677 midterm

order statistic

Uniform Distribution

Consider the uniform distribution $U(a, b)$. The PDF and CDF are given by:

- PDF: $f(x) = \frac{1}{b-a}$, for $a \leq x \leq b$
- CDF: $F(x) = \frac{x-a}{b-a}$, for $a \leq x \leq b$

For the k-th order statistic $X_{(k)}$ from a sample of size n:

- PDF: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1-F(x)]^{n-k}f(x)$

Applying the CDF and PDF of the uniform distribution:

- PDF of $X_{(k)}$: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}\left(\frac{x-a}{b-a}\right)^{k-1}\left(1-\frac{x-a}{b-a}\right)^{n-k}\frac{1}{b-a}$

Exponential Distribution

For the exponential distribution with rate λ, the PDF and CDF are:

- PDF: $f(x) = \lambda e^{-\lambda x}$, for $x \geq 0$
- CDF: $F(x) = 1 - e^{-\lambda x}$, for $x \geq 0$

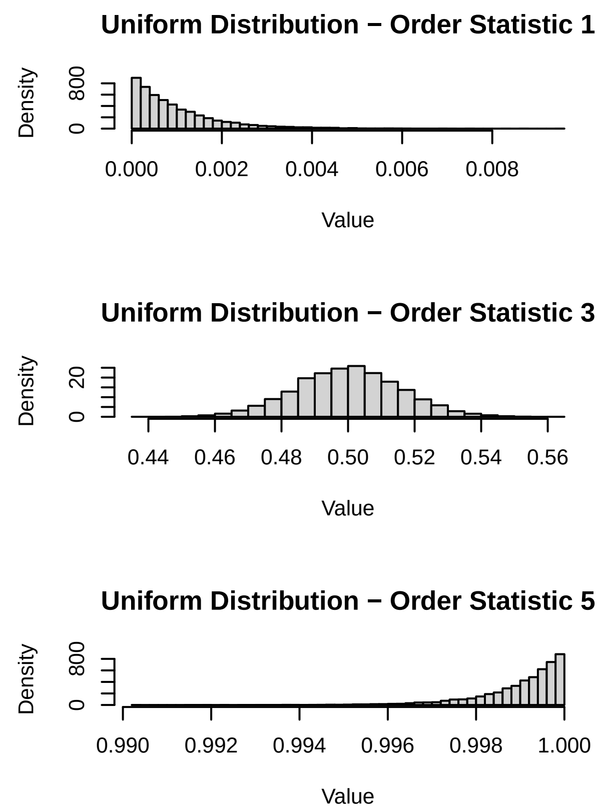
The PDF of the k-th order statistic $X_{(k)}$:

- PDF of $X_{(k)}$: $f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[1-e^{-\lambda x}]^{k-1}e^{-\lambda x[n-k]}\lambda e^{-\lambda x}$

