应用随机过程

第2章 随机过程基本概念

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- 2.4 宽平稳过程的性质
  - 自相关函数与自相关系数的区别

宽平稳过程的自相关函数

设$R_X(\tau)$为平稳过程$\{X_t : t \in T\}$自相关函数，则有

$1^0\ R_X(0) \geq 0.$ $\quad \because R_X(0) = E\left(X_t^2\right) = \psi^2 \geq 0.$

$2^0\ R_X(\tau)$是偶函数，即 $R_X(\tau) = R_X(-\tau).$

$$\because R_X(\tau) = E\left(X_t X_{t+\tau}\right) = E\left(X_{t+\tau} X_t\right) = R_X(-\tau).$$

$3^0\ \forall \tau,\ \left|R_X(\tau)\right| \leq R_X(0).$

$$\because E\left(X_t \pm X_{t+\tau}\right)^2 \geq 0 \quad \Leftrightarrow E\left(X_t^2 \pm 2X_t X_{t+\tau} + X_{t+\tau}^2\right) \geq 0$$

$$\Leftrightarrow 2R_X(0) \pm 2R_X(\tau) \geq 0 \ \Leftrightarrow R_X(0) \geq \left|R_X(\tau)\right|.$$

宽平稳过程的自相关函数

4^0 若$\{X_t\}$满足$X_t = X_{t+T}$，则称为周期平稳过程，其中T为过程的周期．且其自相关函数必为周期函数，周期也是T．

$$\because R_X(\tau+T) = E(X_t X_{t+\tau+T}) = E(X_t X_{t+\tau}) = R_X(\tau).$$

5^0 若$\{X_t\}$含有一个周期分量，则自相关函数也含有一个同周期的周期分量.

宽平稳过程的自相关函数

6^0 $R_X(\tau)$在R上连续的充要条件为$R_X(\tau)$在$\tau=0$处连续.

$\Rightarrow$ 显然

$$\Leftarrow \left[R_X(\tau+\Delta\tau)-R_X(\tau)\right]^2$$

$$=\left[E\left(X_t X_{t+\tau+\Delta\tau}\right)-E\left(X_{t+\Delta\tau}X_{t+\tau+\Delta\tau}\right)\right]^2$$

$$=\left\{E\left[X_{t+\tau+\Delta\tau}\left(X_t-X_{t+\Delta\tau}\right)\right]\right\}^2 \le E\left(X_{t+\tau+\Delta\tau}\right)^2 E\left(X_t-X_{t+\Delta\tau}\right)^2$$

$$=2R_X(0)\left[R_X(0)-R_X(\Delta\tau)\right]\to 0$$

$$\because \lim_{\Delta\tau\to 0} R_X(\Delta\tau)=R_X(0) \quad \therefore \lim_{\Delta\tau\to 0} R_X(\tau+\Delta\tau)=R_X(\tau).$$

宽平稳过程的自相关函数

7^0 非负定性：对任意有限个$t_1,\cdots,t_n\in T$,和任意的实数$a_1,\cdots,a_n\in R$，有

$$\sum_{i=1}^{n}\sum_{j=1}^{n}R_X\left(t_i,t_j\right)a_ia_j\geq 0.$$

$$\because \sum_{i=1}^{n}\sum_{j=1}^{n}R_X\left(t_i,t_j\right)a_ia_j=\sum_{i=1}^{n}\sum_{j=1}^{n}E\left(X_{t_i}X_{t_j}\right)a_ia_j$$

$$=E\left(\sum_{i=1}^{n}a_iX_{t_i}\sum_{j=1}^{n}a_jX_{t_j}\right)$$

$$=E\left(\sum_{i=1}^{n}a_iX_{t_i}\right)^2\geq 0.$$

宽平稳过程的自相关系数

1）定义

$$\rho(\tau)=\frac{C_X(\tau)}{C_X(0)}=\frac{R_X(\tau)-\mu^2}{\sigma^2}$$ 称为宽平稳过程$\{X_t\}$的自相关系数.

物理意义：过程在相距为τ的任意两个时刻的随机变量之间的线性相关程度.

2）性质

1^0 $\rho(0)=1.$

2^0 $\forall\tau,\ |\rho(\tau)|\le 1.$

$$\boxed{\begin{aligned}&C_X(t,t+\tau)\\&=E(X_t-EX_t)(X_{t+\tau}-EX_{t+\tau})\\&=R_X(\tau)-\mu^2\end{aligned}}$$