

## EM

goal:  $\max_{\theta} \log p(x, z | \theta)$

lower bound:  $L(p', \theta) = \sum p'(z | x, \theta) \log \frac{p(x, z | \theta)}{p(z | x, \theta)}$

E:  $\max_{p'} L(p', \theta)$

M:  $\max_{\theta} L(p', \theta)$

this should also have x

add this to notes: this averages over z, so we can maximize it (like assigning z)

$\frac{\partial L}{\partial \theta}$

newton:  $\theta = \theta - (\nabla^2 L)^{-1} \nabla L$

exact line search: min over stepsize

backtracking line search: keep doubling

convexity defs, while below some  $q, prox$

1. Hess
2. Jensen's
3. tangent line

strongly convex:

$$f(x_2) \geq f(x_1) + \nabla f(x_1)^T (x_2 - x_1) + \frac{m}{2} \|x_2 - x_1\|_2^2$$

density estimation (reg.)  
 $p(y | x, \theta) = \sum p(z^i | x, \theta) \cdot p(y | z^i, x, \theta)$   
mixing prop mixture comp.

ex. GMM:  $p(x | \theta) = \sum \pi_i \mathcal{N}(x | \mu_i, \Sigma_i)$

ex. lin reg:  $p(y | z^i = 1, x, \theta) = \mathcal{N}(y | \beta^T x, \sigma^2)$

## basics

Sigmoid(s) =  $\frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$

softmax( $z_1, \dots, z_n$ ) =  $\left[ \frac{e^{z_1}}{\sum e^{z_i}}, \dots \right]$

$\ln(xy) = \ln(x) + \ln(y)$

$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

canonical:  $\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp((x-\mu)^T \Sigma^{-1} (x-\mu))$

moment:  $\Lambda = \Sigma^{-1}$ ,  $\eta = \Sigma^{-1} \mu$   
 $\exp(a + \eta^T x - \frac{1}{2} x^T \Lambda x)$

## stat

$\hat{\theta}_{MLE} = \arg\max_{\theta} p(x | \theta)$

$\hat{\theta}_{MAP} = \arg\max_{\theta} p(x | \theta) p(\theta)$

$\hat{\theta}_{Bayes} = \int p(\theta | x) p(\theta) d\theta$

## decisions

$EU(a | e) = \sum_{s'} P(s' | s, a) \cdot U(s')$

$MEU(a | e) = \max_a EU(a | e)$

$VPI(T) = E_T[MEU(a | e, t)] - MEU(a, e)$

mdps

val it:  $U(s) = R(s) + \gamma \sum_{s'} P(s' | s, a) \cdot U(s')$

$\pi^*(s) = \arg\max_{\pi} U^{\pi}(s)$

policy it:  $U(s) = R(s) + \gamma \sum_{s'} P(s' | s, a) \cdot U(s')$

passive rl

ADP:  $(R(s' | s, a), R(s)) \rightarrow \text{Bellman}$

TD:  $s \rightarrow s': U^{\pi}(s) = U^{\pi}(s) + \alpha [R(s) - U^{\pi}(s) + \gamma U^{\pi}(s')]$

active rl

ADP:  $\dots$

$Q(s, a) = R(s) + \gamma \sum_{s'} P(s' | s, a) Q(s', a)$

TD:  $Q(s, a) = Q(s, a) + \alpha [R(s) - Q(s, a) + \gamma \max_{a'} Q(s', a')]$

SARSA:  $Q(s, a) = Q(s, a) + \alpha [R(s) - Q(s, a) + \gamma Q(s', a')]$

$U(s) = \text{value}(s) = \max_a Q(s, a)$

Linear

PSD:  $x^T A x \geq 0 \forall x$

gradient:  $\mathbb{R}^n \rightarrow \mathbb{R}$  yields  $\mathbb{R}^n$

Jacobian:  $\mathbb{R}^m \rightarrow \mathbb{R}^n$  yields  $\frac{\partial f_i}{\partial x_1} \dots \frac{\partial f_i}{\partial x_m}$

Hessian:  $\mathbb{R}^n \rightarrow \mathbb{R}$  yields  $\frac{\partial^2 f}{\partial x_1^2} \dots \frac{\partial^2 f}{\partial x_1 \partial x_m}$

Frobenius norm:  $\|A\|_F = \sqrt{\sum A_{ij}^2} = \sqrt{\sum \lambda_i^2}$

spectral/ $L_2$  norm:  $\|A\|_2 = \sigma_{\max}(A)$

$\lambda_{\max}(A) = \sup_{x^T x = 1} x^T A x$

diagonalization:  $Q \Lambda Q^T \sim Q$  orthonormal cols eigenvectors

SVD:  $U D V^T \sim U$  cols eigenvectors of  $XX^T$

pca:  $\text{Cov}(x) = U D V^T \sim U$  cols are PCs

$\sum \lambda_i = \sum \text{Var}(x_i)$

## search

$h(n)$  admissible:  $h(n) \leq \text{cost}(n \rightarrow \text{goal})$

$h(n)$  consistent:  $h(n) - h(n') \leq \text{cost}(n \rightarrow n')$

AC-3: apply constraints, recalc neighbors, repeat

## info theory

$H(x) = -\sum p(x) \log p(x)$

$H(y | x) = -\sum_x p(x) \sum_y p(y | x) \log p(y | x)$

$D(P || Q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$

$I(x; y) = \sum_{x,y} p(x, y) \log \frac{p(x, y)}{p(x) p(y)}$

- 2 diagrams  
- Jensen's:  $E[f(x)] \geq f[E(x)]$  when  $f$  convex

Linear reg.

sgd:  $\hat{\theta} = \hat{\theta} + \alpha \nabla_{\theta} J$   
 $0 < \alpha < \frac{1}{2\lambda_{\max}} [x^T x]$

add duality

Logistic reg.

$P(x=1 | x) = \sigma(w^T x)$   
cross-entropy:  $-\sum p(y) \log p^*(y)$

sums

$\min_{u, b} \frac{1}{2} \|w\|_2^2 + C \sum e_i$

s.t.  $e_i \geq 0 \forall i$

$y_i (w^T x_i - b) \geq 1 - e_i \forall i$

binary:  $\min_{u, b} \frac{1}{2} \|w\|_2^2 + C \sum \max(1 - y_i (w^T x_i - b), 0)$

decision tree

info gain:  $H(\text{Parent}) - \text{weighted ave } H(\text{children})$

nearest neighbor

- k-d tree  
- locality sensitive hashing