Normal distribution

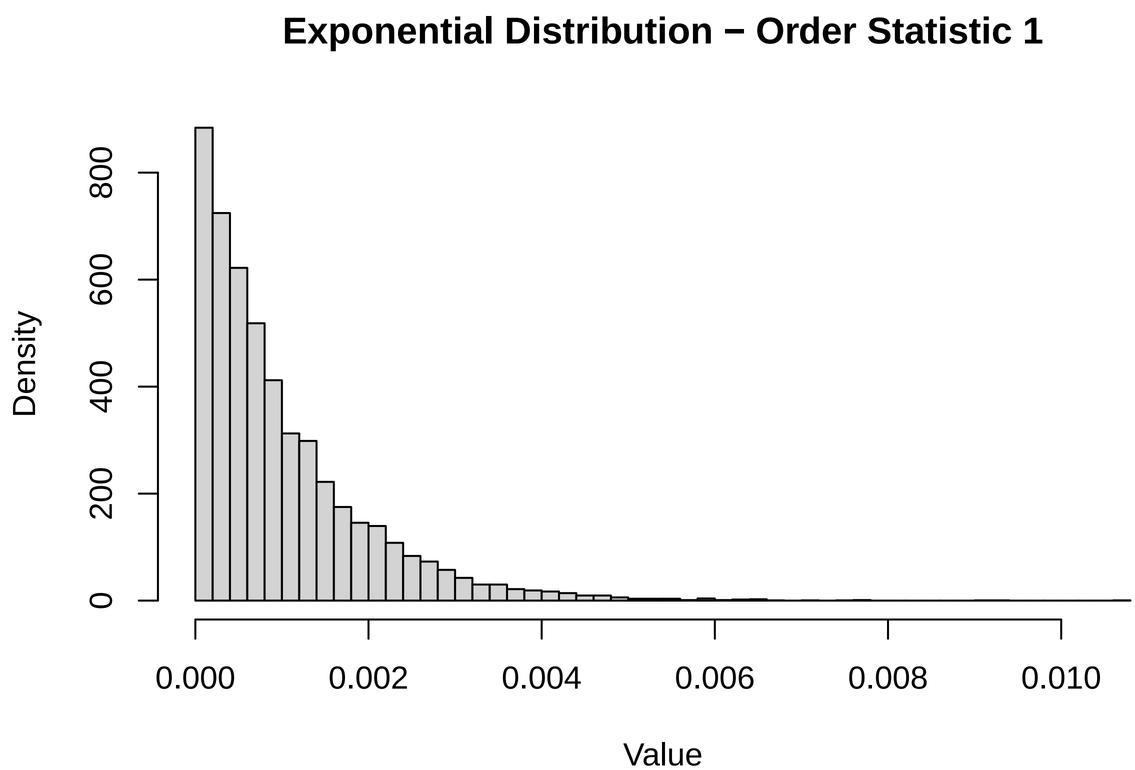
For the normal distribution, deriving the distribution of order statistics analytically is more complex due to the non-linearity of its CDF. Instead, we often use numerical methods or simulations to study the order statistics of normal samples

