basics

- sigmoid(z) $= \frac{1}{1+e^{-z}} = \frac{e^z}{e^z+1}$
- $p(x|\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right]$

linear algebra

- SVD: $A = U\Sigma V^T$
- PSD matrices: $A \succeq 0 \Rightarrow A = UDU^T$, D are evals, U are evecs
- PCA: $A = U\Sigma V^T$, row of V^T are principal components, project onto them to reduce dimension of data
- Use PCA to find direction of maximum variance in data; X^TX is covariance

info theory

- $H(X) = -\sum p(x)\log p(x)$
- $H(Y|X) = -\sum_x p(x)\sum_y p(y|x)\log\ p(y|x)$
- $D(p||q) = \sum_x p(x)\log\frac{p(x)}{q(x)}$
- $I(X;Y) = \sum_X\sum_y p(x,y)\log\frac{p(x,y)}{p(x)p(y)}$
- Jensen's: $E[f(X)] > f(E[X]) \sim$ f convex

optimization

- $0 \preceq \underbrace{mI}_{\text{strong convexity}} \preceq \nabla^2 f(x) \preceq \underbrace{MI}_{\text{smoothness}}$
- exact line search: min over stepsize
- backtracking line search: keep doubling while below some approx
- newton's: $\theta^{(t+1)} = \theta^{(t)} - \nabla^2 f(\theta^{(t)})^{-1}\nabla f(\theta^{(t)})$
- strongly convex:
  $f(x_2) \geq f(x_1) + \nabla f(x_1)^T(x_2 - x_1) + m/2||x_2 - x_1||_2^2$

Uninformed Search/$A^\star$

- Uninformed Search
  - Depth First Search (DFS): search of deepest/leftmost solution (not complete)
  - Breadth First Search (BFS): search of shallowest/leftmost solution (complete)
  - Uniform Cost Search (UCS): search expanding minimum cost (compelte and optimal)
- Informed Search, use heuristics to estimate goal distance
  - Greedy Search: expand node closets to goal
  - $A^\star$: search according to heuristic
  - $A^\star$ tree search with admissibility $h(n) \leq h^\star(n)$ will yield optimal solution
  - $A^\star$ graph search with consistency $h(A) - h(C) \leq cost(A,C)$ will yield optimal solution

Constraint Satisfaction Problems

- Consist of variables (X_i), one domain per var (D_i), constraints (C).
- k-consistency: for any set of $k-1$ vars and for any consistent assignment to those variables, a consistent value can be assigned to the kth variable. Path-consistency is 3-consistent, arc-consistency is 2-consistent, node consistency 1-consistent.
- Backtracking search occurs when a path leads to an inconsistent assignment. Tricks can be employed here: Minimum-remaining values heuristic breaks assignment ties, conflict directed back-jumping can choose good points to backtrack to,
- Tree-structured CSPs can be topologically sorted: then, solved with a forward pass.

Game Trees

- $\alpha - \beta$ pruning: makes solving minimax trees more efficient
- For large trees, use evaluation functions to estimate minimax value at a state
- Rational agents follow principle of maximum utility

Decision theory / VPI

- $EU(\alpha, \mathbf{e}) = \max_{\mathbf{a}} \sum_{\mathbf{s'}} \mathbf{P}(\mathbf{RESULT}(\mathbf{a}) = \mathbf{s'} | \mathbf{a}, \mathbf{e}) \mathbf{U}(\mathbf{s'})$
- $EU(\alpha_{e_j}, \mathbf{e}, \mathbf{e_j}) = \max_{\mathbf{a}} \sum_{\mathbf{s'}} \mathbf{P}(\mathbf{RES}(\mathbf{a}) = \mathbf{s'} | \mathbf{a}, \mathbf{e}, \mathbf{e_j}) \mathbf{U}(\mathbf{s'})$
- $VPI_{\mathbf{e}}(E_j) = (\sum_k P(E_j = e_{jk} | \mathbf{e}) \mathbf{EU}(\alpha_{\mathbf{e_{jk}}} | \mathbf{e}, \mathbf{E_j} = \mathbf{e_{jk}}) - \mathbf{EU}(\alpha | \mathbf{e})$

