EM

goal: max $\log p(x, z|\theta)$

lower bound: $L(p', \theta) = \sum_z p'(z|x,\theta) \log \frac{p(x,z|\theta)}{p(z|x,\theta)}$

E: $\max_{p'} L(p', \theta)$

M: $\max_{\theta} L(p', \theta)$

opt

$0 \preceq mI \preceq \nabla^2 f \preceq MI$

newton: $\theta = \theta - (\nabla^2 f)^{-1} \nabla f$

exact line search: min over stepsize

backtracking line search: keep doubling while below some approx

convexity defs.

1. Hess
2. Jensen's
3. tangent line

strongly convex:

$f(x_2) \geq f(x_1) + \nabla f(x_1)^T (x_2 - x_1) + \frac{m}{2} \|x_2 - x_1\|_2^2$

density estimation (reg.)

$p(y|x,\theta) = \underbrace{\sum_i p(z^i = 1|x, \beta)}_{\text{mixing prop}} \cdot \underbrace{p(y|z^i=1, x, \theta_i)}_{\text{mixture comp.}}$

ex. GMM: $p(x|\theta) = \sum_i \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$

ex. lin reg: $p(y|z^i = 1, x, \theta) = \mathcal{N}(y|\beta_i^T x, \sigma_i^2)$

basics

$\text{Sigmoid}(s) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$

$\text{softmax}(z_1, \ldots, z_n) = \left[\frac{e^{z_1}}{\sum e^{z_i}} \cdots\cdots\right]$

$\ln(xy) = \ln(x) + \ln(y)$

$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$

canonical: $\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left((x-\mu)^T \Sigma^{-1} (x-\mu)\right)$

moment: $\Lambda = \Sigma^{-1}$, $\eta = \Sigma^{-1}\mu$

$\exp(a + \eta^T x - \frac{1}{2} x^T \Omega x)$

stat

$\hat{\theta}_{MLE} = \arg\max_\theta p(x|\theta)$

$\hat{\theta}_{MAP} = \arg\max_\theta p(x|\theta) p(\theta)$

$\hat{\theta}_{Bayes} = \int \theta \, p(\theta|x) \, d\theta$

decisions

$EU(a|e) = \sum_{s'} P(s'|s,a) \cdot U(s')$

$MEU(a|e) = \max_a EU(a|e)$

$VPI(T) = E_T[MEU(a|e,t)] - MEU(a,e)$

mdps

val it: $U(s) = R(s) + \gamma \overbrace{\max_a \sum_{s'} P(s'|s,a) \cdot U(s')}^{MEU(a|e)}$

$\pi^*(s) = \arg\max_\pi U^\pi(s)$

policy it: $U(s) = R(s) + \gamma \overbrace{\sum_{s'} P(s'|s,a) \cdot U(s')}^{EU(a|e)}$

passive rl

ADP: $(P(s'|s,a), R(s)) \to$ Bellman

TD: $s \to s'$: $U^\pi(s) = U^\pi(s) + \alpha[R(s) - U^\pi(s) + \gamma U^\pi(s')]$

active rl

Error term (- should be on right)

ADP: ~~$U(s) \to Q(s,a)$~~

$Q(s,a) = R(s) + \gamma \sum_{s'} P(s'|s,a) \overbrace{\max_{a'} Q(s',a')}^{U(s')}$

TD: $Q(s,a) = Q(s,a) + \alpha[R(s) - Q(s,a) + \gamma \overbrace{\max_{a'} Q(s',a')}^{U(s')}]$

SARSA: $Q(s,a) = Q(s,a) + \alpha[R(s) - Q(s,a) + \gamma Q(s',a')]$

$U(s) = \text{Value}(s) = \max_a Q(s,a)$

Linear

PSD: $x^T A x \geq 0 \; \forall x$

gradient: $\mathbb{R}^? \to \mathbb{R}$ yields $\mathbb{R}^?$

Jacobian: $\mathbb{R}^m \to \mathbb{R}^n$ yields $\frac{\partial f_1}{\partial x_1} \cdots \frac{\partial f_1}{\partial x_m}$

Hessian: $\mathbb{R}^m \to \mathbb{R}$ yields $\frac{\partial f}{\partial x_1^2} \cdots \frac{\partial f}{\partial x_1 \partial x_m}$

Frobenius norm: $\|A\|_F = \sqrt{\sum A_{ij}^2} = \sqrt{\sum \lambda_i^2}$

spectral/L_2 norm: $\|A\|_2 = \sigma_{max}(A)$

$\lambda_{max}(A) = \sup \frac{x^T A x}{x^T x}$

diagonalization: $Q \Lambda Q^T \sim Q$ orthonormal cols eigenvecs

svd: $UDU^T \sim U$ cols eigenvecs of XX^T

pca: $\text{Cov}(x) = UDU^T \sim U$ cols are PCs

$\sum \lambda_i = \sum \text{Var}(x_i)$

search

$h(n)$ admissible: $h(n) \leq \text{cost}(n \to \text{goal})$

$h(n)$ consistent: $h(n') - h(n) \leq \text{cost}(n \to n')$

AC-3: apply constraints, readd neighbors, repeat

info theory

$H(x) = -\sum p(x) \log p(x)$

$H(Y|X) = -\sum_x p(x) \sum_y p(y|x) \log p(y|x)$

$D(p^{(x)} \| q^{(x)}) = \sum_x p(x) \log \frac{p(x)}{q(x)}$

$I(X;Y) = \sum_x p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$

- 2 diagrams
- Jensens: $E[f(x)] \geq f[E[X]]$ when f convex

Linear reg.

sgd: $\hat{\theta} = \hat{\theta} + \alpha \nabla f_\theta$

$0 < \alpha < \frac{1}{2} \lambda_{max}[X^T X]$

Logistic reg.

$P(Y=1|x) = \sigma(w^T x)$

cross-entropy: $-\sum p(y) \log \hat{p}(\hat{y})$

sums

$\min_{w,b} \frac{1}{2}\|w\|_2^2 + C \sum_i e_i$

s.t. $e_i \geq 0 \; \forall i$

$y_i(w^T x_i - b) \geq 1 - e_i \; \forall i$

binary: $\min_{w,b} \frac{1}{2}\|w\|_2^2 + C \sum_i \max(1 - y_i(w^T x_i - b), 0)$

decision tree

info gain: H(Parent) - weighted ave H(children)

nearest neighbor

- K-d tree
- Locality sensitive hashing