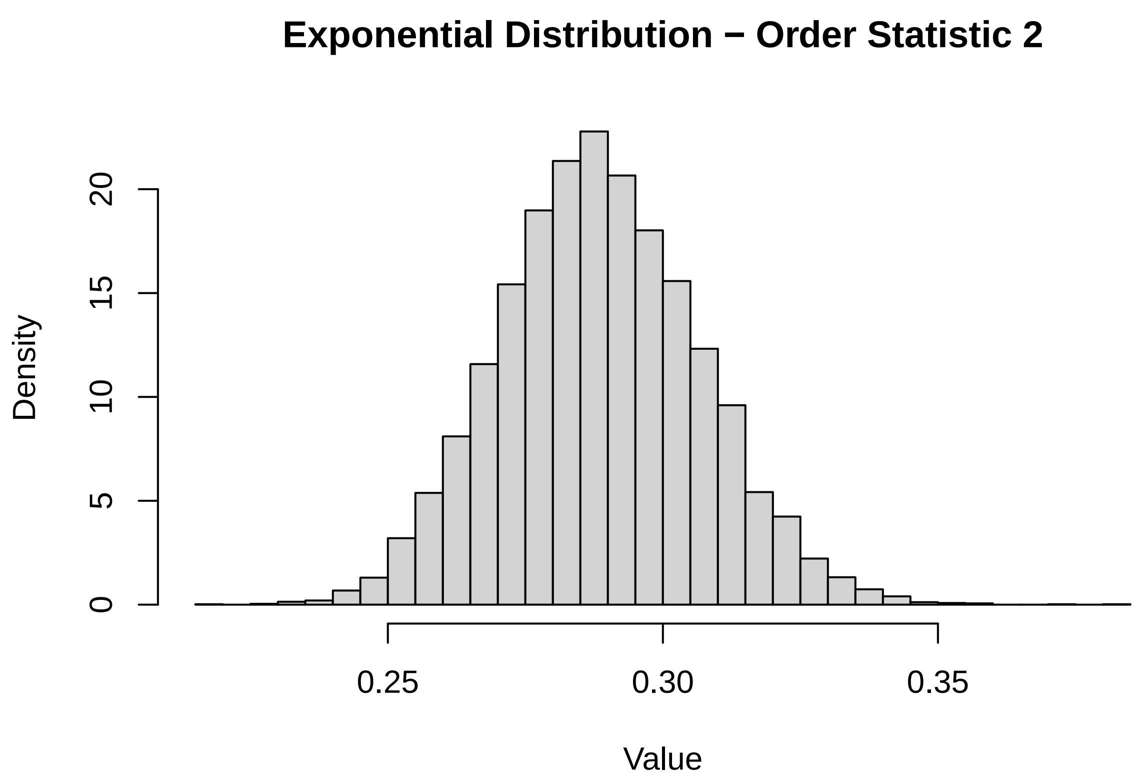
Simulation of uniform

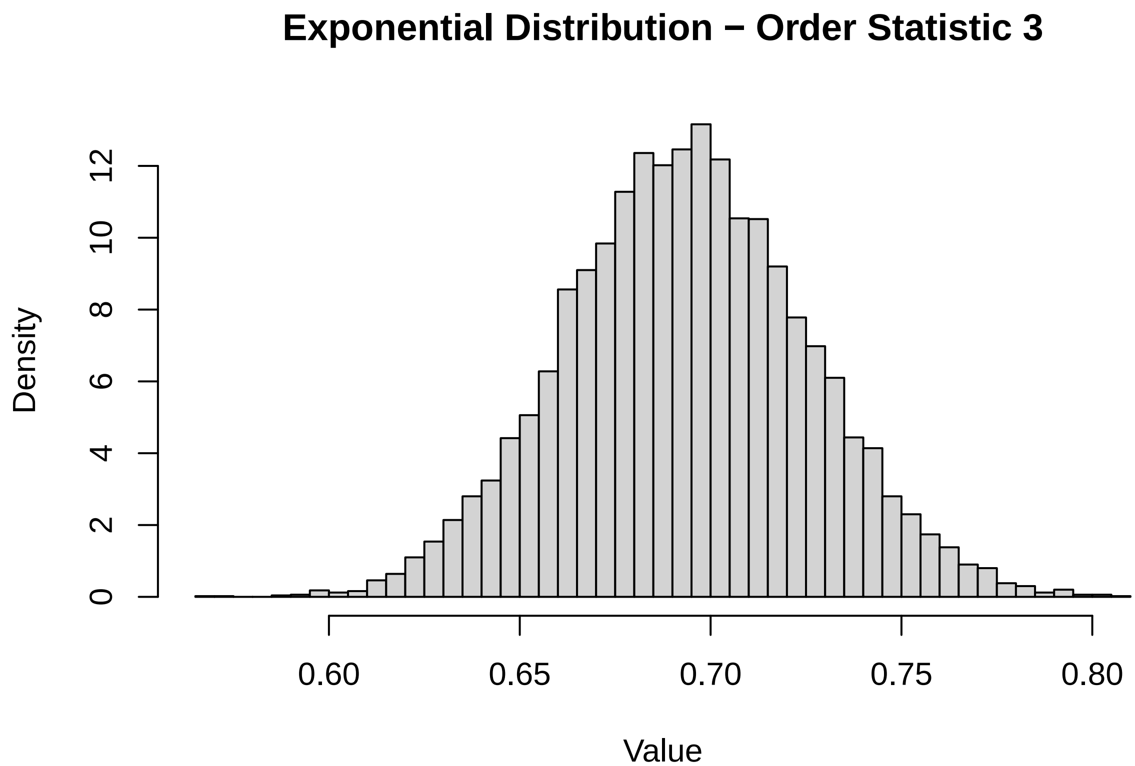


