677 midterm

order statistic

Uniform Distribution

Consider the uniform distribution $U(a, b)$. The PDF and CDF are given by:

- PDF: $f(x) = \frac{1}{b-a}$, for $a \leq x \leq b$
- CDF: $F(x) = \frac{x-a}{b-a}$, for $a \leq x \leq b$

For the k-th order statistic $X_{(k)}$ from a sample of size n:

- PDF: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1-F(x)]^{n-k}f(x)$

Applying the CDF and PDF of the uniform distribution:

- PDF of $X_{(k)}$: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}\left(\frac{x-a}{b-a}\right)^{k-1}\left(1-\frac{x-a}{b-a}\right)^{n-k}\frac{1}{b-a}$

Exponential Distribution

For the exponential distribution with rate λ, the PDF and CDF are:

- PDF: $f(x) = \lambda e^{-\lambda x}$, for $x \geq 0$
- CDF: $F(x) = 1 - e^{-\lambda x}$, for $x \geq 0$

The PDF of the k-th order statistic $X_{(k)}$:

- PDF of $X_{(k)}$: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[1-e^{-\lambda x}]^{k-1}e^{-\lambda x[n-k]}\lambda e^{-\lambda x}$

Normal distribution

For the normal distribution, deriving the distribution of order statistics analytically is more complex due to the non-linearity of its CDF. Instead, we often use numerical methods or simulations to study the order statistics of normal samples