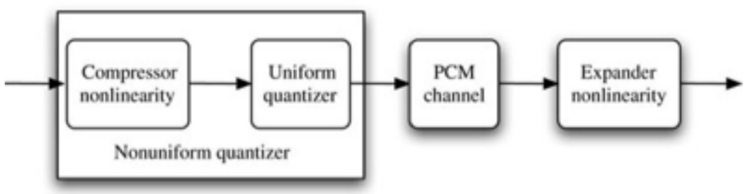
      $$y = \begin{cases} \frac{A}{1+\ln A} \frac{m}{m_p} & 0 \leq \frac{m}{m_p} \leq \frac{1}{A} \\ \frac{1}{1+\ln A}(1 + \ln \frac{Am}{m_p}) & \frac{1}{A} \leq \frac{m}{m_p} \leq 1 \end{cases}$$

    - ***u-law:***

      $$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \frac{\mu m}{m_p}\right) \quad 0 \leq \frac{m}{m_p} \leq 1$$

  - ***Diagrams:***

Can be interpreted as uniform quantization on a compressed (transformed) signal



- ***Transmission Bandwidth of PCM:***

For a binary PCM,we assign n binary bits to each of the L quantization levels.

$$L = 2^n \quad or \quad n = \lceil \log_2 L \rceil$$

The message signal band-limited to B Hz requires a minimum of $2B$ samples per second due to the sampling theorem,thus we require a total of $2nB$ bit/s, or, $2nB$ pieces of information per second.

Because a unit bandwidth can transmit a maximum of two pieces of information per second, so that a minimum channel of bandwidth is

$$B_T = nB \quad Hz$$

In summary

$$2 \times message \quad bandwidth = 2B \leq samples \quad per \quad second$$
$$n \times (samples \quad per \quad second) \leq 2 \times (minimum \quad channel \quad bandwidth)$$
$$\Rightarrow 2nB \leq 2B_T \Rightarrow B_T \geq nB$$

Delta Modulation

- ***Motivation:***
  1. To save channel bandwidth, we want to use as small number of bits as possible to represent a quantized sample.
  2. It's difficult to implement high speed A/D converter with many levels.
  3. If message signal vary slowly in time: represent the difference between the amplitudes of successive samples.

Differential PCM(DPCM)

Consider a message signal which has derivatives of all orders at t. Then we can use the Taylor series for this signal

$$m(t+T_s) = m(t) + T_s\dot{m}(t) + \frac{T_s^2}{2!}m^{(2)}(t) + \ldots \approx m(t) + T_s\dot{m}(t)$$
$$for \quad T_s \quad small$$

We can predict a future signal value at *t+Ts* using signal at t.

Next, we denote the kth sample of m(t) by m[k]

$$m(kT_s) = m[k], m(kT_s \pm T_s) = m[k \pm 1]$$
$$setting \quad t = kT_s \quad and \quad \dot{m}(kT_s) \approx \frac{m(kT_s) - m(kT_s - T_s)}{T_s}$$
$$\Rightarrow m[k+1] \approx m[k] + T_s(\frac{m[k] - m[k-1]}{T_s}) = 2m[k] - m[k-1]$$

The equations above show that we can approximate the (k+1)th sample using the two previous samples[kth and (k-1)th)]. Therefore we can do better approximation using more previous samples, as is shown in the following:

$$m[k] \approx a_1 m[k-1] + a_2 m[k-2] + \ldots + a_N m[k-N]$$
$$\hat{m}[k] = a_1 m[k-1] + a_2 m[k-2] + \ldots + a_N m[k-N], \quad predicted \quad value$$
$$d[k] = m[k] - \hat{m}[k], \quad the \quad difference$$

In DPCM, we transmit not the present sample m[k], but d[k]. At the receiver, instead of the past samples m[k-1],m[k-2],..., as well as d[k], we have their quantized versions mq[k-1],mq[k-2],.../