Double-Sideband Amplitude Modulation

- Amplitude of the carrier is varied in proportion to the baseband(message) signal.
- ***Modulation:***

$$message \quad signal: m(t)$$
$$carrier \quad signal: cos(\omega_c t)$$
$$modulated \quad signal: m(t)cos(\omega_c t) = \frac{1}{2}m(t)e^{-j\omega_c t} + \frac{1}{2}m(t)e^{j\omega_c t}$$
$$Spectrum: M(\omega) \to \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$$

- ***Demodulation: Synchronous/Coherent demodulation***

$$m(t)cos(\omega_c t) \times cos(\omega_c t) = \frac{1}{2}m(t)cos(2\omega_c t) + \frac{1}{2}m(t)$$
$$spectrum: \frac{1}{4}[M(\omega+2\omega_c) + M(\omega-2\omega_c)] + \frac{1}{2}M\omega$$

Amplitude Modulation

- ***Motivation:*** coherent demodulation of DSB-SC requires the receiver to generate a carrier. To simplify receiver, we use AM.
- ***Modulation:***

$$Message \quad signal: m(t)$$
$$Carrier \quad signal: cos(\omega_c t)$$
$$Modulated signal: (A + m(t))cos(\omega_c t)$$
$$Spectrum: M(\omega) \to \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)] + \frac{A}{2}[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$

> *Drawback: Extra transmit power*

- ***Demodulation: Envelope detector***

  To ensure correct envelope detector, we must have

$$A + m(t) > 0 \quad for \quad all \quad t$$
$$Define: \mu - Modulation \quad index, \mu = \frac{m_p}{A}$$

  Envelope detection requires that

$$0 \le \mu \le 1$$

- ***Side Band and Carrier Power***
  - Advantage of envelope detection is achieved under the expense of extra energy

$$\phi_{AM}(t) = Acos(\omega_c t) + m(t)cos(\omega_c t), where \quad Acos\omega_c t \to carrier, m(t)cos\omega_c t \Rightarrow sideband$$
$$P_c = \frac{1}{T}\int_{\frac{T_c}{2}}^{\frac{T_c}{2}} A^2 cos^2\omega_c t dt = \frac{A^2}{2}, P_s = \frac{1}{2}\overline{m^2(t)} = \frac{1}{2}P_m$$

  - Power efficiency is as follows

$$\eta = \frac{P_s}{P_c + P_s} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \le \frac{1}{3}$$
$$Given \quad that \quad when \quad m(t) = cos\omega_m t, \quad \eta \quad reaches \quad maximum$$

Quadrature Amplitude Modulation

- ***Motivation:***

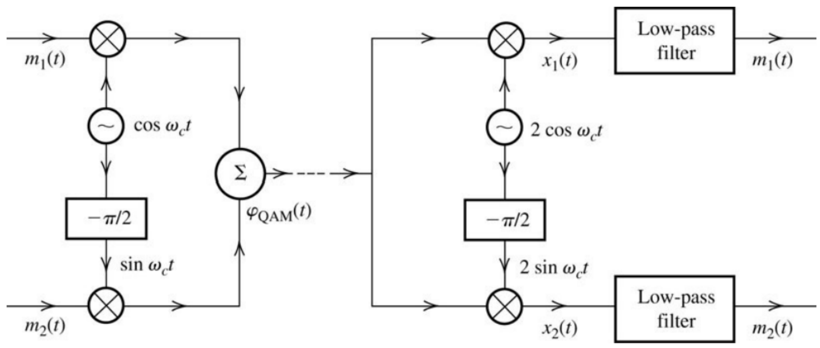
  Using DSB-SC,

$$if \quad m(t) \quad real \to M(\omega) = M^*(-\omega)$$
$$Modulated \quad signal \quad \Phi(t) \quad real \to \Phi(\omega) = \Phi^*(-\omega)$$

We can tell that **only half** of the frequency spectrum of **message signal** and **one fourth** of the frequency spectrum of the **modulated signal** carries information.

