mmellinger66@gmail.com

UC Irvine Statistics 131a - Summer 2013

12:20

Sample Spaces — Ω

Examples

1) Coin Tossing

Ω = {H,T} — If you toss a fair coin once

Ω = {HH, HT, TH, TT} — If you toss a fair coin twice

Ω = {HH...H, HH...HT, HH...TH, ..., T...T} — If you toss a fair coin multiple times

16:20

Any experiment with two possible outcomes will have a similar sample space: Ω = {S,F} — success/failure

18:48

2) Roll a die: Ω = {1, 2, 3, 4, 5, 6}

Roll two dice: Ω = {11, 12, 13, ..., 16, 21, 22, ..., 66}

20:20

3) # α-particles emitted in 1 hour from your smoke detector

$\Omega = \{0, 1, 2, 3, 4, \ldots\} = \mathbb{N}$ – set of natural numbers

23:02

4) Waiting times (e.g. bus, car at stop light) Ω = [0,∞) —excludes ∞

25:03

5) Poker hands — Ω = {AS KS QS JS 2D, ...}

Events

26:25

Sample spaces are sets. If Ω is a sample space and $A \subseteq \Omega$, we call $\underline{A}$ an <u>event</u>

28:05

Examples

1) A = {three heads when tossing a coin six times} = {HHHTTT, HHTHTT, HHTTHT, ..., TTTHHH}

29:35

How many outcomes in this event? All possible outcomes is 2^6.

30:02

slots: __ __ __ __ __ __ Can place an H or T in slot

How many ways can you choose 3 slots for H's? The rest don't matter.

$\binom{6}{3}$ 6 choose 3 $\binom{6}{3} = \frac{6!}{3!\,3!}$

32:08

2) A = {straight flush} — Consecutive cards all in same suit

How many outcomes in this event? 10x4 = 40

37:00

3) Wait times — A = [0,1) = wait time is less than 1 hour

38:00

4) In an experiment with two outcomes, S and F, which are performed n times.

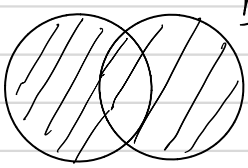
A = {k successes were observed} $k \in \{0, 1, 2, \ldots, n\}$

40:45

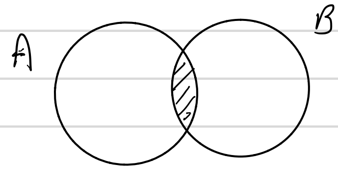
Combining Sets

If A and B are events

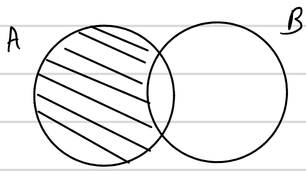
Union: $A \cup B = \{x \in \Omega : x \in A \text{ or } x \in B\}$



Intersection: $A \cap B = \{x \in \Omega : x \in A \text{ and } x \in B\}$

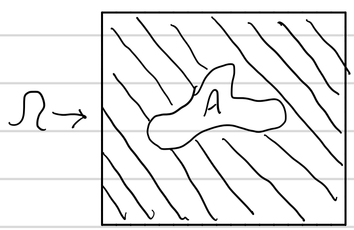


A minus B: $A \setminus B = \{x \in \Omega : x \in A \text{ and } x \notin B\}$

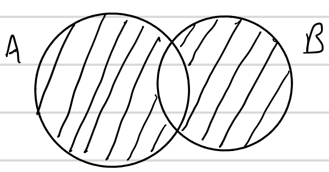


44:18

$A^c = \{x \in \Omega : x \notin A\}$



$A \triangle B = A \setminus B \cup B \setminus A$ – symetric difference of A & B



∅ = empty set - Has no elements

47:30

Definition: A probability is an assignment of a value to each event in a sample space. The value must lie in [0,1] and P(A) denotes the value assigned to A.

And if $A_1, A_2, \ldots, A_n$ are mutually disjoint events, then

50:00

$$A_i \cap A_j = \emptyset,\ i \neq j \qquad P\left(\bigcup_{j=1}^{n} A_j\right) = \sum_{j=1}^{n} P(A_j) \text{ and } P(\Omega) = 1$$

51:57

Is the empty set an event? Yes!