In this chapter,I choose s order statistics that is:0th,25th,50th,75th,100th

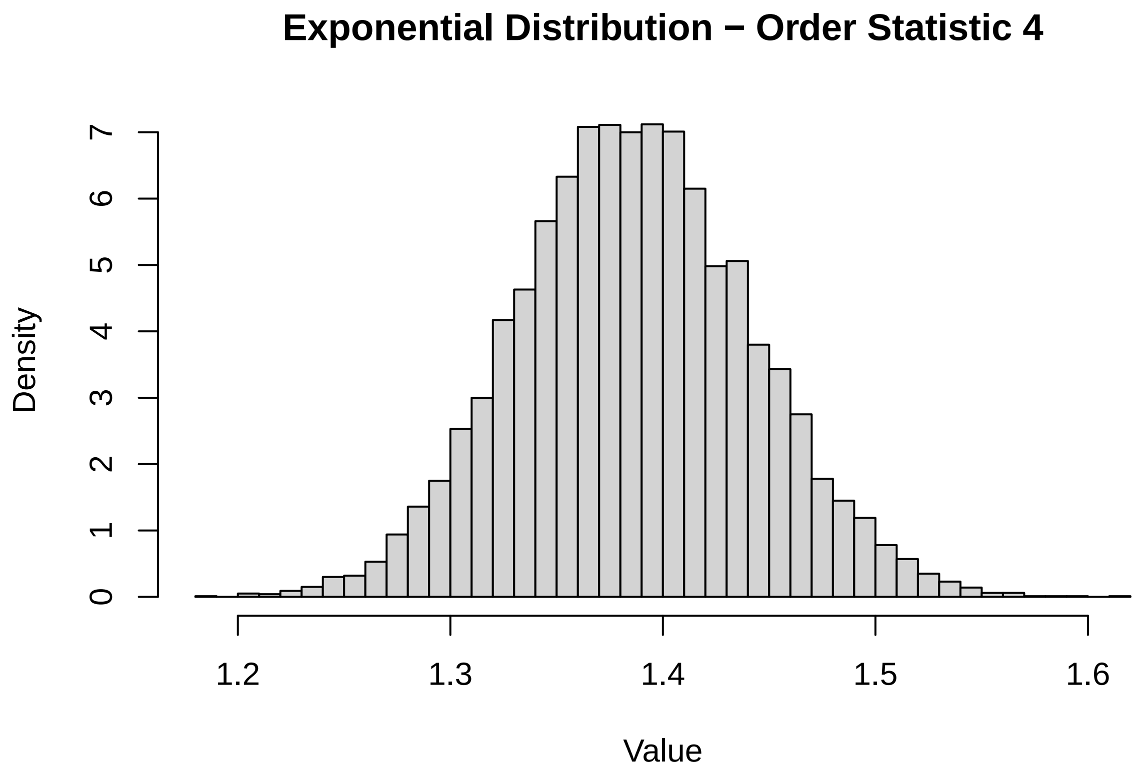
Simulation of exponetianal