- ***Modulation:***

$$basband \quad signals: m_1(t), m_2(t)$$
$$Modulated \quad signal: \Phi(t) = m_1(t)cos\omega_c t + m_2(t)sin\omega_c t$$
$$Spectrum: \Phi(\omega) = \frac{1}{2}(M_1(\omega-\omega_c) + M_1(\omega+\omega_c)) + \frac{j}{2}(M_1(\omega-\omega_c) + M_1(\omega+\omega_c))$$



- ***Demodulation***

$$Channel \quad 1: \Phi(t)2cos\omega_c t = 2m_1(t)cos^2\omega_c t + 2m_2(t)cos\omega_c t sin\omega_c t = m_1(t) + m_1(t)cos2\omega_c t + m_2(t)sin2$$
$$Channel \quad 2: \Phi(t)2sin\omega_c t = 2m_1(t)sin\omega_c t cos\omega_c t + 2m_2(t)sin^2\omega_c t = m_1(t)sin2\omega_c t - m_2(t)cos2\omega_c t + m_2(t)sin2$$

- ***Complex View of Modulation:***

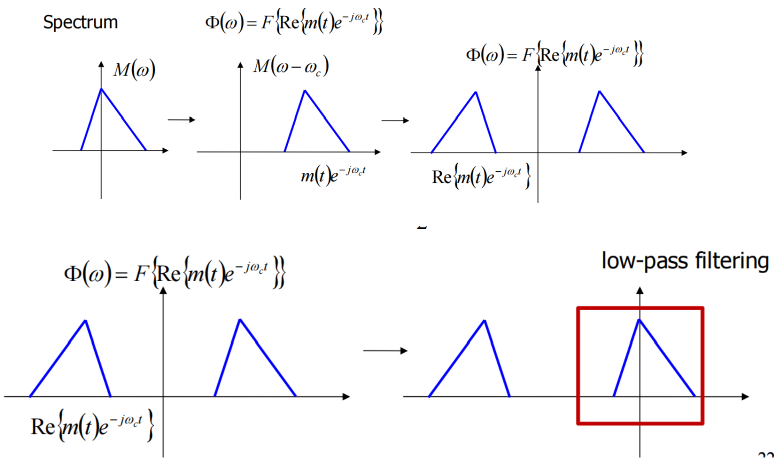
$$Message \quad signal: m(t) = m_1(t) + jm_2(t) \rightarrow M(\omega) \neq M*(-\omega)$$

  - This implies that both positive and negative spectra carry information.

$$Carrier \quad signal: cos\omega_c t = Re\{e^{-j\omega_c t}\}$$
$$Modulated \quad signal: Re\{m(t)e^{-j\omega_c t}\} = m_1(t)cos\omega_c t + m_2(t)sin\omega_c t$$
$$Spectrum: \Phi(\omega) = F\{Re\{m(t)e^{-j\omega_c t}\}\}$$

  - ***Demodulation***:

$$\Phi(t) \rightarrow \Phi(t)e^{j\omega_c t} \Rightarrow \Phi(\omega) \rightarrow \Phi(\omega+\omega_c) \rightarrow lowpass - filter \rightarrow \frac{1}{2}M(\omega)$$



Single Sideband Modulation

- ***Modulation:***

$$Step \quad 1: m(t)cos\omega_c t \rightarrow \Phi(\omega) = \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$$
$$Step \quad 2: Bandpass \quad filter \quad to \quad remove \quad either \quad the \quad LSB \quad or \quad USB.$$

- ***Hilbert Transform***

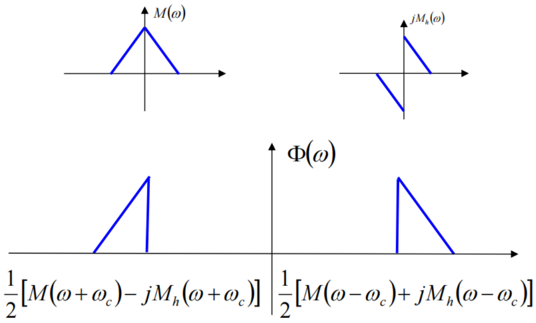
$$Hilbert \quad tranform: H(\omega) = -jsgn(\omega) = \begin{cases} j & \omega \le 0 \\ -j & \omega > 0 \end{cases}$$

$$H(\omega) \overset{F}{\leftrightarrow} h(t) = F^{-1}(-jsgn(\omega)) = \frac{1}{\pi t}$$