P(∅) = 0

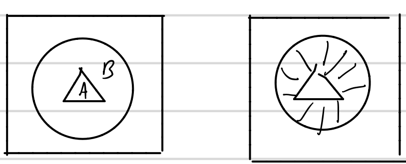
$\Omega =$ 

$$A \cup A^c = \Omega$$
$$A \cap A^c = \emptyset$$

53:40 $A_1 = A,\ A_2 = A^c$

$$1 = P(\Omega) = P(A \cup A^c) = P(A) + P(A^c) \Rightarrow \boxed{P(A^c) = 1 - P(A)}$$

Suppose we have two events. $A \subset B \subset \Omega$

$\Omega =$ 

What can we say about these probabilities? $P(A), P(B)$?

$$P(A) \leq P(B)?$$

Use union of disjoint events: $A \cap B \setminus A = \emptyset$
$A \cup B \setminus A = B$

$$P(A) + P(B \setminus A) = P(A \cup B \setminus A) = P(B)$$

Since $P(B \setminus A) \geq 0$, $\boxed{P(A) \leq P(B)}$

1:02:25

Counting

This gives a way to compute probabilities of events when the probability of each event is the same.

1:04:26

Which outcome is more likely if you toss a fair coin 7 times? HHHTTTH or HTHTHTH?

They have the same probability. All sequences of length 7 are the same.

1:06:15 If Ω is a sample space with a probability P such that

little omega $P(w) = P(w')$, $\forall$ $w, w' \in \Omega$ — They all have the same probability

for all

1:06:35 If $A \subset \Omega$, $P(A) = \frac{\# A}{\# \Omega}$ # A = number of elements of A

1:07:30 Could Ω be infinite?

$$\Omega = \{w_1, w_2, \dots\}$$ w - omega

$$P(w_i) = P(w_j) \quad \forall\, i, j$$

$$A = \{w_i\},\ A_i \cap A_j = \emptyset$$

$$\bigcup_{i=1}^{n} A_i = \Omega$$ — mutually disjoint

$$n \cdot P(w_1) = \sum_{i=1}^{n} P(w_i) = P\left(\bigcup_{i=1}^{\infty}\right) = P(\Omega) = 1 \qquad n = \#\Omega$$

$$P(w_1) = \frac{1}{n}$$

If Ω is infinite then $\frac{1}{n}$ is 0. Probability would be 0. $\boxed{\Omega \text{ must be finite}}$

1:11:40 All the outcomes in A would look something like this:

$$A = \{w_1, w_2, \dots, w_k\} = \bigcup_{i=1}^{k} \{w_i\}$$ — a disjoint union of events

$$P(A) = \sum_{i=1}^{k} P(w_i) = \sum_{i=1}^{k} \frac{1}{\#\Omega} = \frac{\#A}{\#\Omega}$$

1:14:20 Multiplication Principle

If experiment 1 has m possible outcomes and experiment 2 has n possible outcomes then experiments 1 and 2 have m times n possible outcomes.

1:16:25 **Example:**

If you toss a coin then roll a die, how many possible outcomes?

2 outcomes n first experiment.
6 outcomes in second experiment

2 x 6 = 12 outcomes

1:17:28

Example: Have 3 distinct jobs and 47 people.

Job1 Job2 Job3

How many people for job1? 47
How many people for job 2? 46 — Don't want to pick the same person again
How many people for job 3? 45 — Don't want to pick the same people again

47 46 45 = # ways to form an ordered sample without replacement from a group of 47

1:27:10 How many ways are there to select r objects, without replacement, from a group of n objects?

r slots: $$\frac{n\ (n-1)\ (n-2) \cdots (n-r+1)}{r!}$$

1:30:30 $$= \frac{n!}{(n-r)!\,r!} = \binom{n}{r} - \text{n choose r}$$ 1:33:25

1:31:45 **Example:** P(full house)

$$\frac{\text{\# ways to get a full house}}{\text{\# five card hands}} = \frac{\quad}{\binom{52}{5}}$$

\# ways to select value of triplet = 13
\# ways to select 3 cards of the given value $= \binom{4}{3} = 4$
\# ways to get 3 of a kind $= \boxed{13 \cdot 4}$ (mult princ)

\# ways to select value for pair = 12
\# ways to select 2 cards of given value $= \binom{4}{2} = 6$
\# ways to get the pair $= \boxed{12 \cdot 6}$ (mult princ)

$$\frac{13 \cdot 12 \cdot 6 \cdot 4}{\binom{52}{5}}$$

1:40:40 **Example:** : P(flush) – All cards of same suit

$$= \frac{\underbrace{4}_{\text{\# suits}} \cdot \binom{13}{3}}{\binom{52}{5}}$$

Full house beats a flush