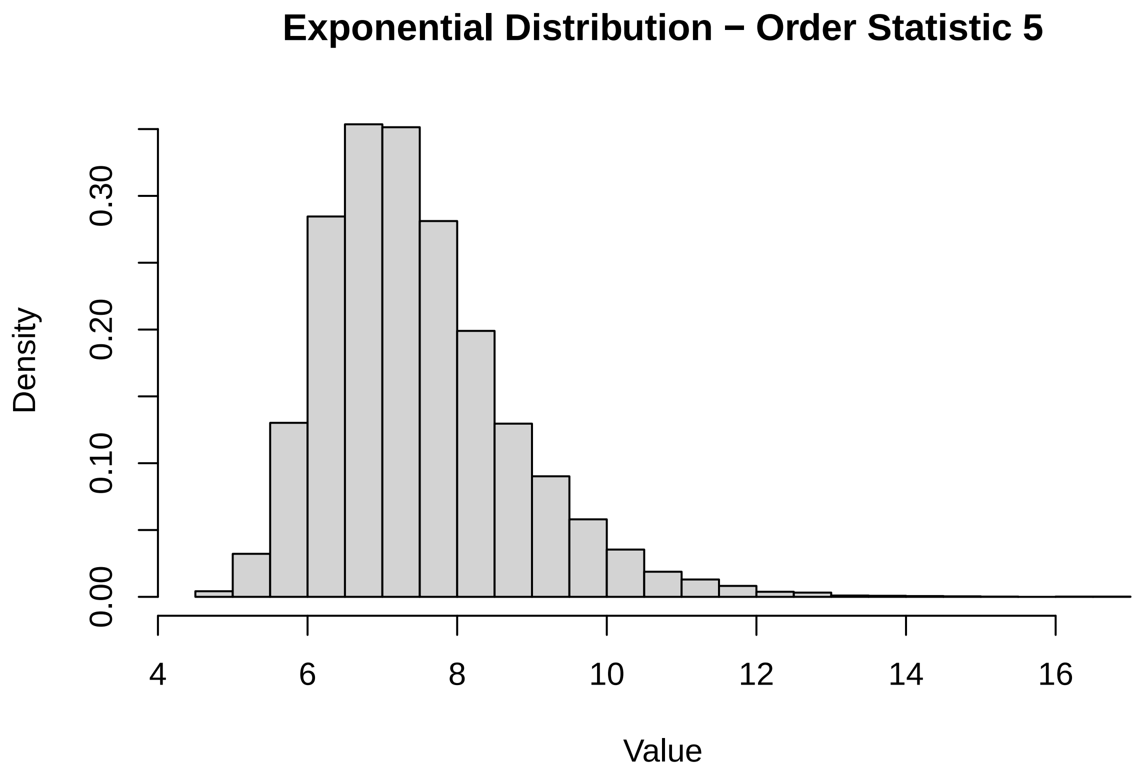




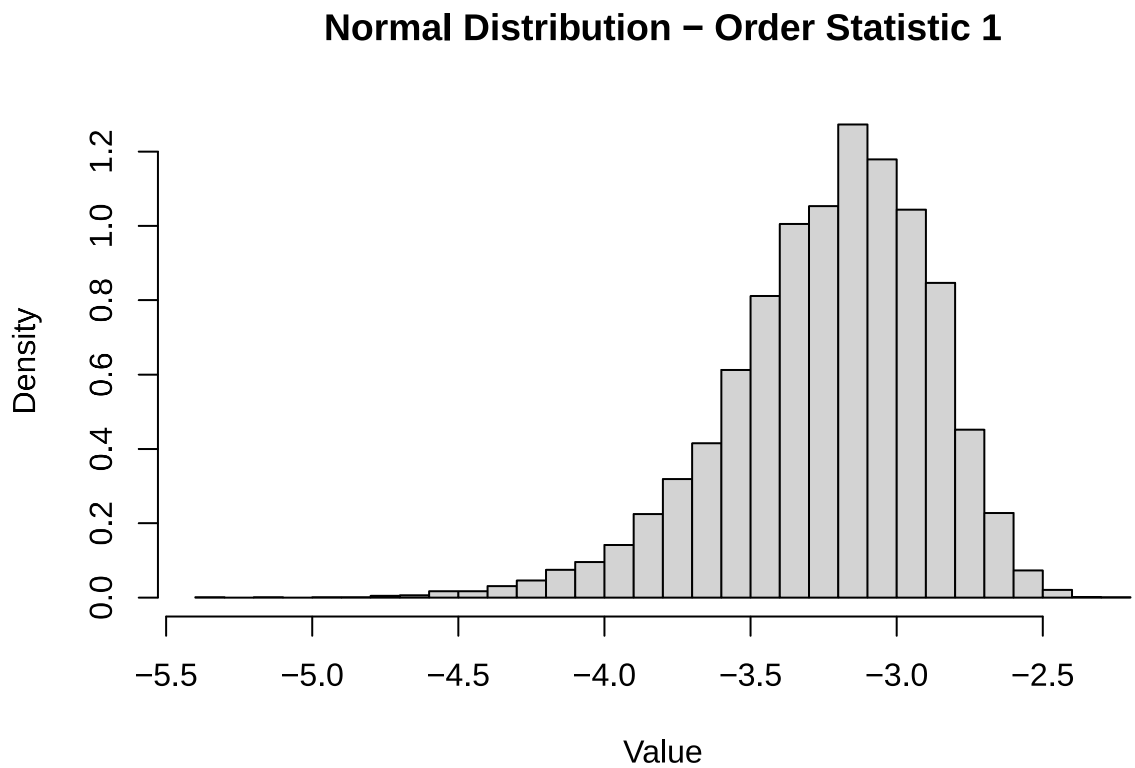


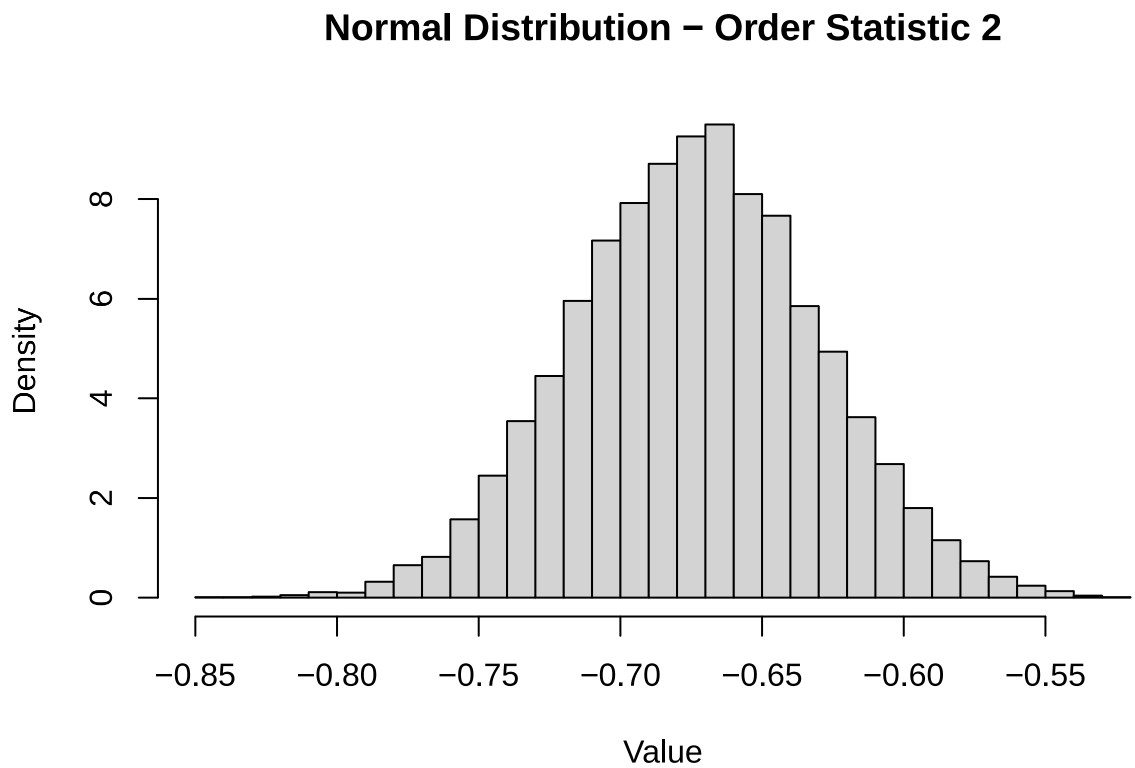
Exponential Distribution − Order Statistic 3
Density
0
2
4
6
8
10
12
0.60
0.65
0.70
0.75
0.80
Value