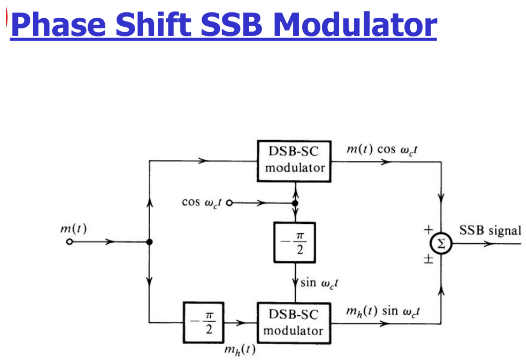
- ***SSB Modulation via Hilbert Transform***
  - ***Modulation***:

$$\Phi(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t, m_h(t) = m(t) * h(t), h(t): Hilbert \quad transform$$

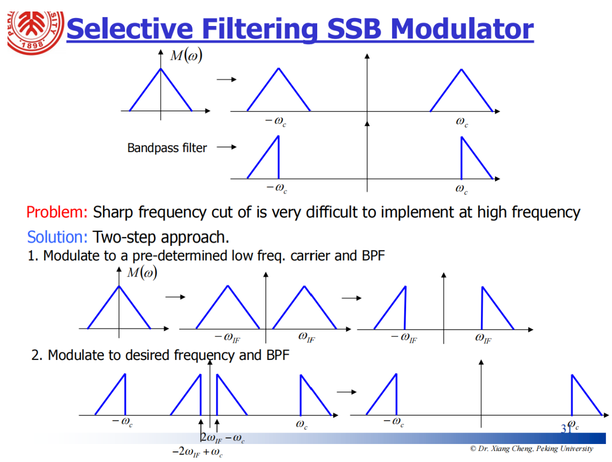
$$\Rightarrow \Phi(\omega) = \frac{1}{2}[M(\omega+\omega_c) - jM_h(\omega+\omega_c)] + \frac{1}{2}[M(\omega-\omega_c) + jM_h(\omega-\omega_c)]$$



  - ***Modulation Using Phase Shift***:



  - ***Modulation Using Selective Filtering***:



  - ***Demodulation***:

1. ***Coherent Demodulation***:

$$\Phi(t)\cos\omega_c t = m(t)\cos^2\omega_c t + m_h(t)\sin\omega_c t\cos\omega_c t = \frac{1}{2}m(t) + \frac{1}{2}[m(t)\cos 2\omega_c t + m_h(t)\sin 2\omega_c t]$$

2. ***Envelope Demodulation:***

$$Acquirements: A \gg m^2(t) + m_h^2(t)$$

$$\Phi(t) + A\cos\omega_c t = (A + m(t))\cos\omega_c t + m_h(t)\sin\omega_c t = K(t)[\cos\theta_t\cos\omega_c t + \sin\theta_t a \sin\omega_c t]$$

$$K^2(t) = (A + m(t))^2 + m_h^2(t) = A^2[1 + \frac{2m(t)}{A} + \frac{m^2(t) + m_h^2(t)}{A^2}]$$

$$\Rightarrow Envelope \approx \sqrt{1 + \frac{2m(t)}{A}} - 1 \approx \frac{m(t)}{A}$$

Vestigial Sideband Modulation

- ***Motivation:***

  It's hard to implement a bandpass filter with sharp edges.

- ***Modulation:***

$$\Phi_{VSB}(\omega) = [M(\omega - \omega_c) + M(\omega + \omega_c)]H_i(\omega), H_i(\omega): transmitter \quad shaping \quad filter.$$

- ***Demodulation:***

$$pass \quad 2\Phi_{VSB}(t)\cos\omega_c t \quad through \quad H_o(\omega), H_o(\omega): receiver \quad shaping \quad filter.$$
$$Spectrum: [\Phi_{VSB}(\omega + \omega_c) + \Phi_{VSB}(\omega - \omega_c)]H_o\omega = M(\omega)[H_i(\omega) + H_i(\omega - \omega_c)]H_o(\omega) + high \quad frequency \quad term.$$
$$Thus \quad we \quad require: [H_i(\omega + \omega_c) + H_i(\omega - \omega_c)]H_o(\omega) = 1$$

  To simplify the filter used by the receiver, we want that

$$H_i(\omega - \omega_c) + H_i(\omega + \omega_c) = 1 \quad for |\omega_c| < 2\pi B$$
$$\Rightarrow H_o(\omega) = 1$$

  so that for the receiver, he only needs to do lowpass