MDPs

- Q(s,a) = expected discounted rewards with s,a
- U(s) = max expected discounted rewards with s
- Bellman: $U(s) = \max_a \sum_{s'} \underbrace{P(s'|s,a)[R(s,a,s') + \gamma U(s')}_{Q(s,a)}]$
- val: iterate bellman, pol: iterate with no max
- $\pi(s) = \underset{a}{\operatorname{argmax}} U^{\pi}(s)$ e

Passive RL

- ADP: learn P(s'|s,a), U(S) $\rightarrow$ plug into Bellman eqn
- TD: $U(s) = (1 - \alpha) U(s) + \alpha[R(s) + \gamma U(s')]$

Active RL

- (TD) Q-learning:
  $Q(s,a) = (1 - \alpha) Q(s,a) + \alpha[R(s) + \gamma \max_{a'} Q(s', a')]$
- SARSA: $Q(s,a) = (1 - \alpha) Q(s,a) + \alpha[R(s) + \gamma Q(s', a')]$
- evaluation functions + policy search
- approximate: $w_i = w_i + \alpha \cdot diff \cdot f_i(s,a)$ (derivative of linear Q(s,a)
- diff = what used to be rhs

logic

- propositional logic: only facts
  - horn clause (at most one positive), definite clause (exactly one positive)
  - TT-ENTAILS: check everything (forward/backward chaining)
  - DPLL - TT-ENTAILS w/ improvements
- first-order logic: facts, objects, relations
  - inference: forward-chaining, backward chaining

Planning

- Goal: to achieve stated goals with a plan of action.
- Developed language: Planning Domain Definition Langauge, PDDL. Consists of *States* (collection of atomic fluents), *Available Actions (s)* (applicable if preconditions are satisfied by s), *Results(s,a)*: specified in terms of what changes, and *Goal Test*. Comparable to search of Ch.7 but broader, more expressive.
- Complexity of planning: planSAT (is there a plan that solves the goal?) and bounded planSAT (plan that solves in $\leq k$ steps?) are both PSPACE, solvable by deterministic Turing Machine in poly time.
- Backwards and Forward search exist, forward more popular because more heuristics exist (ignore preconditions heuristic, ignore delete lists heuristcs, etc.)

Knowledge Representation

- Ontology: question of whether things exist, and how to organize facts about the world
- Upper ontologies break existence into hierarchies (exception-ridden without gradiations to express non-absolutes.)
- General Ontology must unify all special purpose domains (may be impossible.)
- Categorization creates hierarchies, which allows qualities to be inherited down to subcategories

linear regression

- convergence criteria: $0 < \alpha < 0.5\lambda_{\max}(X^T X)$
- normal eq: $\hat{w} = (X^T X)^{-1} X^T y$
- schur comp. like 2d det but divide by inverse (e.g. M/A)