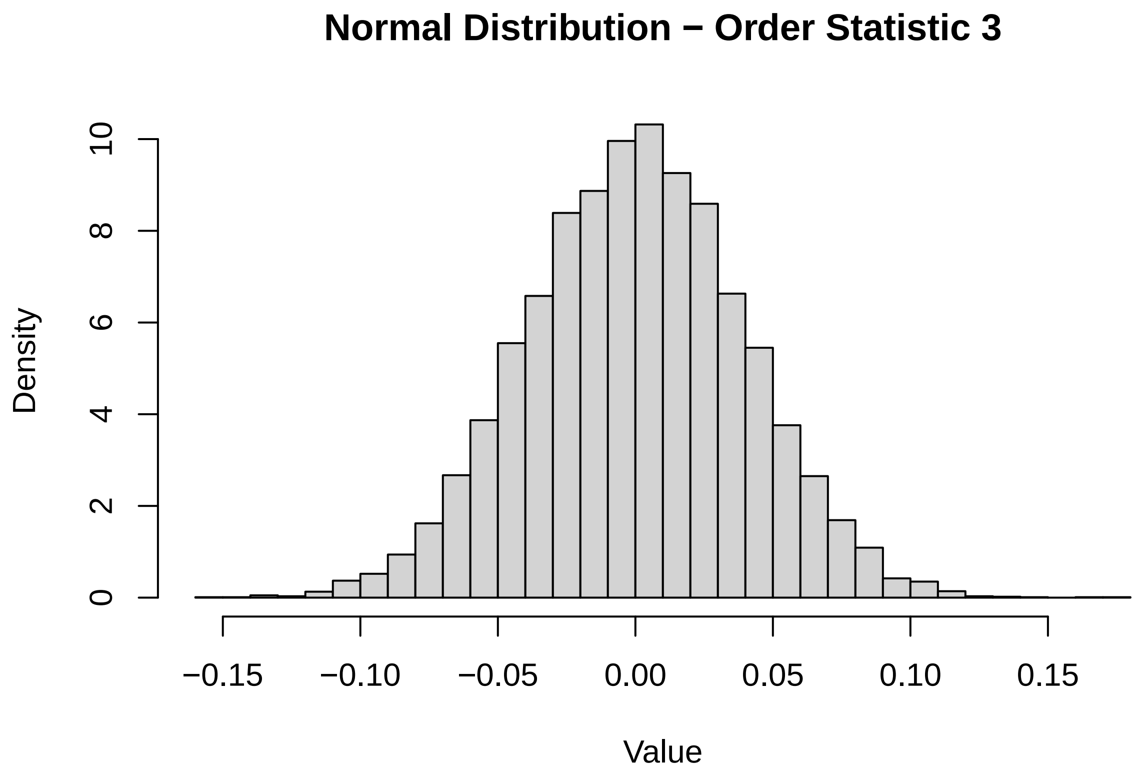

Exponential Distribution − Order Statistic 4
Density
0
1
2
3
4
5
6
7
1.2
1.3
1.4
1.5
1.6
Value

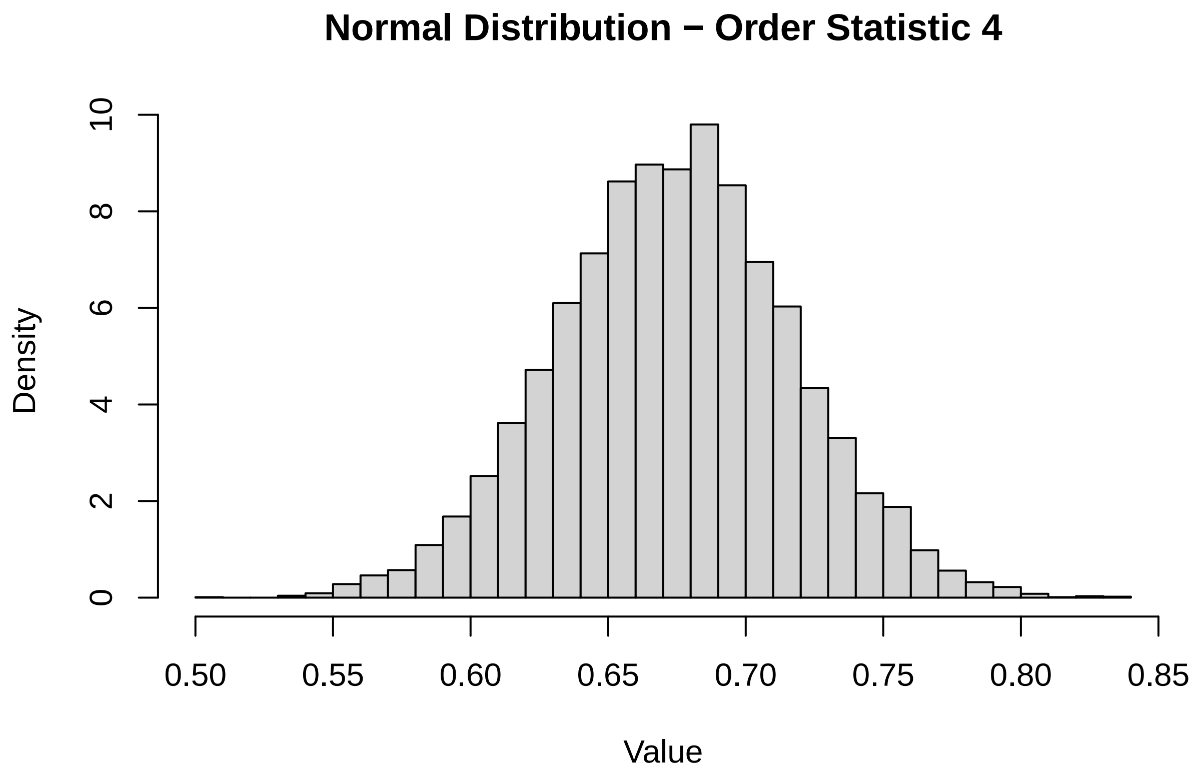


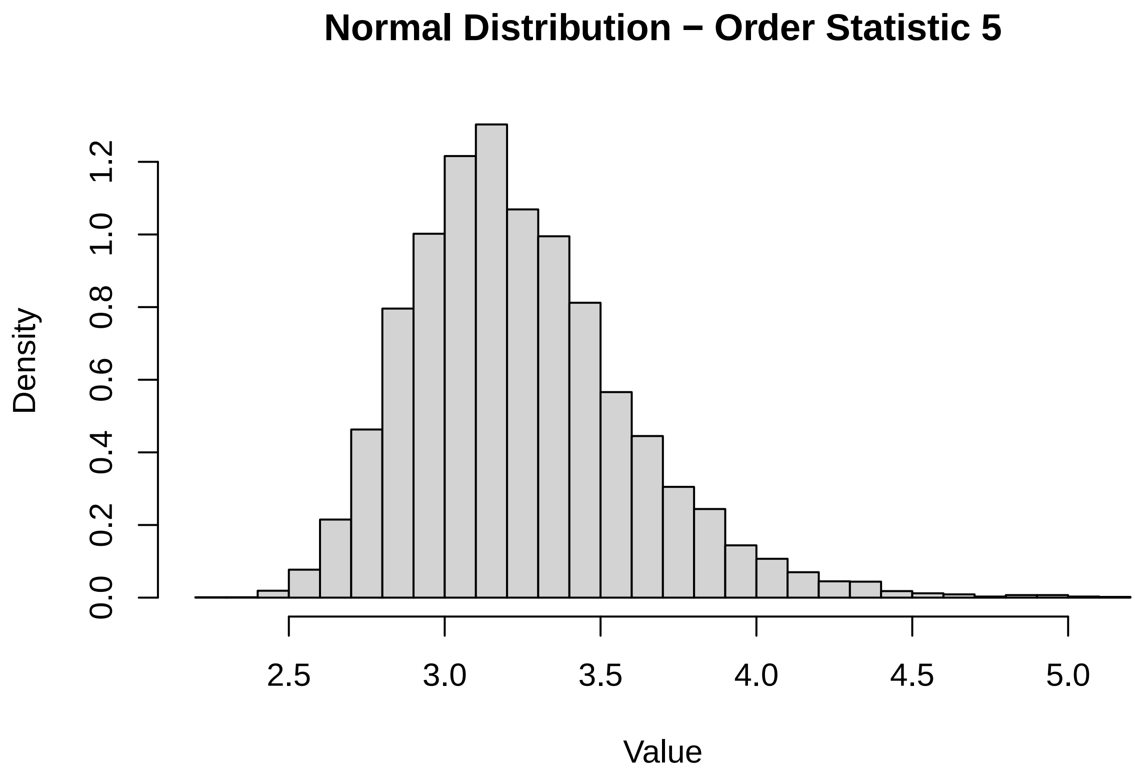
Simulation of normal


Normal Distribution − Order Statistic 1
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−5.5
−5.0
−4.5
−4.0
−3.5
−3.0
−2.5
Value


Normal Distribution − Order Statistic 2
Density
0
2
4
6
8
−0.85
−0.80
−0.75
−0.70
−0.65
−0.60
−0.55
Value


Normal Distribution − Order Statistic 3
Density
Value
0
2
4
6
8
10
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15


Normal Distribution − Order Statistic 4
Density
0
2
4
6
8
10
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
Value


Normal Distribution − Order Statistic 5
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2.5
3.0
3.5
4.0
4.5
5.0
Value