Independence and Factorization

- $P(x_1, ..., x_n) = \prod_i P(X_i | Parents(X_i))$
- Node is conditionally independent of all other nodes given its parents, children, and children's parents (Markov Blanket)
- $X_A \perp X_C | X_B \Rightarrow p(x_A, x_C | x_B) = p(x_A|x_B)p(x_C|x_B)$ or $p(x_A|x_B, x_C) = p(x_A|x_B)$
- Edges do not imply dependence, but lack of edges imply independence
- d-separation: use bayes ball algorithm to find conditional independencies
- Undirected GM: $\prod_{C \in cliques} \phi(C)$ potential function
- filtering: $\underbrace{P(X_{t+1}|e_{1:t+1})}_{\text{new state}} = \alpha \underbrace{P(e_{t+1}|X_{t+1})}_{\text{sensor}} \cdot \sum_{x_t} \underbrace{P(X_{t+1}|x_t)}_{\text{transition}} \cdot \underbrace{P(x_t|e_{1:t})}_{\text{old state}}$
- smoothing: $\underbrace{P(X_{t+k+1}|e_{1:t})}_{\text{new state}} = \sum_{x_{t+k}} \underbrace{P(X_{t+k+1}|x_{t+k})}_{\text{transition}} \cdot \underbrace{P(x_{t+k}|e_{1:t})}_{\text{old state}}$
- mle (viterbi): $\underbrace{\max_{x_{1:t}} P(x_{1:t}, X_{t+1}|e_{1:t+1})}_{\text{mle x}} = \alpha \underbrace{P(e_{t+1}|X_{t+1})}_{\text{sensor}} \cdot \max_{x_t} \left[\underbrace{P(X_{t+1}|x_t)}_{\text{transition}} \cdot \underbrace{\max_{x_{1:t-1}} P(x_{1:t-1}, x_{t+1}|e_{1:t})}_{\text{max prev state}} \right]$

Clustering, Mixture of Gaussians, k-means

- ex. GMMs: $p(x|\theta) = \sum_i \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$
- ex. mixture of linear regressions:
  $p(y|x, \theta) = \sum_i \underbrace{\pi_i(x, \xi)}_{\text{mixing prop.}} \cdot \underbrace{\mathcal{N}(y|\beta_i^T x, \sigma_i^2)}_{\text{mixture comp.}}$

EM Algorithm

- $L(q, \theta) = \sum_z q(z|x) \log \frac{p(x,z|\theta)}{q(z|x)}$
- Expectation: assign to observed X's the unobserved (latent) variable classes. With q as averaging distribution, $q^{(t+1)} = argmax_q L(q, \theta^(t))$
- Maximization: holding assignments constant, compute MAP or MLE estimates of cluster parameters. $\theta^{(t+1)} = argmax_\theta L(q^{t+1}, \theta)$

logistic regression

- $P(Y = 1|x, w) = \text{sigmoid}(w^T x)$
- minimizes cross-entropy: $-\sum_z P(y = z) \log P(\hat{y} = z)$

decision trees

- info gain: $H(parent) - E[H(children)]$

svms

- soft-margin:

$$\min_{w,b,\xi} \quad \frac{1}{2}||w||_2^2 + C\sum_i \xi_i \tag{1}$$

$$s.t. \quad y_i(w^T x_i - b) \geq 1 - \xi_i \ \forall i \tag{2}$$

$$\xi_i \geq 0 \ \forall i \tag{3}$$

- binary: $\min_{w,b} \quad \frac{1}{2}||w||^2 + C\sum_i \max(1 - y_i(w^T x_i - b), 0)$

duality

- primal: $p^* = \min\ f_0(x)$
  $s.t.\ f_i(x) \leq 0$
  $h_i(x) = 0$
- dual: $d^* = \max_{\lambda,\nu} \overbrace{\inf_x \underbrace{f_0(x) + \sum \lambda_i f_i(x) + \sum \nu_i h_i(x)}_{\text{Lagrangian } L(x,\lambda,\nu)}}^{\text{dual function } g(\lambda,\nu)}$
  $s.t.\ \lambda \succeq 0$

kernel methods

- matrix XX^T has each element be dot product

statistical concepts

- $\overbrace{p(\theta|x)}^{\text{posterior}} = \frac{\overbrace{p(x|\theta)}^{\text{likelihood}}\overbrace{p(\theta)}^{\text{prior}}}{p(x)}$
- $\hat{\theta}_{MLE} = argmax_\theta\, p(x|\theta)$
- $\hat{\theta}_{MAP} = \underset{\theta}{argmax}\, p(\theta|x)$
- $\hat{\theta}_{Bayes} = \int \theta\, p(\theta|x) d\theta$